Analytical And Finite Element Studies on Free Vibration of Aluminum Alloy Plate and Control Passive with Damping Orthotropic Patches

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Abstract: - This paper presents studies made on fundamental flexural frequencies of isotropic alloy aluminum rectangular thin plate with various boundary conditions. The first four vibration modes and corresponding frequencies and mode shapes of this plate were obtained by using analytical method, based on Rayleigh-Ritz energy approach and finite element method (FEM) by using ABAQUS code. The convergence and accuracy of the numerical solution was examined. A comparison was made between the analytical values and the finite element solution. The effect of geometrical parameters on natural vibrations of the isotropic rectangular plate was investigated. The values of frequency parameter of the plate having different (length/width) ratios in case of ($a/b \geq 1$) were studied. The results obtained by the analytical approach are in good agreement with the FE solutions. Also, the paper presents simulation results of testing of the plate with passive vibration control realized through patches attached to the plate.

Keywords: - flexural vibration, finite element method, isotropic plate, modal analysis

1. INTRODUCTION

Rectangular isotropic thin plates with various boundary conditions have found a wide range of application in civil, marine, aeronautical, automobile industry and mechanical engineering applications [1]. As the vibratory movements are at the root of many problems that can lead to the ruin of structures, the knowledge and the understanding of the vibratory behavior of isotropic plates has become an important and crucial parameter that must be taken into account when designing structural elements.

Over the past four decades, a lot of research has been carried out on the study of vibration characteristics of isotropic rectangular plates. Many of research have been dedicated to the study of structural vibrations and methods to reduce the resonant amplitudes, including analyses which determine how to modify the structure to avoid coupling resonant frequencies with excitation sources. There are number of solutions on free vibration of rectangular plates in the natural frequencies with a wide range of support conditions. The most widely known are those of Warburton [2] and Leissa [3]. Grootenhuis, O'Boy, Krylov and Hosseini have proposed an exact solution for free flexural vibration of rectangular thick plates using third order shear deformation plate theory [4-6]. Pouladkhan have determined natural frequencies and shape modes of thin rectangular plates by using modal analysis [7]. Analytical solutions are generally available only for simple cases such as plates with simply supported. For other various boundary conditions, one of the most commonly used methods in vibration analysis of thin plates is the Rayleigh-Ritz energetic approach, based on using appropriate functions associated to various boundary conditions to describe the lateral deflection of the plates [1]. The earlier studies on free vibration characteristics of isotropic thin rectangular plates using Rayleigh-Ritz method are those of. Hanna and Leissa who developed an approach based on higher-order shear deformation plate theory of Reddy to analyze free vibration of fully free rectangular [8]. Dozio used a trigonometric Ritz method for general vibration analysis of rectangular Kirchhoff plates [9]. Vescovini et al. used the Ritz method to estimate the free vibration and buckling analysis of highly anisotropic plates [10].

Numerical approaches such as finite element method were adopted by many researchers to study the vibratory behavior of thin isotropic plates. Ramu and Mohantyb have provided a suitable study on free vibration of rectangular plate structures using finite element method [11]. Werfalli and Karoud have conducted a free vibration analysis of rectangular plates using Galerkin based finite element method [12]. Hatiegan analyzed by finite element method thin clamped plates of different geometric forms [13]. Rock and Hinton developed a new finite element to analyze free vibration and transient response of thick and thin plates [14]. Vibration attenuation can be achieved by attaching patches elements on to the structure. This enables a compact vibration damping method without adding significant mass and volumetric occupancy, unlike the bulky mechanical dampers. The application of patches for reducing vibrations and structure borne noise has been studied by many researchers in the past few years [15-17].

This paper presents studies made on fundamental flexural frequencies of thin isotropic aluminum alloy rectangular plate by using Rayleigh approximation method and finite element method (FEM). Three types of boundary conditions, all edges clamped (CCCC), all edges simply supported (SSSS) and two edges clamed and two edges simply supported (SCSC) are investigated.

A comparison between results obtained with both methods has been done. Also, the effect of factor shape upon frequencies of the plate was carried out. The values of the factor shape having different (length/width) ratios in case of ($a/b \geq 1$) were examined. In addition, vibration control of the structure is realized through patches bonded to the plate. Simulations and numerical computations of the structure are performed in ABAQUS.

2. RAYLEIGH-RITZ APPROACH

2.1 Theoretical formulations

It is considered a thin homogeneous orthotropic plate of a constant thickness, with a uniformly distributed mass. Denote the length, width and total thickness of the rectangular plate by a, b, and h. According to the classical thin plate theory of Kirchhoff the fundamental relations are written, taking into account the absence of transverse loads (q=0) and supposing that the plate is not including membrane-bending coupling [18].

$$u_0 = 0, \quad v_0 = 0, \tag{1}$$

$$D_{11}\frac{\partial^4 w_0}{\partial x^4} + 2(D_{12} + D_{66})\frac{\partial^4 w_0}{\partial x^2 \partial y^2} + D_{22}\frac{\partial^4 w_0}{\partial y^4} + \rho\frac{\partial^2 w_0}{\partial t^2} = I_{xy}\left(\frac{\partial^4 w_0}{\partial x^2 \partial t^2} + \frac{\partial^4 w_0}{\partial y^2 \partial t^2}\right) \tag{2}$$

Assuming that the rotatory inertia terms can be neglected ($I_{xy}=0$), the equation of motion Eq. (2) reduces to:

$$D_{11}\frac{\partial^4 w_0}{\partial x^4} + 2(D_{12} + D_{66})\frac{\partial^4 w_0}{\partial x^2 \partial y^2} + D_{22}\frac{\partial^4 w_0}{\partial y^4} + \rho\frac{\partial^2 w_0}{\partial t^2} = 0, \tag{3}$$

In this expression, D_{ij} are the bending and twisting stiffness, $w_0(x,y)$ is the deflection, ρ and ω are mass density and circular frequency, respectively. To solve this differential equation: Eq. (3), consider a solution of the form:

$$w_0(x, y, t) = w_0(x, y)e^{i\omega\omega} \tag{4}$$

where ω the angular frequency of the vibrations is, leads, by substituting the expression (4) into Eq. (3), to:

$$D_{11}\frac{\partial^4 w_0}{\partial x^4} + 2(D_{12} + D_{66})\frac{\partial^4 w_0}{\partial x^2 \partial y^2} + D_{22}\frac{\partial^4 w_0}{\partial y^4} - \rho\omega^2 w_0 = 0 \tag{5}$$

In case of simply supported edges, the boundary conditions are given as:

- Edges x=0 and x=a :

$$w_0 = 0, \quad M_x = -D_{11}\frac{\partial^2 w_0}{\partial x^2} - D_{12}\frac{\partial^2 w_0}{\partial y^2} = 0, \tag{6}$$

- Edges y=0 and y=b :

$$w_0 = 0, \quad M_y = -D_{12}\frac{\partial^2 w_0}{\partial x^2} - D_{22}\frac{\partial^2 w_0}{\partial y^2} = 0, \tag{7}$$

And $w_0(x, y)$ can be put in the form:

$$w_0(x, y) = C_{mn}\sin m\pi\frac{x}{a}\sin m\pi\frac{y}{b}, \tag{8}$$

Substituting this expression witch satisfying the support conditions into Eq. (5) yields:

$$\left[\frac{m^4\pi^4}{a^4}D_{11} + 2\frac{m^2n^2\pi^4}{a^2b^2}(D_{12} + 2D_{66}) + \frac{n^4\pi^4}{b^4}D_{22} - \rho\omega^2\right]C_{mn} = 0, \tag{9}$$

For a nonzero value of C_{mn}, the expression of the natural frequencies becomes:

$$\omega_{mn}=\frac{\pi^2}{a^2}\sqrt{\frac{1}{\rho}\left(m^4D_{11}+2m^2n^2R^2(D_{12}+2D_{66})+n^4R^4D_{22}\right)} \quad (10)$$

The deformed shape of the plate corresponding to the natural frequency ω_{mn} is given by Eq. (8). In the case of other boundary conditions, it is not possible to solve Eq. (3) directly. The determination of the natural frequencies then requires using approximate methods. Using the Rayleigh-Ritz approach the frequency equation may be derived from the expression of maximum strain energy of bending:

$$U_{dmax}-E_{cmax}=\frac{1}{2}\int_{x=0}^{a}\int_{y=0}^{b}\left[\begin{array}{l} D_{11}\left(\frac{\partial^2 w_0}{\partial x^2}\right)^2+2D_{12}\frac{\partial^2 w_0}{\partial x^2}\frac{\partial^2 w_0}{\partial y^2} \\ +D_{22}\left(\frac{\partial^2 w_0}{\partial y^2}\right)^2+4D_{66}\left(\frac{\partial^2 w_0}{\partial x\partial y}\right)^2 \\ -\rho\omega^2 w_0^2 \end{array}\right]dxdy, \quad (11)$$

The displacement is assumed to be an infinite series of admissible shape functions in the x and y directions.

$$w_0(x,y)=\sum_{m=1}^{M}\sum_{n=1}^{N}A_{mn}X_m(x)Y_n(y) \quad (12)$$

where $X_m(x)$ and $Y_n(y)$ are appropriate shape functions along x and y axes that must satisfy the boundary conditions. A_{mn} are the unknown numerical coefficients of the functions. The assumed displacement functions defining the deflection of the plate are given in the form of series functions, as follow [18]:

- Clamped (CCCC):

$$w_0(x,y)=A_{mn}\left(1-\frac{x^2}{a^2}\right)\left(1-\frac{y^2}{b^2}\right)xy\sin\left(m\pi\frac{x}{a}\right)\sin\left(n\pi\frac{y}{b}\right) \quad (13)$$

- Simply supported (SSSS):

$$w_0(x,y)=\sum_{m=1}^{M}\sum_{n=1}^{N}A_{mn}\sin\left(m\pi\frac{x}{a}\right)\sin\left(n\pi\frac{y}{b}\right), \quad (14)$$

- Simply supported-clamped (SCSC):

$$w_0(x,y)=\left(1-\frac{x^2}{a^2}\right)\left(1-\frac{y^2}{b^2}\right)\sum_{m=1}^{M}\sum_{n=1}^{N}A_{mn}\sin\left(m\pi\frac{x}{a}\right)\sin\left(n\pi\frac{y}{b}\right) \quad (15)$$

Finally, the Rayleigh approximation of the natural frequency of the mode (m, n) can be written in the form:

$$\omega_{mn}=\frac{1}{a^2}\sqrt{\frac{D_{11}}{\rho}\left(c_1^4+2(\alpha_{12}+2\alpha_{66})R^2c_2+\alpha_{22}R^4c_3^4\right)} \quad (16)$$

where:

$$\alpha_{12}=\frac{D_{12}}{D_{11}},\ \alpha_{66}=\frac{D_{66}}{D_{11}},\ \alpha_{22}=\frac{D_{22}}{D_{11}}, \quad (17)$$

In the case of an isotropic plate:

$$D_{11}=D_{22}=D_{12}+2D_{66}=D, \quad (18)$$

The expression for the natural frequencies can be written in the form:

$$\omega_{mn}=\frac{1}{a^2}\sqrt{\frac{D}{\rho}\left(c_1^4+2R^2c_2+R^4c_3^4\right)}, \quad (19)$$

where ρ is the mass density and D the flexural rigidity of the plate as defined in Eq. (20) with E being the Young's modulus and υ the Poisson's ratio:

$$D=\frac{Eh^3}{12(1-\nu^2)}, \quad (20)$$

c_1, c_2 and c_3 are the coefficients introduced in the expression (19) for the natural frequencies of the bending vibrations of an orthotropic rectangular plate [18].

And the natural frequency of free vibrations has the form:

$$f_{mn}=\frac{\omega_{mn}}{2\pi} \quad (21)$$

Table 1. Coefficients c_1, c_2 and c_3 introduced in the expression of the angular frequency in case of clamped edges (CCCC):

(m, n)	c_1	c_3	c_2
(1,1)	1.5π	4.730	151.3
(2,1)	2.5π	4.730	12.3 $c_1(c_1-2)$
(3,1)	3.5π	4.730	12.3 $c_1(c_1-2)$
(1,2)	1.5π	2.5π	12.3 $c_3(c_3-2)$
(4,1)	3.5π	4.730	12.3 $c_1(c_1-2)$

Table 2. Coefficients c_1, c_2 and c_3 introduced in the expression of the angular frequency in case of simply supported edges (SSSS):

(m, n)	c_1	c_3	c_2
(1,1)	$m\pi$	$n\pi$	$m^2n^2\pi^4$
(2,1)	$m\pi$	$n\pi$	$m^2n^2\pi^4$
(3,1)	$m\pi$	$n\pi$	$m^2n^2\pi^4$
(1,2)	$m\pi$	$n\pi$	$m^2n^2\pi^4$
(4,1)	$m\pi$	$n\pi$	$m^2n^2\pi^4$

Table 3. Coefficients c_1, c_2 and c_3 introduced in the expression of the angular frequency in case of simply supported / clamped edges (SCSC):

(m, n)	c_1	c_3	c_2
(1,1)	1.5π	$n\pi$	$12.3n^2\pi^2$
(2,1)	2.5π	$n\pi$	$n^2\pi^2 c_1(c_1-2)$
(3,1)	3.5π	$n\pi$	$n^2\pi^2 c_1(c_1-2)$
(1,2)	1.5π	$n\pi$	$12.3n^2\pi^2$
(4,1)	3.5π	$n\pi$	$n^2\pi^2 c_1(c_1-2)$

2.2 Results and discussion

2.2.1 Effect of boundary conditions and of shape factor

This analysis was performed for an aluminum alloy plate (AG4-5083) of length a=200 mm, width b=100 and thickness of 1 mm. The plate has elastic modulus E = 71000 MPa, Poisson's ratio $\nu = 0.33$ and mass density ρ = 2660 kg/m^3. The fundamental natural frequency can be obtained by letting m=1 and n=1. Using MATLAB, with the Eq. (18) and for the first four vibration modes and for all considered applied boundary conditions, angular and natural frequencies are computed then listed in Table 5, Table 6 and Table 7.

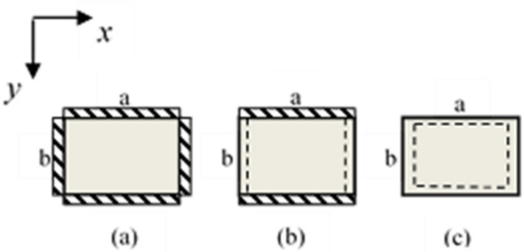

Figure 1. Schematic of the considered boundary conditions: (a) clamped (CCCC), (b) simply supported / clamped (SCSC) (c) simply-supported (SSSS)

It can be seen in Figure.2 that the minimum frequency level is obtained at SSSS boundary condition and the maximum frequency level is obtained at CCCC boundary condition because of its constraints on the degree of freedom. The SCSC boundary condition falls between these two configurations. In all cases of boundary conditions, the natural frequency increases with respect to increase in the mode number.

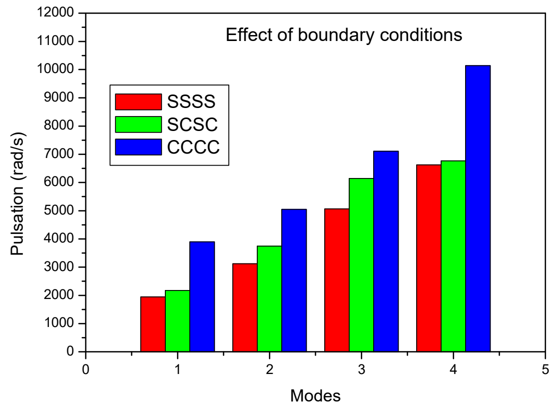

Figure 2. Natural frequencies of the plate with respect to boundary conditions

To confirm the reliability and the accuracy of the results obtained by Rayleigh approach, a parametric study is conducted in order to examine the effect of shape factor upon natural frequencies of the rectangular plate with various boundary conditions.

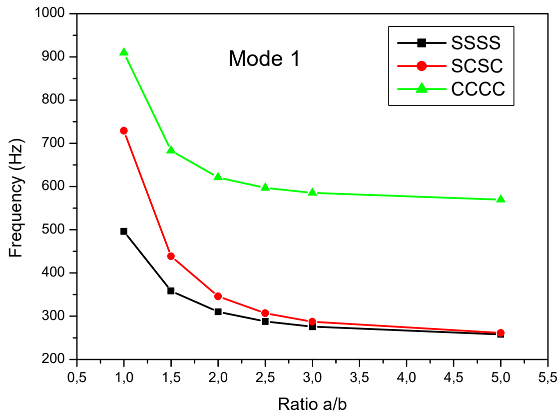

Figure 3. Variation of frequencies with respect to ratio a/b for various boundary conditions in case of mode 1

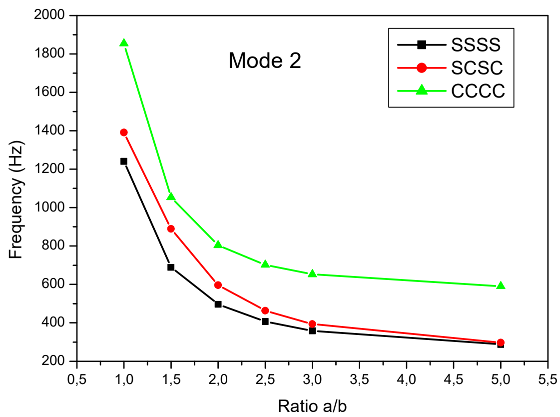

Figure 4. Variation of frequencies with respect to ratio a/b for various boundary conditions in case of mode 2

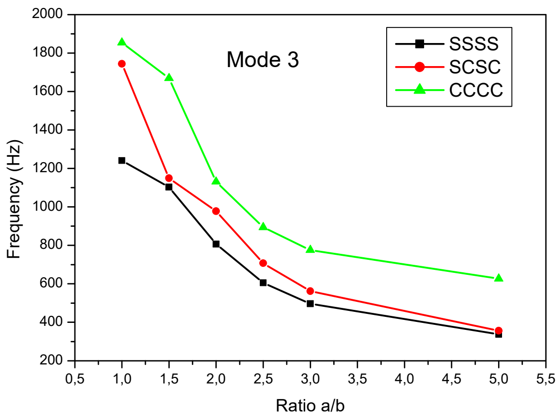

Figure 5. Variation of frequencies with respect to ratio a/b for various boundary conditions in case of mode 3

The values of natural frequencies of the aluminum alloy plates having different (length/width) ratios in case of ($a/b \geq 1$) and for the boundary conditions used in this study, corresponding to the first four natural modes are respectively listed in Table 8. According to Figure 3, Figure 4 and Figure 5, we can see a rapidly decreasing parabolic rate for the interval a/b=1 to a/b=2. In case of (length/width) ratios a/b>1.5 an asymptotic attenuation is imposed. This can be explained by a transfer of rigidity going from the case finite plate into the case of infinite plate.

3. FINITE ELEMENT ANALYSIS

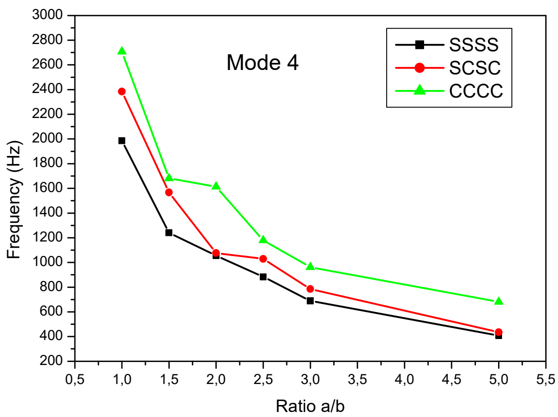

Figure 6. Variation of frequencies with respect to ratio a/b for various boundary conditions in case of mode 4

Nowadays, numerical simulation has become the most efficient and the most widely used calculation tool for predicting the mechanical behavior of structures during commissioning and improving the operation of industrial systems and processes. The Finite element method (FEM) is a powerful computational technique widely used for numerical simulation and optimization of structural geometry, based on the concept that one can replace any continuum by an assemblage of simply shaped elements with well defined force displacement and material relationships. In this paper, the geometric and FE model is carried out using ABAQUS code which is one of the most popular FEM software and has been used for wide range of study. Extracting accurate results in ABAQUS depend on defining the boundary conditions, steps of the solution, type, and size of meshes carefully.

The boundary conditions which are applied to the edges are shown in Figure 7 and can be defined as follows:

- Clamped boundary condition:

$U_1 = U_2 = U_3 = UR_1 = UR_2 = UR_3 = 0$

- Simply supported boundary condition:

$U_1 = U_2 = U_3 = 0$

where U_1, U_2 and U_3 are displacements along x, y and z directions, respectively. UR_1, UR_2, and UR_3 are rotations about x, y and z directions, respectively.

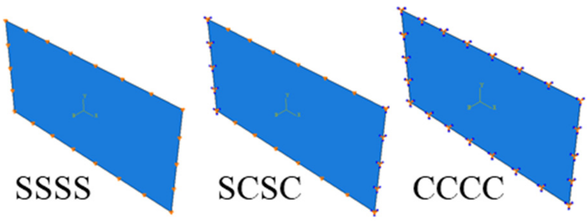

Figure 7. View of the plate with applied boundary conditions SSSS, SCSC and CCCC

The plate geometry and the boundary conditions are the same presented and used in section 2.2. The plate is discretized into a finite number of rectangular elements. The element S4R, defined by four nodes was employed as shown in Figure 8.

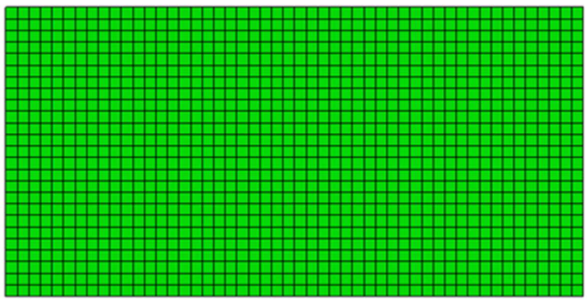

Figure 8. Finite element meshes of studied plate

However, it is always good practice to perform a mesh convergence study. To perform this study, we chose a mesh of approximate global size respectively: AGS=20,10,5,4 and 3. Obtained results for the fundamental mode (m=1 and n=1), in case of simply supported plate (SSSS) are presented in Table 4. We can notice that the nearest result into the analytical approach reference result (f_{11}=310.2161 Hz) is when global approximate size is equal to 4. Using results extracted from ABAQUS, the first four vibration modes, for the applied boundary conditions are presented in Table 5, Table 6 and Table 7.

Table. 4 Approximate global size used to achieve optimal mesh for the plate

AGS	Number of mesh	f_{11} (1/s)
20	50	321.52
10	200	312.80
5	800	310.59
4	1250	310.29
3	2211	310.03

It can be seen from Table 5, Table 6 and Table 7, the agreement between results obtained with Rayleigh-Ritz approach and finite element method, with a maximum error of 0.7819 %. Natural frequencies are found to increase with an increase in the mode number. The first four vibration modes of the plate are shown in Figure 9, in case of simply supported plate (SSSS).

Obtained results are compared with natural and angular frequencies obtained by using Rayleigh-Ritz approach. The relative error is computed and estimated according to the Eq.22 and listed in the same Table.

$$e\% = \frac{|FEM - Analytical|}{|Analytical|} x100 \qquad (22)$$

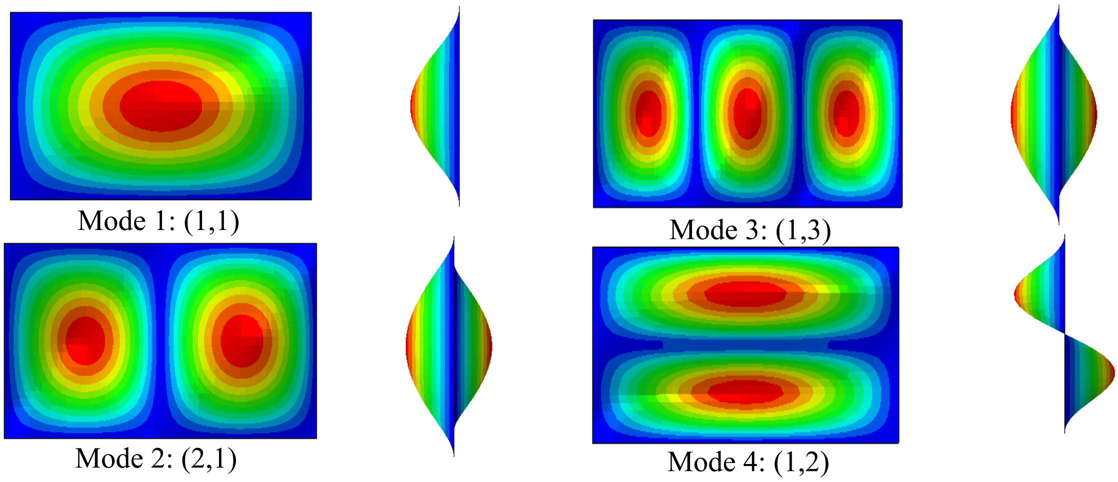

Figure 9. Mode Shapes of the plate with simply supported boundary conditions

Table 5. Angular and natural frequencies of the simply supported rectangular aluminum alloy plate obtained by using Rayleigh-Ritz approach and FEM

		Aluminum alloy plate				
N°	(m, n)	Rayleigh f_{mn} (Hz)	FEM f_{mn} (Hz)	Rayleigh ω_{mn} (rad/s)	FEM ω_{mn} (rad/s)	Error %
1	(1,1)	310.2161	310.29	1949.1455	1949.6095	0.0238
2	(2,1)	496.3458	496.08	3118.6328	3116.9625	0.0535
3	(3,1)	806.5619	807.30	5067.7784	5072.4154	0.0915
4	(1,2)	1054.7349	1061.10	6627.0948	6667.0879	0.6034

Table 6. Angular and natural frequencies of the clamped/simply supported aluminum alloy plate obtained by using Rayleigh-Ritz approach and FEM

		Aluminum alloy plate				
N°	(m, n)	Rayleigh f_{mn} (Hz)	FEM f_{mn} (Hz)	Rayleigh ω_{mn} (rad/s)	FEM ω_{mn} (rad/s)	Error %
1	(1,1)	346.0466	344.24	2174.2753	2162.9237	0.5220
2	(2,1)	596.1644	594.89	3745.8115	3737.8041	0.2137
3	(3,1)	977.7890	975.86	6143.6298	6131.5092	0.1972
4	(1,2)	1076.4430	1077.20	6763.4914	6763.4908	0.0703

Table 7. Angular and natural frequencies of the clamped rectangular aluminum alloy plate obtained by using Rayleigh-Ritz approach and FEM

		Aluminum alloy plate				
N°	(m, n)	Rayleigh f_{mn} (Hz)	FEM f_{mn} (Hz)	Rayleigh ω_{mn} (rad/s)	FEM ω_{mn} (rad/s)	Error %
1	(1,1)	619.7562	619.95	3894.0434	3895.2607	0.0312
2	(2,1)	803.5135	801.79	5048.6244	5037.7951	0.2144
3	(3,1)	1130.8182	1128.70	7105.1405	7091.8312	0.1873
4	(1,2)	1613.8198	1601.20	10139.9292	10060.6363	0.7819

Table 8. Angular and natural frequencies of the aluminum alloy plates with respect to ratio a/b for used boundary conditions

			SSSS			SCSC			CCCC		
a/b	N°	Mode	Rayleigh f_{mn} (Hz)	FEM f_{mn} (Hz)	% Error	Rayleigh f_{mn} (Hz)	FEM f_{mn} (Hz)	% Error	Rayleigh f_{mn} (Hz)	FEM f_{mn} (Hz)	% Error
1	1	(1,1)	496.3483	492.74	0.7269	729.1007	729.03	0.0096	909.4468	906.77	0.2943
	2	(2,1)	1240.8645	1237.50	0.2711	1391.1650	1380.8	0.7450	1854.0410	1859.50	0.2944
	3	(1,2)	1240.8645	1237.50	0.2711	1743.9229	1757.9	0.8014	1854.0410	1859.50	0.2944
	4	(2,2)	1985.3833	1976.80	0.4323	2384.6577	2388.9	0.1778	2707.2843	2736.50	1.079
1.5	1	(1,1)	358.4719	358.31	0.0451	438.5582	436.90	0.3781	682.9605	680.73	0.3265
	2	(2,1)	689.3692	689.39	0.0030	889.8615	890.64	0.0874	1053.3524	1051.10	0.2138
	3	(1,2)	1102.9907	1108.80	0.5266	1149.2883	1148.00	0.1120	1669.1923	1679.00	0.5875
	4	(3,1)	1240.8645	1245.30	0.3574	1567.4839	1570.70	0.2051	1681.0403	1682.60	0.0927
2	1	(1,1)	310.2161	310.29	0.0238	346.0466	344.24	0.5220	621.0276	619.95	0.1735
	2	(2,1)	496.3458	496.08	0.0535	596.1644	594.89	0.2137	803.5135	801.79	0.2144
	3	(3,1)	806.5619	807.30	0.0915	977.7890	975.56	0.2279	1130.8182	1128.70	0.1873
	4	(1,2)	1054.7349	1061.10	0.6034	1076.4430	1077.20	0.0703	1613.8198	1601.20	0.7819
2.5	1	(1,1)	287.8805	288.08	0.0692	307.0078	305.33	0.5465	596.8895	596.62	0.0451
	2	(2,1)	407.0035	406.82	0.0450	463.6720	461.80	0.4037	701.7807	700.80	0.1397
	3	(3,1)	605.5419	605.48	0.0102	707.1119	703.86	0.4598	894.0052	892.21	0.2008
	4	(4,1)	883.4955	884.75	0.1419	1029.5061	1027.6	0.1851	1179.0023	1176.80	0.1867
3	1	(1,1)	275.7476	276.02	0.0987	287.3220	285.85	0.5123	585.1707	585.45	0.0477
	2	(2,1)	358.4719	358.41	0.0172	393.7640	391.74	0.5140	652.4158	652.00	0.0637
	3	(3,1)	496.3458	496.16	0.0374	561.7554	558.40	0.5973	775.5837	774.48	0.1423
	4	(4,1)	689.3692	689.64	0.0392	785.2589	782.40	0.3640	961.6770	959.74	0.2014
5	1	(1,1)	258.0998	258.48	0.1473	261.2215	260.50	0.2762	569.7762	571.38	0.2814
	2	(2,1)	287.8805	288.10	0.0762	297.3082	295.69	0.54428	590.7115	591.50	0.1334
	3	(3,1)	337.5151	337.55	0.0103	356.1557	353.22	0.8242	626.7474	627.20	0.0722
	4	(4,1)	407.0035	406.90	0.0254	435.0695	432.16	0.6687	6.8091745	680.95	0.0047

4. PASSIVE VIBRATION CONTROL WITH DAMPING PATCHES

This section presents simulations and research results of testing the passive vibration controle of the aluminum alloy rectangular isotropic plate analyzed previously with damping patches attached to the top surface. For various boundary conditions: SSSS, SCSC and CCCC. The same plate of dimensions 200x100x1 mm^3 is modeled in ABAQUS and meshed with S4R elements. Three rectangular patches denoted P1, P2 and P3 made of one layer of glass/epoxy orthotropic material of sections representing respectively 1, 5 and 10 % of the total plate section, are glued to the center of the plate as shown in Figure 10. Also, the patch was modeled with S4R elements. The material properties of the patches used are listed in Table 9. Note that E_1, E_2 and E_3 are Young's moduli, G_{12}, G_{13} and G_{23} shear moduli, υ_{12}, υ_{13} and υ_{23} Poisson's ratio.

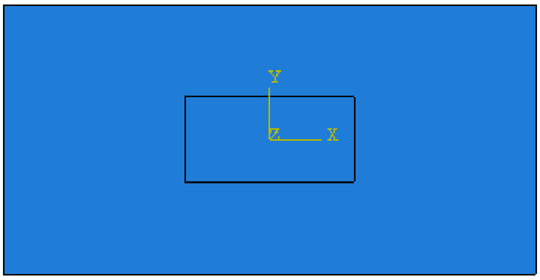

Figure 10. Aluminum alloy plate with attached orthotropic glass/epoxy patch

Table 9. Mechanic and physic properties of the glass/epoxy patches

	glass/epoxy
ρ (Kg/m^3)	2040
E_1 (MPa)	45178.86
E_2 (MPa)	13852.18
E_3 (MPa)	13854.28
ν_{12}	0.2656
ν_{13}	0.2656
ν_{23}	0.2209
G_{12} (MPa)	4803.66
G_{13} (MPa)	4804.05
G_{23} (MPa)	3435.43

All elastic properties were estimated after calculating the homogenized properties by using EasyPBC open-source ABAQUS CAE interface plugin [19], in case of volume fraction of fibers V_f=0.6. Exploiting data extracted from ABAQUS, the natural frequencies of vibration corresponding to the first four modes are obtained with and without damping patches and listed in Table 10, Table 11 and Table 12. The relative error between natural frequencies of the plate with and without damping patches is computed and presented in the same Tables. It is observed that for the configurations of considered boundary conditions, the patch P3 which covers 10% of the surface area of the plate offers the minimized vibration level with a reduction in frequency to reach 19.4577 %, in case of simply supported/ clamped plate. It is clear that with increasing the surface area of the patch we can better control the frequency level. In this investigation we also examined the influence of the patch geometry on the passive controle vibration. We consider three simply supported plates with rectangular, square and circular patches attached to the center as shown in Figure 11 and covered 10% of the global surface area. It can be seen from the results presented in Table 13 that the rectangular patch gives the maximum reduction in frequency for modes 1 and 2. The square patch offers maximum reduction in frequency in case of modes 3 and 4.

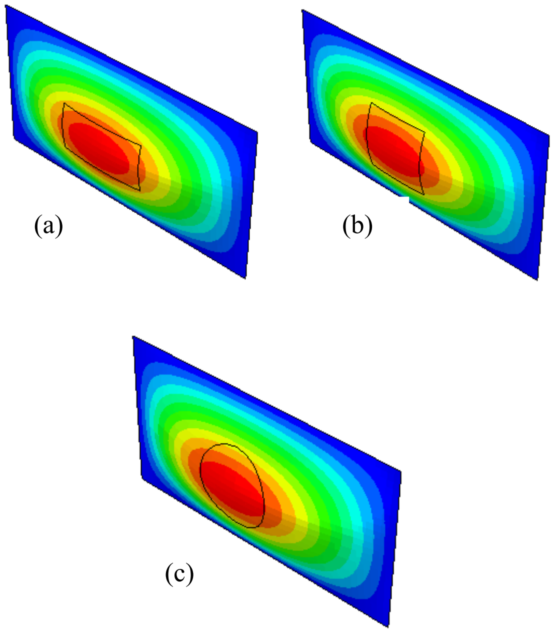

Figure 11. Different patches shapes attached to the plate:(a) rectangular, (b) square and (c) circular

Table 10. Natural frequencies of vibration corresponding to the first four modes obtained with and without damping patches case of simply supported

Mode	Frequency Plate only	Frequency Plate with patch (P1)	Difference (P1) %	Frequency Plate with patch (P2)	Difference (P2) %	Frequency Plate with patch (P3)	Difference (P3) %
1	310.29	306.67	1.1666	294.46	5.1016	283.55	8.6177
2	496.08	495.93	0.0302	492.21	0.7801	483.36	2.5641
3	807.30	804.16	0.3889	789.11	2.2531	785.98	2.6409
4	1061.10	1060.70	0.0376	1052.10	0.8481	1032.3	2.7141

Table 11. Natural frequencies of vibration corresponding to the first four modes obtained with and without damping patches case of simply supported/clamped plate

Mode	Frequency Plate only	Frequency Plate with patch (P1)	Difference (P1) %	Frequency Plate with patch (P2)	Difference (P2) %	Frequency Plate with patch (P3)	Difference (P3) %
1	344.24	339.63	1.3391	324.11	5.8476	311.08	17.6301
2	594.89	594.65	0.0403	588.42	1.0875	574.70	18.7480
3	975.86	973.15	0.2777	957.47	1.8844	954.99	19.4577
4	1077.20	1076.70	0.0464	1067.10	0.9376	1045.50	11.3451

Table 12. Natural frequencies of vibration corresponding to the first four modes obtained with and without damping patches case of clamped plate

Mode	Frequency Plate only	Frequency Plate with patch (P1)	Difference (P1) %	Frequency Plate with patch (P2)	Difference (P2) %	Frequency Plate with patch (P3)	Difference (P3) %
1	619.95	608.58	1.8340	573.02	7.5699	545.06	12.0800
2	801.79	801.29	0.0623	790.29	1.4342	766.59	4.3901
3	1128.70	1122.30	0.5670	1101.60	2.4009	1098.10	2.7110
4	1601.20	1600.20	0.0624	1569.40	1.9860	1545.20	3.4973

Table 13. Natural frequencies of vibration corresponding to the first four modes obtained with various geometry of damping patches case of simply supported plate

Mode	Frequency Plate without patch	Frequency plate with rectangular patch (P3)	Frequency plate with square patch (P3)	Frequency plate with circular patch (P3)
1	310.29	283.55	284.28	284.54
2	496.08	483.36	489.83	490.02
3	807.30	785.98	781.25	781.88
4	1061.10	1032.3	1010.80	1016.30

4. CONCLUSIONS

The first part of this work provides a modal analysis of aluminum alloy isotropic thin rectangular plate in order to determine its natural frequencies and mode shapes by using analytical method based on Rayleigh-Ritz energy approach and finite element method based on ABAQUS software. Both obtained results were compared. The effect of boundary conditions and shape factor were examined. The second part of this study presents simulations results of testing of the plate with passive vibration control with orthotropic patches.

The following conclusions are made based on the above study.

- The FEM study showed acceptable results in comparison with analytical solution.
- This work reviewed the robustness and capability of the shell element (S4R) provided by ABAQUS software in case of modal analysis.
- The fully clamped plate has the highest level frequency; the simply supported plate has the lowest level. The simply/clamped plate fall between these two configurations.
- For the four first natural modes, the frequencies obtained with Rayleigh energy approach and FE method agree closely with those of the conventional finite element method, with a maximum error of 1,079 %.
- For various configurations of considered boundary conditions, vibration attenuation can be achieved by attaching patches elements on to the plate.
- With increasing the surface area of the patch we can control better the plate frequency level.
- The patch geometry has big influence on the passive controle of plate because the shape of damping patches is defined according to the nodal patterns of the corresponding mode shapes of vibrating plate.
- In case of rectangular plates, rectangular patch gives maximum reduction in frequency for modes 1 and 2. The square

patch offers maximum reduction in frequency in case of modes 3 and 4.

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