Generalized Differential Quadrature Method for Studying the Out off Plane Vibrations of Curved Pipes Conveying Fluid

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Abstract: - A curved pipe bent in the arc of a circle conveying fluid is investigated. The flowing fluid is considered as a non-compressible and heavy. The considered pipe is assumed cantilevered at its both ends. The Generalized Differential Quadrature Method is employed to investigate the dynamic out off plane stability of the pipe. The obtained numerical results show the dependence of the critical fluid velocity on the maximal central angle of the pipe.

Keywords: - curved pipe, fluid, circular frequency, flow velocity, GDQM

1. INTRODUCTION

The straight and curved pipes conveying fluid are used in various fields of engineering. The vibrations of these tubes are an important topic for research.

Svetlitsky [1] was the first to study the out-of-plane vibrations of curved tubes with flowing fluid. Jung and Chung [2] apply Hamilton's principle to obtain the equations describing the in- and out-of-plane vibrations of extensible curved pipes. The Galerkin method is used to solve them.

Aubad [3] applies the finite element method to study the dynamics of curved pipes conveying fluid supported by linear elastic supports. He determines the circular frequency of the system on dependence on its stiffness, location of the supports, as well as on the geometric and physical characteristics of the pipe and the fluid.

Melo and Castro [4] study the in- and out-of-plane vibrations of curved tubes. Two approaches are applied. In the first one the pipe is examined based on the beam theory, while in the second approach the study is based on the shell theory.

Gongmin Liu et al. [5] apply the transfer matrix method to investigate the dynamics of curved pipes. The circular frequencies of the oscillations are determined.

D. Lolov and Sv. Lilkova-Markova [6] employed Galerkin's method to study the free out-of-plane vibrations of a planar curved tube conveying fluid.

2. PROBLEM FORMULATION

The present paper investigates the out-of-plane dynamic stability of curved pipes, conveying fluid. The static scheme of the pipe under consideration is shown in Fig. 1. The pipe is bent in the form of an arc of a circle with radius R. The material of the pipe is linear elastic with Young's modulus E and shear modulus G. The cross-sectional parameters are area A, axis moment of inertia I and polar moment of inertia I_c. The fluid flowing in the pipe is heavy and non-compressible. u is the displacement on the normal of the curve, w that on the tangent and v is the out of plane displacement.

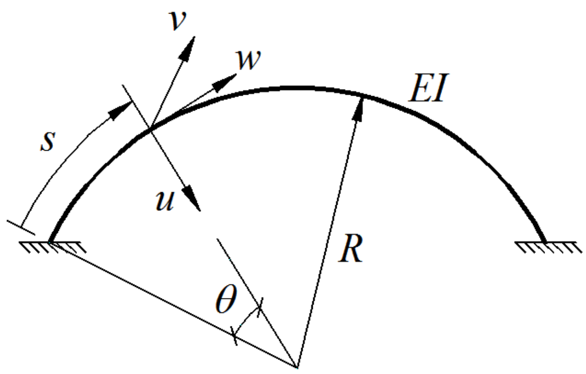

Figure 1. Static scheme of the investigated pipe conveying fluid

The differential equation of the free out-of-plane vibrations of the pipe, shown in [6], is

$$EI\frac{\partial^4 v}{\partial s^4}+\left(MV^2-\frac{GI_c}{R^2}\right)\frac{\partial^2 v}{\partial s^2}+2MV\frac{\partial^2 v}{\partial s\,\partial t}+ , \quad +(M+m)\frac{\partial^2 v}{\partial t^2}=0 \tag{1}$$

where t is the time and s is the curvilinear abscissa. The mass of the pipe per unit length is denoted by m and the mass of the fluid per unit length of the pipe by M. V is the flow velocity.

For convenient reasons the curvilinear abscissa s in (1) is expressed by the central angle θ. The following equation is obtained

$$\frac{EI}{R^4}\frac{\partial^4 v}{\partial \theta^4}+\left(MV^2-\frac{GI_c}{R^2}\right)\frac{1}{R^2}\frac{\partial^2 v}{\partial \theta^2}+\frac{2MV}{R}\frac{\partial^2 v}{\partial \theta\,\partial t}+ , \quad +(M+m)\frac{\partial^2 v}{\partial t^2}=0 \tag{2}$$

The function of the displacement v is presented as a product of two functions:

$$v(\theta,t)=C(\theta)e^{\omega t}, \tag{3}$$

where ω is the circular frequency

Then the equation (2) will be

$$\frac{EI}{R^4}\frac{d^4C}{d\theta^4}+\left(\frac{MV^2}{R^2}-\frac{GI_c}{R^4}\right)\frac{d^2C}{d\theta^2}+ , \quad +\frac{2M\,V\,\omega}{R}\frac{dC}{d\theta}+(M+m)\,\omega^2C=0 \tag{4}$$

New dimensionless parameters are introduced:

$$\phi=\frac{C}{R};\ u=VR\sqrt{\frac{M}{EI}};\ \beta=\frac{M}{M+m}; , \quad k^2=\frac{GI_c}{EI};\ \Omega=R^2\omega\sqrt{\frac{(M+m)}{EI}} \tag{5}$$

After transformations the differential equation (5) takes the forms:

$$\frac{d^4\phi}{d\theta^4}+\left(u^2-k^2\right)\frac{d^2\phi}{d\theta^2}+2\,u\,\Omega\sqrt{\beta}\,\frac{d\phi}{d\theta}+\Omega^2\phi=0 \tag{6}$$

,

The dimensionless differential governing equation of motion (6) can be transformed into a system of algebraic equations by means of the Generalized Differential Quadrature Method (GDQM) [7], [8]. The basic idea of the method is to approximate a derivative of a function at any discrete point of a domain as a weighted linear sum of function values at all discrete points, expressing mathematically as follows.

$$\left.\frac{d^n\,\phi(\theta)}{d\,\theta^n}\right|_{\theta=\theta_i}=\sum_{j=1}^{m}\beta_{ij}^{(n)}\phi(\theta_j),\quad i=1,...,m, \tag{7}$$

where m is the total number of the sampling points of the chosen grid on the axis of the pipe, $\beta_{ij}^{(n)}$ is the weighting coefficient corresponding to the n^{th} order derivative at point i.

In the present paper the Chebyshev-Gauss-Lobatto point distribution is assumed [8]:

$$\theta_i=\frac{1}{2}\left[1-\cos\left(\frac{i-1}{m-1}\pi\right)\right],\quad i=1,...,m, \tag{8}$$

The weighting coefficients are calculated by means of Lagrange interpolating functions. For the first derivative, the weighting coefficients are calculated as [8]:

$$\beta_{ij}^{(1)}=\frac{L^{(1)}(\theta_i)}{(\theta_i-\theta_j)\,L^{(1)}(\theta_j)},\quad i,j=1,...,m,\ i\neq j, \tag{9}$$

$$\beta_{ii}^{(1)}=-\sum_{j=1,j\neq i}^{m}\beta_{ij}^{(1)},\quad i,j=1,...,m, \tag{10}$$

while for higher order derivatives, ones get iteratively

$$\beta_{ij}^{(n)}=n\left(\beta_{ii}^{(n-1)}\beta_{ij}^{(1)}-\frac{\beta_{ij}^{(n-1)}}{\xi_i-\xi_j}\right), , \quad i,j=1,...,m;\ i\neq j;\ n=2,..,(m-1) \tag{11}$$

$$\beta_{ii}^{(n)}=-\sum_{j=1,j\neq i}^{m}\beta_{ij}^{(n)}, , \quad i,j=1,...,m;\ n=2,..,(m-1) \tag{12}$$

where the first derivative of Lagrange interpolating polynomials at each point θ_k in equation (9) is defined as

$$L^{(1)}(\xi_k)=\prod_{l=1,l\neq k}^{m}(\xi_k-\xi_l),\quad k=1,...,m, \tag{13}$$

The Lagrange interpolating polynomials in conjunction with Chebyshev-Gauss-Lobatto sampling points of equation (8) ensures convergence, so that the increasing number of sampling points leads to an error decrease.

Through the GDQM the governing equation (6) is rewritten in discrete form at the points $i = 3,4,...,(m-2)$.

$$\sum_{j=1}^{m}\beta_{ij}^{(4)}\phi(\theta_j)+(u^2-k^2)\sum_{j=1}^{m}\beta_{ij}^{(2)}\phi(\theta_j)+ \\ +2\,u\,\Omega\,\sqrt{\beta}\sum_{j=1}^{m}\beta_{ij}^{(1)}\phi(\theta_j)+\Omega^2\phi(\theta_j)=0, \tag{14}$$

The boundary conditions for the pipe shown in Fig 1 are:

$$\phi(\theta_1)=\phi(\theta_m)=0, \tag{15}$$

$$\left.\frac{d\,\phi(\theta)}{d\,\theta}\right|_{\theta=\theta_1}=\left.\frac{d\,\phi(\theta)}{d\,\theta}\right|_{\theta=\theta_m}=0, \tag{16}$$

Equation (14), can be rewritten in the following matrix form:

$$\left[B^{(4)}+(u^2-k^2)B^{(2)}+2\,u\,\Omega\,\sqrt{\beta}B^{(1)}\right]\delta+ \\ +\Omega^2 I\delta_d=0, \tag{17}$$

in (17)

$$B_{ij}^{(4)}=\beta_{ij}^{(4)};\quad B_{ij}^{(2)}=\beta_{ij}^{(2)};\quad B_{ij}^{(1)}=\beta_{ij}^{(1)}, \\ i=3,4,...,(m-2),\ j=1,...,m, \tag{18}$$

$$\delta=\{\phi(\theta_1),...,\phi(\theta_m)\}^T, \tag{19}$$

$$\delta_d=\{\phi(\theta_3),...,\phi(\theta_{m-2})\}^T, \tag{20}$$

The four boundary conditions (15) and (16) are also written in a matrix form.

$$K_b\delta=0, \tag{21}$$

where

$$K_b=\begin{vmatrix}1 & 0 & 0 & ... & 0\\ \beta_{11}^{(1)} & \beta_{12}^{(1)} & \beta_{13}^{(1)} & ... & \beta_{1m}^{(1)}\\ 0 & 0 & 0 & ... & 1\\ \beta_{m1}^{(1)} & \beta_{m2}^{(1)} & \beta_{m3}^{(1)} & ... & \beta_{mm}^{(1)}\end{vmatrix}, \tag{22}$$

The discrete field (17) can be combined with the boundary conditions (21) into m algebraic equations with m unknown nodal displacements as follows

$$\begin{vmatrix}K_b\\ B^{(4)}+(u^2-k^2)B^{(2)}+2\,u\,\Omega\,\sqrt{\beta}B^{(1)}\end{vmatrix}\delta+ \\ +\begin{vmatrix}0\\ \Omega^2 I\delta_d\end{vmatrix}=0, \tag{23}$$

In order to calculate the natural frequencies of the pipe, equation (23) is reorganized of the following form

$$\left(\begin{vmatrix}K_{bb} & K_{bd}\\ B_{db}^{(4)} & B_{dd}^{(4)}\end{vmatrix}+(u^2-k^2)\begin{vmatrix}0 & 0\\ B_{db}^{(2)} & B_{dd}^{(2)}\end{vmatrix}+ \\ +2\,u\,\Omega\,\sqrt{\beta}\begin{vmatrix}0 & 0\\ B_{db}^{(1)} & B_{dd}^{(1)}\end{vmatrix}\right)\begin{vmatrix}\delta_b\\ \delta_d\end{vmatrix}+\Omega^2\begin{vmatrix}0 & 0\\ 0 & I\end{vmatrix}\begin{vmatrix}\delta_b\\ \delta_d\end{vmatrix}=\begin{vmatrix}0\\ 0\end{vmatrix}, \tag{24}$$

where

$$\delta_b=\{\phi(\theta_1),\phi(\theta_2),\phi(\theta_{m-1}),\phi(\theta_m)\}^T, \tag{25}$$

$$K_{bb}=\begin{vmatrix}1 & 0 & 0 & 0\\ \beta_{11}^{(1)} & \beta_{12}^{(1)} & \beta_{1(m-1)}^{(1)} & \beta_{1m}^{(1)}\\ 0 & 0 & 0 & 1\\ \beta_{m1}^{(1)} & \beta_{m2}^{(1)} & \beta_{m(m-1)}^{(1)} & \beta_{mm}^{(1)}\end{vmatrix}^T, \tag{26}$$

$$K_{bd}=\begin{vmatrix}0 & 0 & ... & 0\\ \beta_{13}^{(1)} & \beta_{14}^{(1)} & ... & \beta_{1(m-2)}^{(1)}\\ 0 & 0 & ... & 0\\ \beta_{m3}^{(1)} & \beta_{m4}^{(1)} & ... & \beta_{m(m-2)}^{(1)}\end{vmatrix}^T, \tag{27}$$

$$B_{db}^{(4)}=\begin{vmatrix}\beta_{31}^{(4)} & \beta_{32}^{(4)} & \beta_{3(m-1)}^{(4)} & \beta_{3m}^{(4)}\\ \beta_{41}^{(4)} & \beta_{42}^{(4)} & \beta_{4(m-1)}^{(4)} & \beta_{4m}^{(4)}\\ ... & ... & ... & ...\\ \beta_{(m-3)1}^{(4)} & \beta_{(m-3)2}^{(4)} & \beta_{(m-3)(m-1)}^{(4)} & \beta_{(m-3)m}^{(4)}\\ \beta_{(m-2)1}^{(4)} & \beta_{(m-2)2}^{(4)} & \beta_{(m-2)(m-1)}^{(4)} & \beta_{(m-2)m}^{(4)}\end{vmatrix}^T, \tag{28}$$

$$B_{dd}^{(4)}=\begin{vmatrix}\beta_{33}^{(4)} & \beta_{34}^{(4)} & ... & \beta_{3(m-2)}^{(4)}\\ \beta_{43}^{(4)} & \beta_{44}^{(4)} & ... & \beta_{4(m-2)}^{(4)}\\ ... & ... & ... & ...\\ \beta_{(m-2)3}^{(4)} & \beta_{(m-2)4}^{(4)} & ... & \beta_{(m-2)(m-2)}^{(4)}\end{vmatrix}^T, \tag{29}$$

$$\delta_b=-K_{bb}^{-1}K_{bd}\delta_d^{\ T}, \tag{30}$$

$$\begin{aligned}&\left(B_{dd}^{(4)} - B_{db}^{(4)}K_{bb}^{-1}K_{bd}\right)\delta_d + \\ &+\left(u^2 - k^2\right)\left(B_{dd}^{(2)} - B_{db}^{(2)}K_{bb}^{-1}K_{bd}\right)\delta_d + \\ &+2\,u\,\Omega\sqrt{\beta}\left(B_{dd}^{(1)} - B_{db}^{(1)}K_{bb}^{-1}K_{bd}\right)\delta_d + \Omega^2 I\delta_d = 0\end{aligned} \quad (31)$$

,

Equation (31) represents an eigenvalue problem. For different values of the non-dimensional velocity u are obtained the non-dimensional natural frequencies Ω. If $\mathrm{Re}\,\Omega < 0$ the system is stable. At $\mathrm{Re}\,\Omega = 0$ the system is at the edge of loss of stability, the corresponding fluid velocity is the critical fluid velocity.

3. RESULTS AND DISCUSSION

Numerical studies have been carried out for the fluid flowing pipe in Fig. 1.

The geometric and the material characteristics of the pipes are: the radius $R = 20\ m$, the inner and the outer radii of the cross-section of the pipes are $r_{inner} = 0{,}095\ m$ and $r_{outer} = 0{,}1\ m$, Young's modulus $E = 210\ GPa$, shear modulus $G = 80\ GPa$, the density of the material of the pipe $\rho = 7800\ kg/m^3$. The density of the flowing fluid is $\rho = 1000\ kg/m^3$.

The critical non-dimensional flow velocity u_{cr} is obtained for different values of $\theta_{\max}$. The results are shown in Fig.2. It is obvious that increasing the $\theta_{\max}$ lowers the critical fluid velocity u_{cr}.

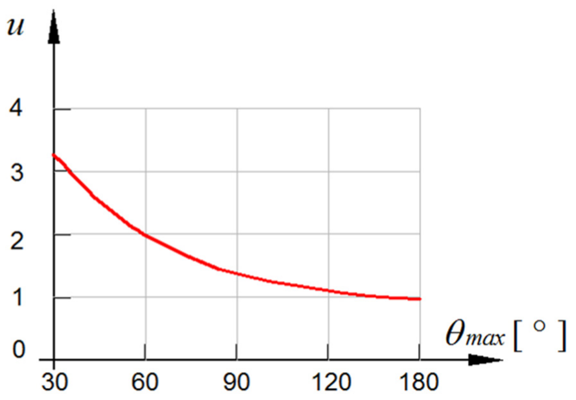

Figure 2. Dependence of the critical non-dimensional fluid velocity on $\theta_{\max}$:

4. CONCLUSIONS

The employed GDQM in the paper allows relatively easy determination of the first natural frequencies of the out off plane vibrations of pipes with axes bent in the arc of circle and with flowing fluid. The method could be competitive with other established approaches for investigating the dynamic stability of the pipes conveying fluid like the Matrix method.

The results show the dependence of the critical non-dimensional fluid velocity of the system on $\theta_{\max}$. Increasing the $\theta_{\max}$ leads to decrease in the critical velocity.

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