The Effect of The Symmetrical Elastic Nonlinearity on Structural Vibrations Transmitted by Dynamic Equipment on The Construction Envelope

Polidor BRATU
IMSAR, street Constantin Mille no. 15, Bucharest 030167, Romania,
e-mail: icecon@icecon.ro

George Lucian NICOLAE
IMSAR, street Constantin Mille no. 15, Bucharest 030167, Romania,
e-mail: nicolaegeorgelucian91@gmail.com

Samir IACOVESCU
IMSAR, street Constantin Mille no. 15, Bucharest 030167, Romania,
e-mail: samir.iacovescu@upit.ro

Dan ION
IMSAR, street Constantin Mille no. 15, Bucharest 030167, Romania,
e-mail: dan.ion@icecon.ro

Andrei COTOBAN
IMSAR, street Constantin Mille no. 15, Bucharest 030167, Romania,
e-mail: andrei.cotoban@gmail.com

Abstract: - The results of the research carried out for elastic systems with symmetrical nonlinearity have been systematized in this article so that they can be disseminated for real models of dynamic equipment in construction endowment. Thus, the ventilation - conditioning systems, water pumping systems, main and interior piping systems from nuclear power plants, platforms with optoelectronic equipment are supported on elastic devices that can have a non-linear behavior.

In this case, it is highlighted that under certain functional and constructive conditions it is possible the occurrence of subharmonic vibrations of order 1/3 of the excitation pulsation. This leads to the generation of structural noise with consequences for the people inside it.

Keywords: - nonlinear vibrations, subharmonic vibrations, symmetrical elasticity

1. INTRODUCTION

It is adopted the model of the system with a degree of freedom with viscoelastic connection where the elastic force is a nonlinear function of the third order in relation to the instantaneous deformation.

In this case, in order to highlight the occurrence of 1/3 subharmonics order, it was used the method of external excitation on the frequency band sought for so that the mechanical work of external excitation be equal to the mechanical work of dissipation. [1-5]

2. EVALUATION OF SUBHARMONIC VIBRATIONS OF ORDER 1/3 OF THE EXCITATION PULSATION

The system is characterized by a coefficient of rigidity expressed by the relationship:

$$k=k_0\,(1+\beta q^2) \quad (1)$$

where for $\beta > O$, the relation (1) characterizes a “strong” elastic element, and for $\beta < O$ - a “soft” elastic element. It is mentioned that the parameter β is measured in m^{-2} , and k_0 în Nm^{-1} . The differential equation of the system is:

$$a\ddot{q} + b\dot{q} + k_0(1+\beta q^2)q = Q_0 \cos\ \omega t \quad (2)$$

where q is the generalized coordinate which can be linear displacement or angular displacement; a- coefficient of inertia (mass m or moment of inertia J); b - viscous amortization factor; k_0 - the rigidity coefficient of the nonlinear elastic system; Q_0 the amplitude of the disturbing factor which may be force or moment; ω - pulsation of the disturbing factor.[6-10]

We change the variable of the form τ = pt, where p is the natural pulsation of the system. In this case we have:

$$\frac{d\tau}{dt} = p, q' = \frac{dq}{d\tau} = \frac{dq}{dt} \cdot \frac{dt}{d\tau} = \dot{q}\frac{1}{p} \quad (3)$$

from where it emerges $\dot{q} = pq'$

For $\ddot{q}$ we perform:

$$q' = \frac{d}{dt}\dot{q} = \frac{d}{dt}(pq') \text{ sau } \ddot{q} = p^2\frac{d}{d\tau}(q') = p^2 q''. \quad (4)$$

Using the obtained relations to change the variable, the equation becomes:

$$ap^2 q'' + bpq' + k_0(1 + \beta q^2)q = \underline{Q}_0 \cos \omega t \quad (5)$$

which we divide by $ap^2 = k_0$ and have:

$$q'' + rq' + (1 + \beta q^2)q = q_0 \cos \omega t \quad (6)$$

where $p = \sqrt{k_0/am}$ is the natural pulsation of the linear elastic system without amortization;

$r = \frac{b}{ap}$ - relative amortization

$q_0 = \frac{Q_0}{k_0}$ - the relative amplitude of the excitation

It is noted $\Omega = \omega/p$ the relative pulsation, so that equation (16.18) can be written as:

$$q'' + rq' + (1 + \beta q^2)q = q_0 \cos \Omega\tau \quad (7)$$

There are analyzed the free unamortized vibrations described by equation (7) where r = 0, so that the equation:

$$q'' + q + \beta q^3 = 0 \quad (8)$$

For the nonlinear differential equation (8) choose the solution (in the first approximation) such as:

$$q = A\sin \Omega\tau$$

which inserted together with its derivative in equation (8), leads to

$$-A\Omega^2 \sin \Omega\tau + A\sin \Omega\tau + \beta A^3 \sin^3 \Omega\tau = 0$$

and by transforming the function $\sin^3 \Omega\tau$ and ordering we have

$$A\left(1 - \Omega^2 + \frac{3}{4}\beta A^2\right)\sin \Omega\tau - \frac{1}{4}\beta A^3 \sin 3\Omega\tau = 0$$

The coefficient of $\sin \Omega\tau$ is canceled and obtained:

$$\Omega^2 = 1 + \frac{3}{4}\beta A^2 \quad (9)$$

The relation (9) represents, in the first approximation, the "amplitude-frequency" characteristic specific to nonlinear systems. It is found that if β> 0 then the relative pulsation Ω increases by A, and if β<0 the relative pulsation decreases by A.

For the second approximation, the solution is chosen

$$q = A\sin \Omega\tau + B\sin 3\Omega\tau \quad (10)$$

which inserted together with its derivative in equation (8) leads to identity.

$$\begin{gathered} A\left(1 - \Omega^2 + \frac{3}{4}\beta A^2\right)\sin \Omega\tau + \left(B - 9B\Omega^2 - \frac{1}{4}\beta A^3\right)\sin 3\Omega\tau + \\ + 3A^2 B\beta \sin^2 \Omega\tau \sin 3\Omega\tau + \\ + 3AB^2\beta \sin \Omega\tau \sin^2 3\Omega\tau + \\ + \beta B^3 \sin^3 3\Omega\tau = 0 \end{gathered}$$

in which the coefficient of the function sin3Ωτ is canceled and it is obtained

$$B = \frac{\beta A^3}{4(1 - 9\Omega^2)} \quad (11)$$

relation that represents the "amplitude - frequency" characteristic in the second approximation.

If we insert (9) in (11) and approximate $\beta A^2 \cong 0$ then we have

$$B = -\beta\frac{A^3}{32}$$

The analysis of the occurrence of the subharmonic vibrations is based on the fact that the mechanical work provided by the excitation, for the main harmonic in free vibration, must be equal to the mechanical work of dissipation of the system. Thus, for the symmetric nonlinear elastic system, the mechanical work provided by the excitation $Q_0 \cos 3\Omega\tau$ and absorbed by the 3rd order harmonic of the form q_3 = Bsin3Ωτ resonating with the excitation, that is Ω = 1, is of the form [11-15]

$$L_3 = \int_0^{2\pi} Q_0 \cos 3\Omega\tau dq_3 \quad (12)$$

in which it is inserted dq_3 = 3Ωbcos3Ωτ and we obtain

$$L_3 = 3BQ_0 \int_0^{2\pi} \cos^2 3\Omega\tau d\tau \quad (13)$$

If Ω = 1, we have

$$L_3 = 3\pi B Q_0 \quad (14)$$

The mechanical work absorbed by the viscous system, on the 1st order component, with the solution $q_1 = A\sin\Omega\tau$,, corresponds to the viscous force $F_1 = b\dot{q} = pbq_1'$ and is given by the relation [16-18]

$$.L_1 = \int_0^{2\pi} F_1 dq_1 \quad (15)$$

where $F_1 = pbA\Omega\cos\Omega\tau$, and $dq_1 = A\omega\cos\Omega\tau d\tau$
For $\Omega = 1$, the relation (15) becomes

$$L_1 = \int_0^{2\pi} pbA^2 \cos^2 \tau d\tau = \pi pbA^2 \quad (16)$$

he condition of occurrence of the subharmonic of 1/3 order is that $L_3 > L_1$, that is

$$3\pi B Q_0 > \pi pbA^2 \quad (17)$$

in which it is inserted (17) and obtained

$$3Q_0 \left|\frac{\beta A^3}{32}\right| > pbA^2 \quad (18)$$

Taking into account the previously made notations and replacing b=mpr in (18) we obtain

$$r < \frac{3}{32} q_0 A\beta \quad (19)$$

The relation (19) represents the condition of occurrence of the subharmonic vibrations of order 1/3. It is found that subharmonics of order 1/3 cannot be primed (triggered) alone, but only due to excitation to resonance, when amplitude increases a lot. In real physical systems it is difficult to prevent the occurrence of natural vibrations due to external excitations, which is why protection conditions are required for the subharmonic response. This requires that relation (19) be modified as follows: [19-22]

$$r > \frac{3}{32} A_1 q_0 \beta \quad (20)$$

where A_1 is the maximum possible amplitude of the fundamental vibrations in the system.[23-25]

3. CONCLUSIONS

For symmetrical nonlinear elastic systems, in the case of leaning the dynamic equipment, it is highlighted the case of appearance of subharmonic vibrations of order 1/3 when relation $32\tau<2q_0 A\beta$ is satisfied.

The avoidance of subharmonic vibrations is possible in this case by adopting the appropriate hysteretic amortization $r = \frac{b}{ap}$ or $r = \frac{b}{m\omega_n} > \frac{3}{32} A q_0 \beta$

REFERENCES

[1] Bratu P., Rheological Model of the Neoprene Elements Used for Base isolation against Seismic Action, *Materiale Plastice*, vol. 46, issue 3, 2009, pp.288-294.

[2] Bratu P., The variation of Natural Frequencies of Road Vibrator-rollers, as a Function of Parameters of Neoprene Vibration Isolation Elements, *Materiale Plastice*, vol. 48, issue 2, 201l, pp.l44-147.

[3] Dobrescu C., Analysis of the dynamic regime of forced vibrations in the dynamic compacting process with vibrating rollers, *ACTA NAPOCENSIS - Applied mathematics, mechanics and engineering*, vol. 61, issue 1 2018, pp.71-76.

[4] Bratu P., Hysteretic Loops in Correlation with the maximum dissipated energy, for linear systems, *Symmetry*, vol. 11, issue 3, 2019, pp.1-12.

[5] Bratu P., Dynamic Rigidity of The Linear Voigt - Kelvin Viscoelastic Systems, *Romanian Journal of Acoustics and Vibration*, vol. 16, issue 2, 2019, pp.184-188.

[6] Dragan N., Studies on the Mechanical Elastic Systems with nonlinear damping. Power and amplitude numerical analysis, *Proceedings of the 10th WSEAS International Conference on AUTOMATION & INFORMATION ICAIW*, 2009, pp.130-133.

[7] Dragan N., Studies on the Mechanical Elastic Systems Dynamics of the Rigid Body with Structural Symmetries. Modal Analysis. Transmitted Forces and Moments, *Proceedings of the 10th WSEAS International Conference on AUTOMATION & INFORMATION ICAIW*, 2009, pp.118-123.

[8] Dragan N., A numerical approach of power analysis for the elastical mechanical systems with non linear damping, *Romanian Journal of Acoustics and Vibration*, vol. 5, issue 1, 2008, pp.17-22.

[9] Drăgan N., Theoretical studies regarding the dynamics of the rigid body with elastic bearings and structural symmetries, excited by harmonical forces and couples, *Annals of the Oradea University, Fascicle of Management and Technological Engineering*, vol. 7 (17), 2008, pp. 217-224.

[10] Drăgan N., The dynamic analysis of the mechanical systems. Calculus of the equivalent dynamic forces and torques, *The Annals of "Dunarea de Jos" University of Galaţi, Fascicle XIV Mechanical Engineering*, 2008, pp. 43-48.

[11] Dragan N. Composite materials used for decreasing the noise level inside the cabins of the mobile technological equipment, *Annals of the Oradea University, Fascicle of Management and Technological Engineering* , vol. 9(19), issue 1, 2010, pp. 3.73-3.82.

[12] Drăgan N. Modal calculus of the reinforced concrete bridges modeled as a rigid solid beared on viscous elastic neoprene supports, *The Annals of "Dunarea de Jos" University of Galaţi, Fascicle XIV Mechanical Engineering* vol. 2, issue 16, 2010, pp.27-34.

[13] Drăgan, N., Instrumental analysis of the reinforced concrete bridges subjected to dynamic forces from the traffic*, The Annals of "Dunarea de Jos" University of Galaţi, Fascicle XIV Mechanical Engineering,* Vol. 2, Issue 16, 2010, pp. 35-40

[14] Drăgan, N. Nonlinear approach of the systems between the dynamic modelling difficulties and the advantages of using it in mechanical designing, *Romanian Journal of Acoustics and Vibration*, Vol. 2, Issue 1, 2010, pp. 2.

[15] Drăgan, N., Dzemyda, L, The innovative concept of dynamic analysis for the movements of the viaduct modeled as solid body with elastic bearings, *Romanian Journal of Acoustics and Vibration*, Volume 8, Issue 1, 2011, pp. 13- 20.

[16] Drăgan, N., Considerations on the nonlinearity grade of the nonlinear mechanical elastic systems, *The Annals of "Dunarea de Jos" University of Galați, Fascicle XIV Mechanical Engineering,* Vol. 1, Issue 1, 2012, pp. 13-18.

[17] Drăgan, N., Considerations on the composite neoprene vibration isolators used for the mechanical systems bearings. The dynamics of the non-linear models*, Annals of the Oradea University*, *Fascicle of Management and Technological Engineering,* vol. XI (XXI), No. 2, 2012, pp. 4.35-4.41

[18] Drăgan, N., Dynamic analysis of reinforced concrete bridges - Modal calculus (eigenvalues, eigenfrequencies)*, Synthesis of Theoretical and Applied Mechanics,* Vol. 9 (2018), no. 4, 2018, pp. 245-254

[19] Vasile O., Active Vibration Control for Viscoelastic Damping Systems under the Action of Inertial Forces, *Romanian Journal of Acoustics and Vibration*, vol. 14, issue 1, 2017, pp.54-58.

[20] Bratu P., Vasile O., Spânu G.C., The analysis of insulation systems based on Hooke - Voigt kelvin dynamic rheological model, *Journal of Vibrational Engineering and Technologies*, vol. 5, issue 3, 2017, pp.255-261.

[21] Vasile O., Predoi, M.V., Dragomir, M., Tiron, A., Furdui ,H. Vibration isolation analysis of electric motors for essential dynamic regimes, *Journal of Vibrational Engineering and Technologies*, vol. 5, issue 3, 2017, pp.239-245.

[22] Vasile O., Dynamic behaviour analysis of Machines in transient regime on torsional vibration stresses, *Romanian Journal of Acoustics and Vibration*, vol. 10, issue 2, 2013, pp.129-134.

[23] Iancu V., Vasile O., Gillich G.R., Modelling and Characterization of Hybrid Rubber-Based Earthquake Isolation Systems, *Materiale Plastice*, vol. 49, Issue 4, 2012, pp. 237-241.

[24] Bratu, P., Vasile, O., Assessment of Dissipated Energy to Harmonic Cycles of Displacement for Visco-Elastic Elastomeric Anti-Seismic Insulators, *Romanian Journal of Acoustics and Vibration*, vol. 9, issue 2, 2012, pp.83-89.

.