Frequency Estimation using Spectral Techniques with the Support of a Deep Learning Method

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Abstract: - In the case of damage detection, it is important to estimate the frequencies accurately. DFT-based methods provide us with amplitude-frequency pairs, but displayed frequencies carry important errors in the case of short signals. On the other hand, the amplitudes displayed for a sinusoidal signal with different time lengths describe approximately a *sinc* function. When involving interpolation to find the maxima of the *sinc* function, to which the real amplitude of the signal corresponds, it is necessary to ensure the existence of at least three points on the main lobe of the *sinc* function. To this aim, we apply zero-padding to the original signal in such a way that its length is doubled. The frequency estimation method proposed in this paper involves an artificial neural network (ANN). The three amplitudes taken from the main lobe determined for all considered signal lengths are the input values used to train the network. The correction term that allows us to evaluate the frequency will be the target. Following the simulations made, it is found that using normalized data sets we can estimate any frequencies, irrespective of the frequency with which the network was trained. It applies to any signal length, and any signal amplitude.

Keywords: - frequency estimation, Discrete Fourier Transform, Artificial Neural Network, *sinc* function

1. INTRODUCTION

Used in various fields such as medicine, engineering, communications, or other domains [1], the frequency changes must be carefully monitored because they indicate important information regarding the improper functioning of some organs [2] or the appearance of some structural defects [3].

If in the case of long signals in time the accurate evaluation of the signal frequency is easy, in the case of short signals the standard methods, based on DFT or FFT do not provide precise results.

Various methods to improve frequency reading have been presented, involving the interpolation of two or three points from the DFT spectrum and the determination of the maximum interpolation curve, which allows to find the correct frequency [4-11].

In this paper, we use zero-padded signals [12] to ensure a higher number of points on the main lobe than they were initially from the DFT. We select the largest amplitude and its two neighbors and use this data for signals of different time lengths to train the Artificial Neural Network (ANN).

The proposed method allows us to accurately evaluate the natural frequencies of generated or measured signals, even if they have different frequencies than the training signals of different amplitudes.

2. THE STANDARD FREQUENCY ESTIMATION ALGORITHM

A vibration signal is acquired with analog sensors and afterward converted into a set of values by an analog-to-digital converter.

Analysis to assess the state of the system implies estimating the natural frequencies of the signal, hence identifying the harmonic components of the signal.

The time-history of a harmonic signal $a(t)$ is expressed, in the analog form, as follows

$$a(t)=A\sin(2\pi f_R t) \quad (1)$$

In this equation, A is the signal amplitude and f_R is the frequency of the signal.

If the signal is sampled with N samples and using a sampling rate f_S, the discrete signal $a[n]$ becomes a sequence

$$a[n] = A\,sin(\,2\pi f_R \frac{n}{f_S}),\; n = 0 \ldots N \quad (2)$$

The DFT converts the sequence $a[n]$ in the frequency domain, resulting in a set of N amplitudes displayed on k=0…N-1 spectral lines. The value calculated for the line k is:

$$A_k = \sum_{n=0}^{N-1} a[n][cos\left(\frac{2\pi}{N}nk\right) - j\,sin\left(\frac{2\pi}{N}nk\right)] \quad (3)$$

where j^2=-1.

Hence, every A_k can be decomposed into a real part $Re\,A_k$

$$Re\,A_k = \sum_{n=0}^{N-1} a[n]\,cos\left(\frac{2\pi}{N}nk\right) \quad (4)$$

and the imaginary part $Im\,A_k$.

$$Im\,A_k = -\sum_{n=0}^{N-1} a[n]\,sin\left(\frac{2\pi}{N}nk\right) \quad (5)$$

The magnitude displayed at the k-th spectral line of the DFT becomes

$$|A_k| = \sqrt{(Re\,A_k)^2 + (Im\,A_k)^2} \quad (6)$$

The spectral lines obtained in DFT are equidistant having the distance between them given by the frequency resolution

$$\Delta f = f_S\,/\,N = 1\,/\,t \quad (7)$$

Because in the first phase we have insufficient points on the main lobe (those with a green square) by using zero-padding these points multiply (those with a red square), which brings improvements to the possibility of using the proposed method.

This ensures that the point with the highest amplitude A_k and its neighbors with amplitudes A_{k-1} and A_{k+1} are on the same lobe. But the correct frequency is not indicated by the spectral line k, but it is in a position between two spectral lines, at a distance δ from the previous spectral line k-1, see Figure 1.

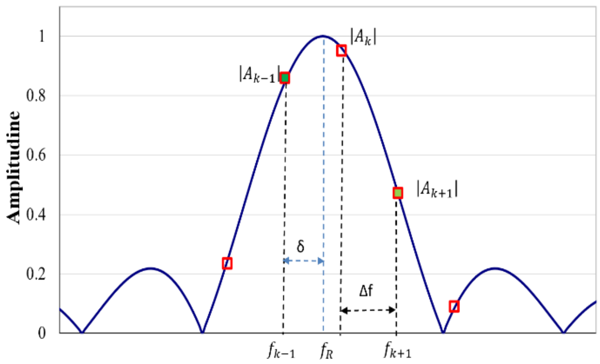

Figure 1. The DFT spectrum of the zero-padded signal

Thus the real frequency can be calculated with the help of the frequency f_{k-1} corresponding to the spectral line k-1 with the help of this correction factor δ as follows:

$$f_R = (k - 1 + \delta)\Delta f \quad (8)$$

Hence, the correction term result as

$$\delta = \frac{f_R - f_{k-1}}{\Delta f} \quad (9)$$

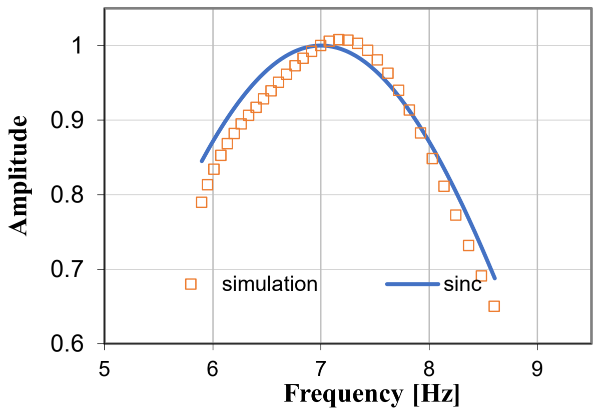

Figure 2. Comparison between the sinc function and maximizers achieved by simulation

We must note that if the signal contains a small number of cycles then the differences shown in Figure 2 make the *sinc* function unusable for interpolation.

3. DATABASE FOR TRAINING THE ANN

We aim to determine the correction coefficient δ for a generated signal having the amplitude A=1 and f_R=7 Hz, involving a sinusoid generated with N=2155 samples taken by a frequency rate of f_S=1000 Hz.

The shortest signal time length used for training is t_{min}=2.106 s, thus generated with N_{min}=2017 samples, while the longest signal time length is t_{max}=2.154 s, being generated with N_{max}=2155 samples.

Between the two time limits, we generate signals by repetitively adding 2 samples to the previous signal.

To double the number of spectral lines, we double the length of the signal in time through the zero-padding procedure.

With the help of an application developed in Python [19], we calculate the DFT for the zero-padded signal and select the maximum amplitude A_k and its two neighbors A_{k-1} and A_{k+1}.

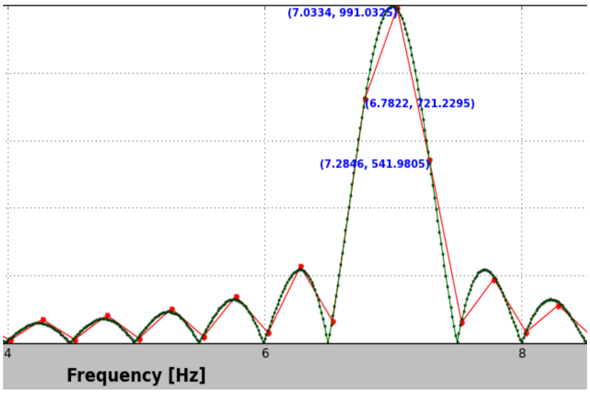

Figure 3. DFT spectrum for the signal with f_R =7Hz

Figure 3 shows the DFT spectrum for the analyzed signal. After selecting the three amplitudes we will normalize these values by dividing them by A_k, thus generalizing the data sets for any amplitude signal. These normalized values will constitute the INPUT values, and as TARGET values we will consider the correction terms δ determined using relation (8).

Examples of these values for signals of different lengths, having the frequency f_R=7Hz are given in Table 1

Table 1. Examples of Input and Target Data

N (samples)	INPUT			TARGET
	A_{k-1}	A_k	A_{k+1}	δ (-)
2155	0.54176623	1	0.74159122	1.163
2153	0.55915886	1	0.72178927	1.135
2151	0.57653578	1	0.70230579	1.107
2137	0.6990033	1	0.57444628	0.911
2135	0.71686704	1	0.55730014	0.883
2133	0.73490225	1	0.54039665	0.855
2111	0.95493054	1	0.36441802	0.547
2109	0.97797202	1	0.34864229	0.519

The complete set of INPUT and TARGET data are given in [19].

4. TRAINING OF THE NEURAL NETWORK

By using the data generated with the methods described in the previous section, a feed-forward backpropagation neural network was developed and trained, through the Matlab software, with its main parameters illustrated in Figure 4.

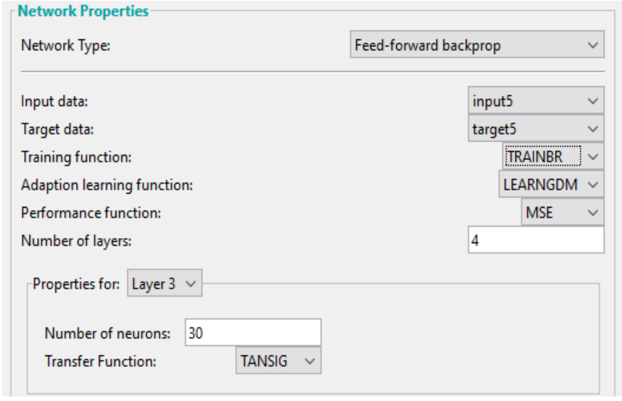

Figure 4. Main configuration of the ANN

Feed-forward neural networks consist of interconnected processing units that are grouped into several neuron layers in accordance with the biological model of the human brain. It contains layers of sensory neurons, layers of processing neurons (cerebral cortex), and layers of motor neurons [20, 21].

The neurons in the first layer are the only ones that receive signals from the outside, and this layer is called the input and is the only layer that contains "degenerate" neurons that have a transfer function.

Artificial neurons in the other layers (intermediate and output) are sigmoidal neurons, and there are no connections between neurons in the same layer [20]. The middle layers are called the hidden layer, and these neurons are directly connected to the neurons in the next layer. A feed-forward neural network can have one or more hidden layers. Feed-forward modeling applications use at least 2 hidden layers. The number of input neurons and the number of output neurons are generally required to be applied experimentally based on simulations [22, 23].

The network is trained using the Bayesian Regularization function, by employing a network of 3 hidden layers, each containing 30 neurons and one output layer, as shown in Figure 5.

In this study, Bayesian learning is used to avoid the phenomenon of network overfitting [24].

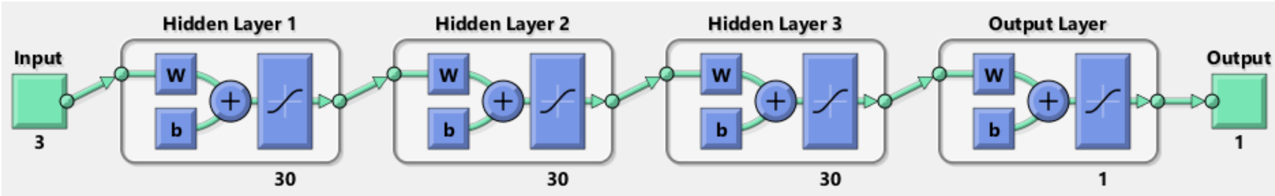

Figure 5. Network architecture

As input data, we have entered the values of the normalized amplitudes A_{k-1}/A_k, A_k/A_k, and A_{k+1}/A_k and the corresponding target values consisting of the correction terms δ for a defined number of acquisition samples.

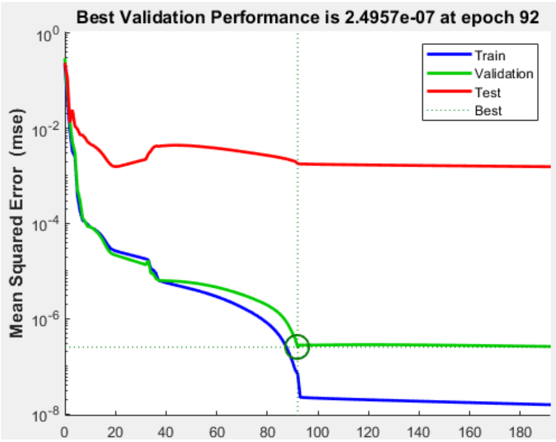

Figure 6. Network performance

The learning evolution and the performance of the network achieved in different moments of the training process are illustrated in Figure 6, in which three characteristic curves are displayed:

- learning rate with a blue line represents the progress / current state of the training at a certain moment in time.

- validation curve with a green line represents validation error, and when the network reaches the predetermined number of 100 validation errors, the drive stops.

- the test curve, marked with a red line, has an informative role.

To illustrate the efficiency of the training network the regression curves are plotted and shown in Figure 7. The resulting total regression coefficient shows the proportionality between the input and output values, therefore for good performance, it should be as close as possible to 1.

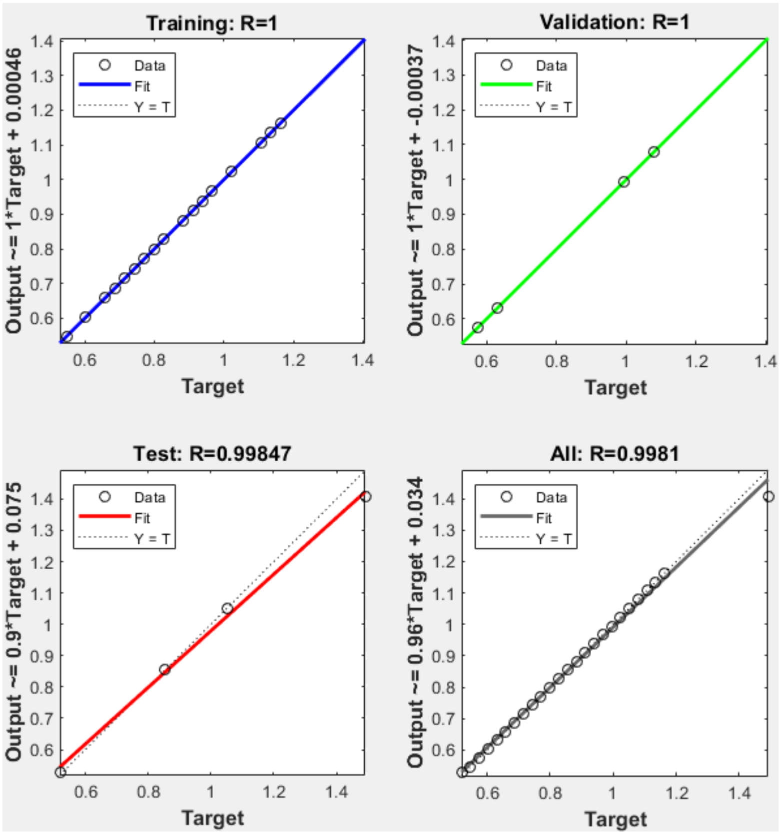

Figure 7. The regression coefficient for the trained network

5. VALIDATION OF THE PROPOSED TECHNIQUES

We test the accuracy of the method by involving four types of data:

- from a signal S_1 with the frequency equal to that of the signal used for training, which is $f_R = 7$ Hz, and the number of samples within the training interval;
- from a signal S_2 with the frequency equal to that of the signal used for training, which is $f_R = 7$ Hz, and the number of samples without the training interval;
- from a signal S_3 with a frequency $f_R = 5$ Hz different from that of the signal used for training and the number of samples within the training interval;
- from a signal S_4 with a frequency $f_R = 5$ Hz different from that of the signal used for training and the number of samples without the training interval.

All test signals are generated with a sampling rate of 1000 Hz, equal to the sampling rate of the signal used for training. Before calculating the DFT, we apply zero padding to double the length of the signal. Afterward, we identify the maximizer A_k and the two neighbors A_{k-1} and A_{k+1}. The three values are normalized, by division with A_k.

After estimating the correction term δ with help of the ANN, we calculate the frequency with the mathematical relation

$$f_O = f_{k-1} + \delta \Delta f \tag{10}$$

The frequencies estimated with the current method, which involves the ANN, are presented in tables 2 - 5.

Table 2. Test data and frequency estimation for the signal with A=1 and with the length in the training interval

N (samples)	Normalized amplitudes			Correction term	Obtained frequency	Error to the obtained frequency	Frequency from DFT	Error to the frequency from DFT
	A_{k-1}	A_k	A_{k+1}	δ (-)	f_O (Hz)	ε_1 (%)	f_k (Hz)	ε_2 (%)
2155	0.95830	1	0.36158	0.5413	4.99914	0.02	5.1055929	2.11
2151	0.99146	1	0.33952	0.519	5.00325	0.07	5.115089	2.30
2139	0.40570	1	0.90810	1.3638	4.99504	0.10	4.909983	1.80
2137	0.41867	1	0.89191	1.3478	4.99597	0.08	4.914579	1.71
2125	0.49574	1	0.79805	1.2377	4.99828	0.03	4.942339	1.15
2123	0.50845	1	0.78297	1.2179	4.99832	0.03	4.946996	1.06
2113	0.57138	1	0.71006	1.1169	4.99808	0.04	4.970414	0.59
2111	0.58386	1	0.69600	1.0966	4.99800	0.04	4.975124	0.50

Table 3. Test data and frequency estimation for the signal with A=1 and with the length outside the training interval

N (samples)	Normalized measured amplitudes			Correction term	Obtained frequency	Error to the obtained frequency	Frequency from DFT	Error to the frequency from DFT
	A_{k-1}	A_k	A_{k+1}	δ (-)	f_O (Hz)	ε_1 (%)	f_k (Hz)	ε_2 (%)
2097	0.67075	1	0.602505	0.9558	4.9978058	0.04	5.0083472	0.17
2093	0.6956	1	0.57734	0.916	4.997849	0.04	5.0179211	0.36
2081	0.77193	1	0.50553	0.7977	4.998245	0.04	5.0468637	0.94
2075	0.81161	1	0.47135	0.739	4.998553	0.03	5.0614605	1.23
2065	0.88158	1	0.41604	0.6411	4.999055	0.02	5.0859772	1.72
2061	0.91133	1	0.39423	0.6019	4.999247	0.02	5.0958505	1.92
2055	0.95832	1	0.36151	0.5412	4.999075	0.02	5.1107325	2.21
2053	0.99146	1	0.33954	0.519	5.00341	0.03	5.1157125	2.31

Table 4. Test data and frequency estimation for the signal with A=2 and with the length in the training interval

N (samples)	Normalized amplitudes			Correction term	Obtained frequency	Error for obtained frequency	Frequency from DFT	Error for frequency from DFT
	A_{k-1}	A_k	A_{k+1}	δ (-)	f_O (Hz)	ε_1 (%)	f_k (Hz)	ε_2 (%)
2155	0.95830	1	0.36158	0.5413	4.99914	0.02	5.105593	2.11
2151	0.99146	1	0.33952	0.519	5.00325	0.07	5.11509	2.30
2139	0.40570	1	0.90810	1.3638	4.99504	0.10	4.905396	1.80
2137	0.41867	1	0.89191	1.3478	4.99597	0.08	4.91458	1.71
2125	0.49574	1	0.79805	1.2377	4.99828	0.03	4.942339	1.15
2123	0.50845	1	0.78297	1.2179	4.99832	0.03	4.946996	1.06
2113	0.57138	1	0.71006	1.1169	4.99808	0.04	4.970414	0.59
2111	0.58386	1	0.69600	1.0966	4.99800	0.04	4.975124	0.50

Table 5. Test data frequency estimation for the signal with A=2 and with the length outside the training interval

N (samples)	Normalized amplitudes			Correction term	Obtained frequency	Error for obtained frequency	Frequency from DFT	Error for frequency from DFT
	A_{k-1}	A_k	A_{k+1}	δ (-)	f_O (Hz)	ε_1 (%)	f_k (Hz)	ε_2 (%)
2097	0.670757	1	0.602505	0.9558	4.997806	0.04	5.008347	0.17
2093	0.69566	1	0.57734	0.916	4.99784	0.04	5.017921	0.36
2081	0.77193	1	0.50553	0.7977	4.99824	0.04	5.046864	0.94
2075	0.81161	1	0.47135	0.739	4.99855	0.03	5.061461	1.23
2065	0.88158	1	0.41604	0.6411	4.99905	0.02	5.085977	1.72
2061	0.91133	1	0.39423	0.6019	4.99924	0.02	5.095850	1.92
2055	0.95832	1	0.36151	0.5412	4.99907	0.02	5.110733	2.21
2053	0.97470	1	0.350551	0.5191	4.99856	0.03	5.115712	2.31

In tables 2 – 5, we also indicate the frequencies calculated using the DFT method. The errors obtained for the two methods are determined as follows: the error ε_1 is found for the frequency obtained with the help of the correction term and the error ε_2 is found for the frequency obtained from the DFT. Analyzing these errors we can conclude that the method is efficient in accurately estimating the frequencies, because the error is below 0.1%, while with DFT the frequencies we obtained significantly higher errors.

4. CONCLUSIONS

The method we propose in this paper to estimate accurately the frequencies of a signal implies using a correction term determined with the help of an ANN. The method can be applied for any frequency, irrespective of the frequency used for training.

This is explainable by the fact that the correction term is a normalized one, the frequency being found after we multiply this term with the frequency resolution.

The results we achieved are extremely precise for all analyzed cases, the error being less 0.1%, both for test signals with the length within or without the range of the training data.

It is worth to be mentioned that the correction term is found automatically by the ANN from information regarding the relationship between the amplitudes on the main lobe of the spectrum.

We have demonstrated in prior research that, because we normalize the amplitudes in the DFT for training, the amplitudes of the analyzed signal must not be known a priori. The next attempts concern the effect of the sampling rate on the correction term.

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