Dynamic Modelling of Vibrating Equipment for Fine Grinding of Granular Material

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Abstract: - The process of fine grinding of the granular materials from crushed stone, marble or brittle solid oxide materials is carried out with vibrating equipment with two or more grinding chambers. The granular material is fed o the fine grinding chamber together with the spherical or cylindrical grinding bodies.

The vibration regime the whole assembly is subjected causes the material to be brought to a powder state with a very high degree of fineness in the range of (5÷10) μm.

The shredded material is used to coat the welding electrodes, the insulation of electrical conductor, the anti-corrosion coatings for special equipment as well as in the chemical and /or pharmaceutical industry.

The study includes the dynamic computational model and the foundation of the dynamic vibration regimes so that the technological grinding process may be realized.

Thus, the calculation formulas for the forced vibration amplitudes and working pulsations are given to achieve the optimal regime for achieving the fine grinding degree at particle size class level required by the industrial process technologies.

Thea results of the dynamic modelling, of the optimization of the calculation of the vibration parameters and of the correlation with the grinding fineness were applied for several equipment at the following companies: Ductil Buzău - Bucharest, Medicines Factory Iași, Cement Factory Bicaz-Tașca all in Romania.

Keywords: - dynamic model, vibrating equipment, fine vibratory grinding, dynamic regime, technological vibration

1. INTRODUCTION

The dynamic model study was carried out in such a way as to ensure the compatibility of the vibratory fine grinding process with the mechanical and rotating force excitation structure at the required parameters of the technological regime. For this purpose, several types of vibrating machines with one, two or more grinding chambers with spherical or cylindrical shredding bodies were analyzed.

Forced vibrations, in the technological regime, are generated by a rotating force at the required pulsation, based on two eccentric masses arranged at the ends of a horizontal shaft driven by a controllable rotating system.

The adopted calculation model is specific to a two-chamber vibrating machine in a mass-balanced system and between the two chamber there is located the rotating shaft with the two eccentric masses.

The entire vibrating assembly is rested on elastomer devices with the necessary elastic properties to isolate the vibrations transmitted to the machine foundation.

On the basis of data from several types of machines for crushed stone, marble, chalk, calcium oxide, iron, etc., the dynamic model was designed and the equations of motion in the vibratory regime were formulated. The dynamic model corresponds to the rigid body with four point of elastic leaning, with geometric and mass symmetry.

The result from the dynamic calculation were compared with the values of the experimentally measured parameters on four categories of vibrating equipment of the same dynamic class. [1,2,3]

2. ASSESSMENT OF DYNAMIC CALCULATION PARAMETERS

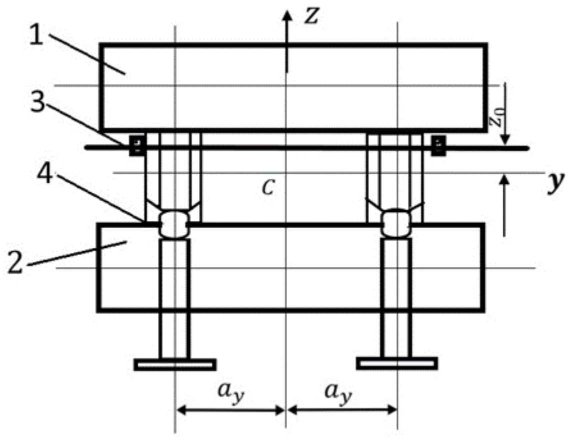
2.1. Dynamic model

The category of specialized technological machines for very fine grinding of granular materials also includes vibrating mills with grinding bodies (bars or balls). These machines are integrated in the technological flows of process industries, where it is necessary to produce very fine powders (5-10 μm) from ferroalloys, small stones or marble chips, chalk, etc.

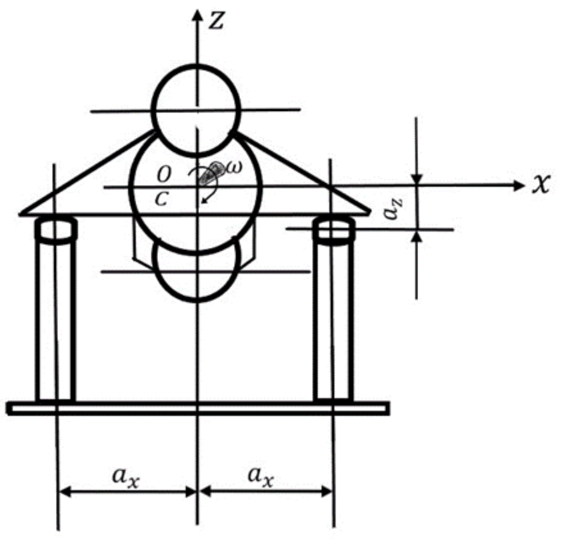
The vibrating mill consists of several main parts (fig.1); as follows:

- grinding chambers *1,2*, identical, arranged symmetrically to the longitudinal axis *Cy,* where *C* is the mass center of the system;
- inertial vibro-generator3, with rotating force whose axis passes through *O* and is parallel to the *Cy* axis, where *O* is the center of the disturbing force;
- vibration leaning system *4*, consisting of elastomeric anti-vibration elements.

The dynamic study of vibrating mills must solve the following fundamental problems, namely: the dynamic regime for technological and transmitted vibrations and the elastic leaning to ensure vibration isolation at the foundation [4,5].



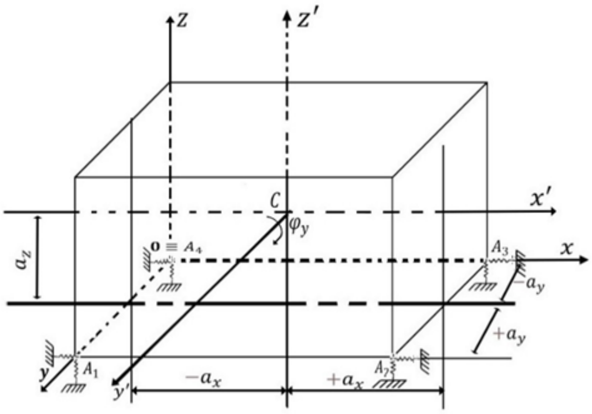
a) Side view



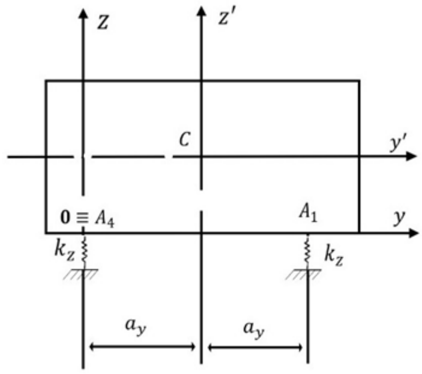
b) Transversal view

Figure 1. Constructive schematic of the equipment

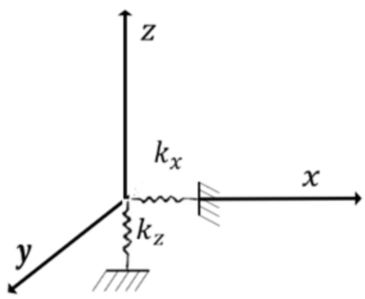
Taking into account the constructive and technological peculiarities of vibrating mills with two grinding chambers, the dynamic calculation model schematized in figure 2 was adopted. [6]



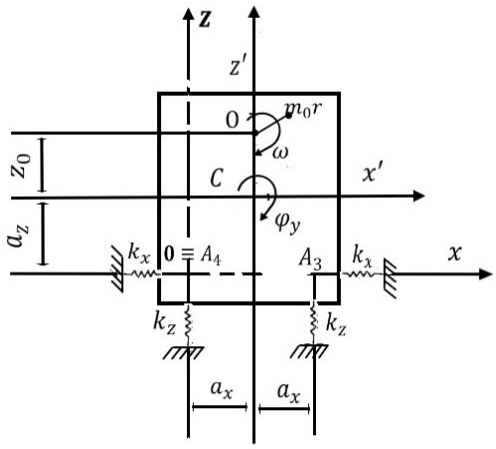
a) Spatial representation (3D)



b) Front representation (2D)



c) Bi-dimensional representation elastic leaning



d) Side representation (2D)

Figure 2. Dynamic model

The machine operates in post-resonance according to all degrees of freedom i.e., $\omega = (3...7)\, p_j$, where ω is the pulsation of the disturbing force and p_j, the Eigen pulsation corresponding to the j order vibration mode, which is why the dissipative forces have been neglected in the calculation model because their influence by the centrifugal force of inertia, whose direction does not pass through the mass center C of the system. Also, the case when the horizontal plane of the elastic leaning elements does not contain the center of mass C of the system was considered. In this case, the calculation model is schematized as a rigid body with two vertical planes of symmetry, elastically leaning on identical elements, perturbed outside the center of mass [7,8,9].

2.2. Dynamic calculation parameters

The displacement vectors of points A1, A2, A3, A4, with respect to the inertial system, considered fixed, Oxyz shall be determined taking into account the coordinates of the points of the elastic elements, as follows:

A1 (-ax , + ay, - az);
A2 (+ax , + ay, - az);
A3 (+ax , - ay, - az);
A4 (-ax , - ay, - az).

Thus, we have:

$$u_1 = \begin{Bmatrix} x \\ 0 \\ z \end{Bmatrix} + \begin{bmatrix} 0 & 0 & \varphi_y \\ 0 & 0 & 0 \\ -\varphi_y & 0 & 0 \end{bmatrix} \begin{Bmatrix} -a_x \\ +a_y \\ -a_z \end{Bmatrix} = \begin{Bmatrix} x \\ 0 \\ z \end{Bmatrix} + \begin{Bmatrix} -\varphi_y a_z \\ 0 \\ \varphi_y a_x \end{Bmatrix}$$

$$u_2 = \begin{Bmatrix} x \\ 0 \\ z \end{Bmatrix} + \begin{bmatrix} 0 & 0 & \varphi_y \\ 0 & 0 & 0 \\ -\varphi_y & 0 & 0 \end{bmatrix} \begin{Bmatrix} a_x \\ a_y \\ -a_z \end{Bmatrix} = \begin{Bmatrix} x \\ 0 \\ z \end{Bmatrix} + \begin{Bmatrix} -\varphi_y a_z \\ 0 \\ -\varphi_y a_x \end{Bmatrix}$$

$$u_3 = \begin{Bmatrix} x \\ 0 \\ z \end{Bmatrix} + \begin{bmatrix} 0 & 0 & \varphi_y \\ 0 & 0 & 0 \\ -\varphi_y & 0 & 0 \end{bmatrix} \begin{Bmatrix} a_x \\ -a_y \\ -a_z \end{Bmatrix} = \begin{Bmatrix} x \\ 0 \\ z \end{Bmatrix} + \begin{Bmatrix} -\varphi_y a_z \\ 0 \\ -\varphi_y a_x \end{Bmatrix}$$

$$u_4 = \begin{Bmatrix} x \\ 0 \\ z \end{Bmatrix} + \begin{bmatrix} 0 & 0 & \varphi_y \\ 0 & 0 & 0 \\ -\varphi_y & 0 & 0 \end{bmatrix} \begin{Bmatrix} -a_x \\ -a_y \\ -a_z \end{Bmatrix} = \begin{Bmatrix} x \\ 0 \\ z \end{Bmatrix} + \begin{Bmatrix} -\varphi_y a_z \\ 0 \\ \varphi_y a_x \end{Bmatrix}$$

The potential deformation energy of the elastic elements is

$$2\mathrm{V} = k_x\,(u_{1x}^2 + u_{2x}^2 + u_{3x}^2 + u_{4x}^2) + k_z\,(u_{1z}^2 + u_{2z}^2 + u_{3z}^2 + u_{4z}^2)$$

where:

$$u_{1x} = u_{2x} = u_{3x} = u_{4x} = x - \varphi_y a_z$$
$$u_{1z} = u_{4z} = z + \varphi_y a_z$$
$$u_{2z} = u_{3z} = z - \varphi_y a_z$$

By replacing u_{jx} , u_{jz}:

$$2V = 4\,k_x\,(x - \varphi_y a_z)^2 + 2\,k_z\,(z + \varphi_y a_x)^2 + 2\,k_z\,(z - \varphi_y a_x)^2, \quad (1)$$

from where the generalized forces corresponding to the elastic forces, are

$$Q_x = -\frac{\partial V}{\partial x} = -4k_x(x - \varphi_y a_z);$$
$$Q_\varphi = -\frac{\partial V}{\partial \varphi_y} = -4k_x a_z x - 4(k_x a_z^2 + k_z a_x^2)\varphi_y;$$
$$Q_z = -\frac{\partial V}{\partial z} = -4k_z z$$

The kinetic energy of the system is

$$2E = m\dot{x}^2 + J_y\,\dot{\varphi}^2{}_y + m\dot{z}^2 \quad (2)$$

and the generalized force corresponding to the inertial disturbing force for x, φ_y and z is

$$Q_{Fx} = P_0 \cos\omega t = \mathrm{m}_0 r \omega^2 \cos\omega t$$
$$Q_{F\varphi} = z_0 P_0 \cos\omega t$$
$$Q_{Fz} = P_0 \sin\omega t$$

For the elastic system model as per figure 2, the system of differential equations of movement is:

$$m\ddot{X} + 4k_x X - 4a_z k_x \varphi_x = P_0 \cos\omega t$$
$$J_y \ddot{\varphi}_y + 4\varphi_y(k_z a_x^2 + k_z a_x^2) - 4k_x a_z X = z_0 P_0 \cos\omega t \quad (3)$$
$$m\ddot{Z} + 4k_z Z = P_0 \sin\omega t$$

in which: m is the total (suspended) mass of the machine; J_y – the moment of inertia of the machine relative to inertia of the machine relative to C_y axis; k_x- the coefficient of rigidity of an elastic element in the C_x direction; k_z-the coefficient of rigidity of an elastic element in the C_z direction; a_x-the distance measured vertically between the horizontal plane containing the elastic elements and the position of the mass center; z_o-the distance measured vertically, in the longitudinal median plane, between the center of mass C and the center of perturbation O; P_o- the amplitude of the inertial disturbance force as $P_o=m_o r\omega^2$; ω-pulsation of the disturbing force; m_o- the total eccentric mass relative to the axis of rotation of the dynamic unbalance elements of the vibrator; r-the distance between the axis of rotation and the center of mass of the dynamic unbalance elements of the vibrator; $m_o r$-the total static moment of the dynamic unbalance elements of the vibrator.[10,11,12]

For the decoupled mode of the vertical translation vibrations, the Eigen pulsations of the system are given by the relations:

$$\omega_z^2 = 4\frac{k_z}{m} \tag{4}$$

For the coupled modes (X, φ_y) of the translation vibrations along axis Cx' and rotation about the Cy' axis, we have:

$$\frac{p_{x\varphi y}^2}{\omega_z^2} = \frac{1}{2}\left\{\frac{k_x}{k_z}\left(1+\frac{a_z^2}{\rho_y^2}\right)+\frac{a_z^2}{\rho_y^2} \pm \sqrt{\left[\frac{k_x}{k_z}\left(1+\frac{a_z^2}{\rho_y^2}\right)+\frac{a_z^2}{\rho_y^2}\right]^2-4\frac{k_x}{k_z}\left(\frac{a_z^2}{\rho_y^2}\right)^2}\right\} \tag{5}$$

where ρ_y is the gyration radius given by formula $J_y=m\rho_y^2$

For the regime of forced vibrations, the solutions of the differentiated equations will be harmonic as the right side, with the amplitudes of the displacements given by the relations:

$$A_x = \frac{m_0 r\omega^2}{4k_z}\frac{\frac{k_x a_z}{k_z\rho_y}\left(\frac{a_z}{\rho_y}-\frac{z_0}{\rho_y}\right)+\left(\frac{a_x}{\rho_y}\right)^2-\left(\frac{\omega}{\omega_z}\right)^2}{\left(\frac{\omega}{\omega_z}\right)^4-\left[\frac{k_x}{k_z}+\frac{k_x}{k_z}\left(\frac{a_z}{\rho_y}\right)^2+\left(\frac{a_x}{\rho_y}\right)^2\right]\left(\frac{\omega}{\omega_z}\right)^2+\frac{k_x}{k_z}\left(\frac{a_x}{\rho_y}\right)} \tag{6}$$

$$A_\varphi = \frac{m_0 r\omega^2}{4k_z\rho_y}\frac{\frac{k_x a_z}{k_z\rho_y}\left(\frac{a_z}{\rho_y}-\frac{z_0}{\rho_y}\right)+\frac{z_0}{\rho_y}\left(\frac{\omega}{\omega_z}\right)^2}{\left(\frac{\omega}{\omega_z}\right)^4-\left[\frac{k_x}{k_z}+\frac{k_x}{k_z}\left(\frac{a_z}{\rho_y}\right)^2+\left(\frac{a_x}{\rho_y}\right)^2\right]\left(\frac{\omega}{\omega_z}\right)^2+\frac{k_x}{k_z}\left(\frac{a_x}{\rho_y}\right)} \tag{7}$$

$$A_z = \frac{m_0 r\omega^2}{4k_z}\frac{1}{1-\left(\frac{\omega}{\omega_z}\right)^2} \tag{8}$$

2.3. Parameters of technological vibrators

In order to achieve the fine grinding degree of granular materials it is necessary to analyze the following technological parameters:

a) The coefficient of vertical throwing of material Γ, also called the machine coefficient, is given by the relation

$$\Gamma = \frac{A_z\omega^2}{g} \tag{9}$$

with values in the range 6...12;

b) the frequency of the technological vibrations must have values within the range (15...50) Hz, depending on the granular material and the grinding fineness;

c) the vertical amplitude of the technological vibrations are in the range (3...10) mm.

For a favorable operation of the machine, it is necessary that the vertical vibratory motion, considered technological motion, be predominant, requiring the analysis of two variants, namely:

- the machine is "centered", that is the center of disturbance O coincides with the center of mass C;
- the machine is "centered" and "balanced", that is the horizontal plane of the center of mass, identical to that of the center of disturbance, contains the anti-vibration elements (C=0 and a_z=0)

In the case of the **centered machine**, the condition z_o = 0 implies the following relations for the amplitudes A_z and A_φ.

$$A_z = \frac{m_0 r\omega^2}{4k_z}\frac{\frac{k_x}{k_z}\left(\frac{a_z}{\rho_y}\right)^2+\left(\frac{a_x}{\rho_y}\right)^2-\left(\frac{\omega}{\omega_z}\right)^2}{D} \tag{10}$$

$$A_\varphi = \frac{m_0 r\omega^2}{4k_{z-y}} : \frac{\frac{k_x a_z}{k_z\rho_y}}{D} \tag{11}$$

where

$$D = \left(\frac{\omega}{\omega_z}\right)^4-\left[\frac{k_x}{k_z}+\frac{k_x}{k_z}\left(\frac{a_z}{\rho_y}\right)^2+\left(\frac{a_x}{\rho_y}\right)^2\right]\left(\frac{\omega}{\omega_z}\right)^2+\frac{k_x}{k_z}\left(\frac{a_x}{\rho_y}\right)$$

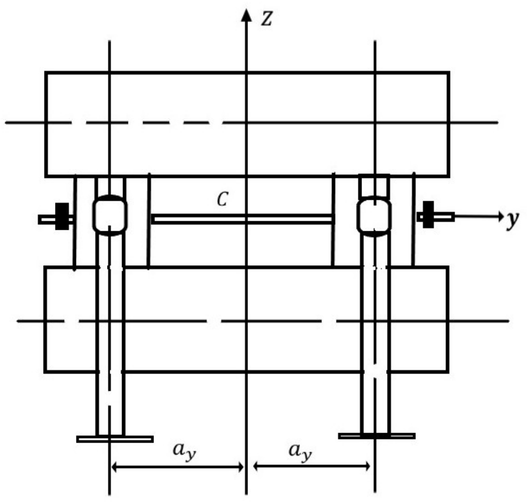
2.4. Insulation of the vibrations transmitted to the foundation

In the case of the **centered and balanced machine** (fig.3) we have the conditions z_o=0 and for a_z=0, implying A_ =0, and for A_x the relation:

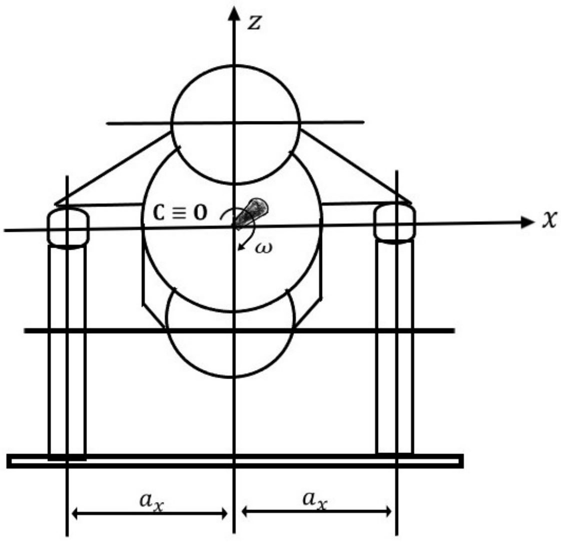
$$A_x = \frac{m_0 r\omega^2}{4k_z}\frac{\left(\frac{a_x}{\rho_y}\right)^2-\left(\frac{\omega}{\omega_z}\right)^2}{D} \tag{12}$$

It is specified that both cases the amplitude A_z is given by the relation (8).

Parameters of the anti-vibration insulation system. The anti-vibration insulation system consists of four rubber elements, of annular cross-section, connected in parallel, stressed in vertical compression and shear in the horizontal plane (fig.3), namely the machine is centered [13,14,15].



a) side view



b) transversal view (front)

Figure 3. Centered and dynamically balanced vibrating equipment

The amplitude of the dynamic force F_{Tx} that is transmitted to the foundation through the four elastic elements on direction x is given by the relation.

$$F_{Tx} = 4k_z\sqrt{A_x^2 + a_z^2 A_\varphi^2} \quad (13)$$

and the degree of proper insulation is calculated as follows:

$$I_x = 1 - \frac{F_{Tx}}{P_0} \quad (14)$$

with values comprised between 0,70 ... 0,95.

3. PARAMETRIC VARIATION CURVES FOR A VIBRATING MACHINE

On the basis of relations (6), (7), (8), (13), (14), (15) and (16), the variation curves for the vibration regime of a vibratory grinding machine with the following technical characteristics were set: m=5000 kg; J_y=1500 kg m^2; k_x=k_y=2·10^6 N/m; a_x=1,0 m; a_y=1,2 m; a_z=0,75 m; z_o=0,15 m; $m_o r$ =1,0 m; ω=(0 ... 500) rad/s.

In Figures 4,5 and 6 the variation curves of the amplitudes A_x, A_y, A_z according to the continuous variation of the excitation pulsation ω, were presented.

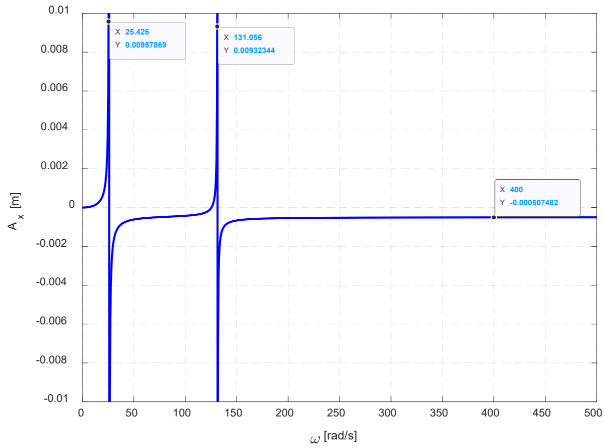


Figure 4. Variation of A_x in relation to ω

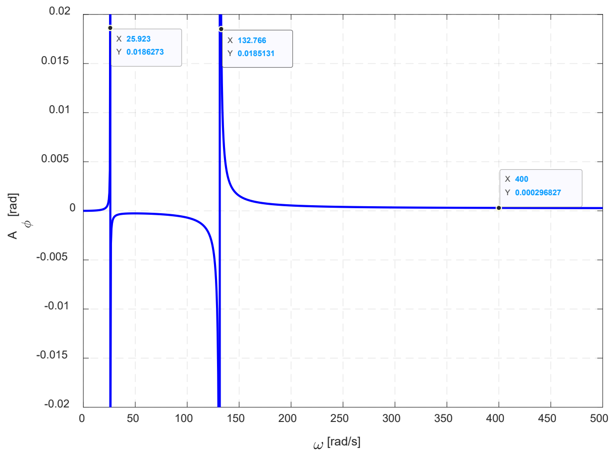


Figure 5. Variation of A_Φ in relation to ω

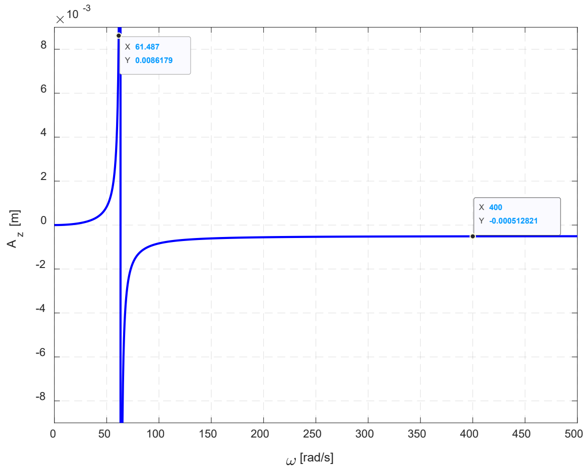


Figure 6. Variation of Az in relation to ω

Figure 7 and 8 present the dynamic forces and in relation to the variation of ω, which are transmitted to the foundation of the vibrating machine.

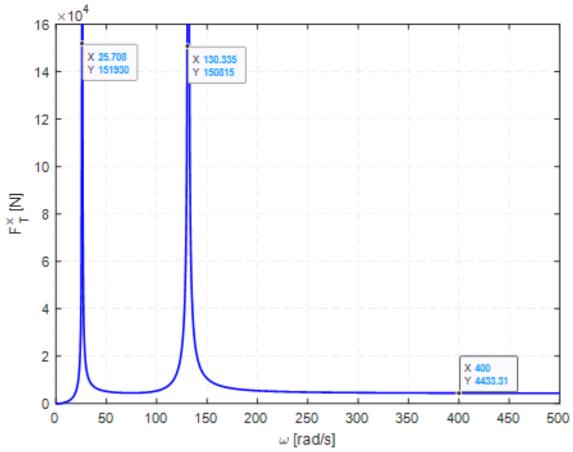


Figure 7. Variation of F_T^x in relation to ω

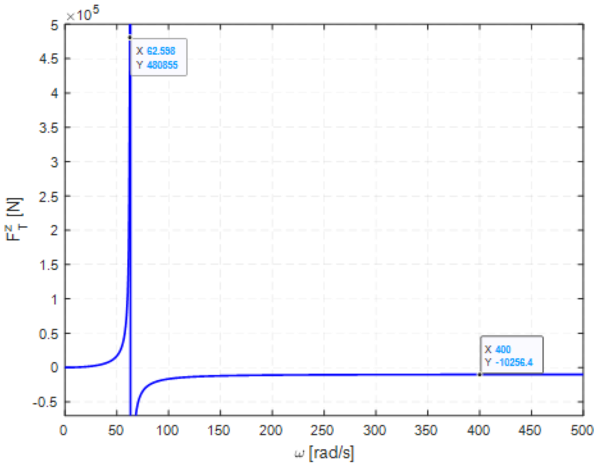


Figure 8. Variation of F_T^z in relation to ω

Figures 9 and 10 show the dimensionless marks I_x and I_z in relation to the variation of ω signifying the degree of dynamic insulation in the x and z degrees of freedom directions.

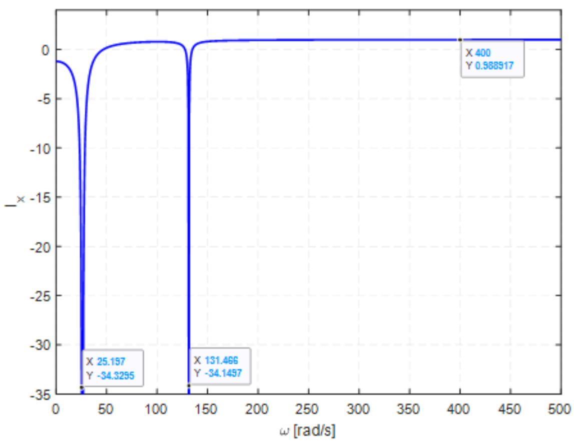


Figure 9. Variation of I_x in relation to ω

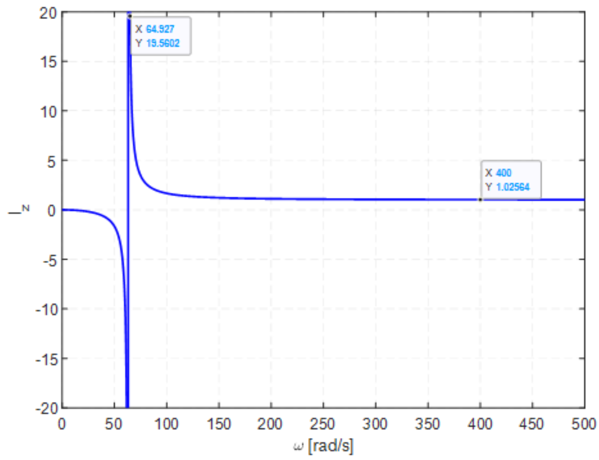


Figure 10. Variation of I_z in relation to ω

The amplitude of the dynamic force that is transmitted to the foundation through the four elastic elements in the z direction is

$$F_{Tz} = 4k_z A_z \tag{15}$$

and the corresponding degree of insulation can be calculated as follows

$$I_z = 1 - \frac{F_{Tz}}{P_0} \tag{16}$$

This shall be included between 0,80 ... 0,95.

4. CONCLUSIONS

On the basis of the calculation relations presented and the values of the required technological parameters the following conclusions can be summarized:

a) the motion of the uncentred and unbalanced machine has three degrees of freedom, two of which are coupled in the *x*, φ_y directions;
b) the most rational technical solution is to adopt the constructive variant with the centered and balanced machine (fig.3) for which the calculation relations (8) and (11) for amplitudes are valid;
c) the vibration insulation parameters can be calculated with the formulae (14) and (16);
d) the most convenient post-resonance dynamic regime results from figures 4...10, for $\omega > 3p_j$, where p_j is the self-pulsation by the j=1,2,3... degree of freedom.

The determination of the dynamic parameters, the maximum disturbing force P_o, the technological amplitude A_z, the technological pulsation ω, is conducted taking into account the synthesis parameter Γ given by formula (9) and by the recommendation that the system should work in post-resonance, that is ω = (3...7) p_j, where p_j is the self-pulsation of the system according to modes *x, z* and φ_y.[16,17]

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