Analysis of Vibration Suppression in Multi-Degrees of Freedom Systems

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Abstract: In the paper, we propose to show that a complex mechanical system, with elastic elements, subject to periodic excitations, can function as a dynamic absorber for some frequencies. A study of this problem allows determining the conditions under which this occurs. Inserting a specially built absorber to absorb a certain frequency can be an expensive decision for the user. This disadvantage can be easily removed if more attention is paid to the design phase. In the paper we present how this can be done through the judicious choice of some constructive parameters and the results are illustrated with an example. The advantage of this approach is the fact that the attachment of an additional dynamic absorber in the system is avoided, which, without changing the response of the already designed system, involves additional costs. The obtained results can be a starting point for the development of interesting applications in the field of suppressing unwanted vibrations in industrial mechanical systems.

Keywords: - multi-degree of freedom; vibration; mechanical system; dynamic absorber (DVA);

1. INTRODUCTION

The problem of vibrations is one of the main problems that mechanical engineers have to solve when designing and building a machine. There are many ways to reduce these vibrations, passive or active. Among all these methods, the use of dynamic vibration absorbers (DVA) stands out, a simple method that involves relatively low costs. The applicability of the process has been demonstrated in mechanical, civil and automotive engineering, aeronautics, naval engineering as well as other fields. The first dynamic absorber was patented by Frahm [1] in 1909 and the theoretical justification of its use is given by Ormondroyd and Den Hartog [2] in 1928. The practical applicability with low costs represents the main advantage of this system, reason for which it was widely used and there are numerous studies on DVA. Obviously, the design of a DVA requires the knowledge and optimization of some parameters.

Interdisciplinary studies that propose the use of magnetic force to achieve a dynamic absorption are addressed in [3]. In this way, a control of this phenomenon is ensured in rotating systems. In the work, the use of Jeffcott vibration absorption systems is proposed. The determination of the optimum parameters for the absorber is done with the classical theory of vibrations. Experimental determinations were made to validate the proposed solutions. The verification of the theoretical results was obtained in this way.

A semi-active suspension of a seat used in a work machine is proposed in [4]. Two possible versions are studied, the first one using a magneto-rheological damper, the second a complex system formed by a magneto-rheological damper and a classic DVA.

In general, a DVA allows the suppression of a single disturbing frequency. A solution that allows the suppression of three frequencies, which is achieved by changing some geometric dimensions, is studied in [5]. In this way, the installation of three DVAs is avoided, an operation that requires additional costs. In this way, DVA can be used, depending on the frequency that is desired to be suppressed, for one of the three frequencies that it is capable of suppressing. The validity of the proposed design is proved using numerical examples. An elastic continuum is used for the seismic impact protection of a construction in [6]. This DVA has several natural frequencies that are in the frequency range of seismic effects. If the natural frequency of the DVA is close to the building's excitation frequency, a good part of the building's oscillation energy is transferred to the DVA and decreases the peak level of the building's frequency response function. in [7] a particular problem is studied, that of preventing the overturning of a rigid block. It is recommended to use a classic pendulum shock absorber for this. The geometric characteristics are made up of parameters that can vary. An experimental validation of the proposed solution is made.

In the case of designing devices that operate at supercritical speeds, from the moment of starting the machine must sweep the entire range of speeds, so it must also pass through the critical ones, an event that can lead to malfunctions. The result is the need to design DVAs that work in the area of resonance pulsations. To achieve this objective, a rotating DVA with a viscoelastic element is studied in [8]. In this way, a smooth transition is obtained when the device is turned on, from zero to supercritical speed.

A numerical simulation for a system with two degrees of freedom equipped with a nonlinear variable frequency DVA is presented in [9].

The use of the magneto-rheological elastomer (a smart materials with elastic property variable in the external magnetic field) were developed in the last period to propose a new solution for DVAs [10,11]. The papers [12,13] present detailed studies on the modeling of DVA systems. Other results regarding the design and tuning of these dynamic absorption systems are presented in [14-20]. A work that signals the possibility of a mechanical system becoming absorbent is [21]. Based on this paper and on different theoretical study of the problem of DVA [22-31], the model of a multi degrees of freedom mechanical system is studied below.

The problem we propose to analyze is whether a mechanical system with several degrees of freedom can become a dynamic absorber for certain frequencies. In this way, the way is opened so that, through a judicious design of a mechanical system with several variable parameters, we can achieve a dynamic absorption of vibrations without using additional elements, i.e. classic DVAs.

Obviously, the design of a system without absorbers is simplified, no longer having to design, tune and realize the DVA.

2. MODEL AND METHOD

In the following, it is considered an elastic mechanical system, which has a vibration movement. Damping is neglected to simplify the presentation. The considered system has 6 degrees of freedom and is excited with a harmonic force with ω pulsation. The system vibrates according to a law of motion given by the system of differential equations [32]:

$$\boldsymbol{M}\ddot{\boldsymbol{X}} + \boldsymbol{K}\boldsymbol{X} = \boldsymbol{F}\cos\omega\boldsymbol{t}\,, \tag{1}$$

In these equations, $\boldsymbol{M}$ represents the mass matrix, $\boldsymbol{K}$ is the stiffness matrix and $\boldsymbol{F}$ is the vector of harmonic excitation forces. It will be assumed that the exciting force acts only on one element of the system, let's say on element 1. The problem we ask ourselves is whether the studied system can become a dynamic absorber for a disturbing frequency. And, if it is possible, what are the conditions that the system must fulfill in order to achieve this. The problem arises of determining the conditions under which this can happen. We partition the matrices that appear in equation (1) as follows:

$$\begin{bmatrix} \boldsymbol{M}_{11} & \boldsymbol{M}_{12} \\ \boldsymbol{M}_{21} & \boldsymbol{M}_{22} \end{bmatrix} \begin{Bmatrix} \ddot{\boldsymbol{X}}_1 \\ \ddot{\boldsymbol{X}}_2 \end{Bmatrix} + \begin{bmatrix} \boldsymbol{K}_{11} & \boldsymbol{K}_{12} \\ \boldsymbol{K}_{21} & \boldsymbol{K}_{22} \end{bmatrix} \begin{Bmatrix} \boldsymbol{X}_1 \\ \boldsymbol{X}_2 \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{F}_1 \\ \boldsymbol{F}_2 \end{Bmatrix} \boldsymbol{cos\,\omega t}\,, \tag{2}$$

The vector $\boldsymbol{X}_1 = \lfloor x_1 \quad x_2 \quad \dots \quad x_p \rfloor$ contains the first p masses of the system (with the dimension $p \times 1$) and $\boldsymbol{X}_1 = \lfloor x_{p+1} \quad \dots \quad x_n \rfloor$ the next *n-p* masses (with the dimension (*n-p)* × *1*). $\boldsymbol{M}_{11}, \boldsymbol{K}_{11}$ have the dimensions $p \times p$, $\boldsymbol{M}_{12}, \boldsymbol{K}_{12}$ have the dimensions $p \times (n-p)$, the matrices $\boldsymbol{M}_{21}, \boldsymbol{K}_{21}$ have the dimension *(n-p) x p*, and finally $\boldsymbol{M}_{22}$, $\boldsymbol{K}_{22}$ the dimension $p \times p$. To solve the Eq.(2) the solution is chosen in the form:

$$\begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{\Phi}_1 \\ \boldsymbol{\Phi}_2 \end{Bmatrix} cos\,\omega t, \quad (3)$$

Differentiating two times, it is obtained:

$$\begin{Bmatrix} \ddot{X}_1 \\ \ddot{X}_2 \end{Bmatrix} = -\omega^2 \begin{Bmatrix} \boldsymbol{\Phi}_1 \\ \boldsymbol{\Phi}_2 \end{Bmatrix} cos\,\omega t, \quad (4)$$

Introducing conditions (3) in Eq. (2), it results:

$$\begin{bmatrix} K_{11} - \omega^2 M_{11} & K_{12} - \omega^2 M_{12} \\ K_{21} - \omega^2 M_{21} & K_{22} - \omega^2 M_{22} \end{bmatrix} \begin{Bmatrix} \boldsymbol{\Phi}_1 \\ \boldsymbol{\Phi}_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}, \quad (5)$$

and, developing:

$$\begin{matrix} \left(K_{11} - \omega^2 M_{11}\right)\boldsymbol{\Phi}_1 + \left(K_{12} - \omega^2 M_{12}\right)\boldsymbol{\Phi}_2 = F_1 \\ \left(K_{21} - \omega^2 M_{21}\right)\boldsymbol{\Phi}_1 + \left(K_{22} - \omega^2 M_{22}\right)\boldsymbol{\Phi}_2 = F_2 \end{matrix}, \quad (6)$$

Our conditions are expressed in the following form: if $F_2 = 0$ (there is no excitation on the second part of the system) it is imposed that $\boldsymbol{\Phi}_1 = 0$ (there is not motion of the first part of the system). Using these conditions in system (6), it is obtained:

$$\left(K_{12} - \omega^2 M_{12}\right)\boldsymbol{\Phi}_2 = F_1, \quad (7)$$

$$\left(K_{22} - \omega^2 M_{22}\right)\boldsymbol{\Phi}_2 = 0. \quad (8)$$

It is denoted:

$$C_{22} = K_{22} - \omega^2 M_{22}. \quad (9)$$

For Eq. (8) to admit a non-zero solution, it must have:

$$P(\omega) = det\left(K_{22} - \omega^2 M_{22}\right) = \{0\}, \quad (10)$$

The Eq. (10) represents a first condition.

Considering the theory of linear equations we can find a normalized solution $\boldsymbol{\Phi}_2$ of Eq. (8) [34-38], be it Y. Considering this solution, any other vector $X = \lambda Y$ verifies the system (8), and (7) can be written:

$$\left(K_{12} - \omega^2 M_{12}\right)\lambda Y = F_1, \quad (11)$$

Eq. (11) are p linear equations that represents p conditions to keep at bodies. It is possible to eliminate λ from system multiplying Eq (11) by Y^T :

$$\lambda Y^T\left(K_{12} - \omega^2 M_{12}\right)Y = Y^T F_1, \quad (12)$$

and obtaining λ:

$$\lambda = \frac{Y^T F_1}{Y^T\left(K_{12} - \omega^2 M_{12}\right)Y}. \quad (13)$$

If it is introduced λ into Eq (11) it obtains:

$$\left(K_{12} - \omega^2 M_{12}\right)\frac{Y^T F_1}{Y^T\left(K_{12} - \omega^2 M_{12}\right)Y} Y = F_1 \quad (14)$$

Eq. (14) represents a set of p conditions but only *p-1* are independents. Condition (10) togheter with (14) represents 1+*p*-1=*p* conditions that must be fulfilled by the parameters of the system to obtain the desired result. So, the first p masses remain at rest.

Consider now a machine that, powered by a motor (usually electric) rotating at a speed ω. It is desired to have no vibration of mass 1, so to dynamically isolate a single mass m_1 and then $p = 1$. The matrix K_{11} has size *(n-1)* × *(n-1)*, the vector F_1 has only one element F, the condition that the geometric, mass and elastic quantities must fulfill are given by $det\left([K_{22}] - \omega^2 [M_{22}]\right) = 0$ (to have zero displacement of mass *1* at frequency ω). So: $X_1 = x_1$,

$$\begin{bmatrix} k_1 - \omega^2 m_1 & K_{12} - \omega^2 M_{12} \\ K_{21} - \omega^2 M_{21} & K_{22} - \omega^2 M_{22} \end{bmatrix} \begin{Bmatrix} x_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ 0 \end{Bmatrix}, \quad (15)$$

or:

$$\left(K_{12} - \omega^2 M_{12}\right)X_2 = F_1, \quad (16)$$

$$\left(K_{22} - \omega^2 M_{22}\right)X_2 = 0, \quad (17)$$

In order to have a non-zero solution of system (16), condition (10) must be fulfilled, that is, the determinant of the system must be equal to zero. Consider a normalized solution of system (16), denote it by Y_2. Then any vector $X_2 = \lambda Y_2$ is a solution of system (16) an Eq.(16) become:

$$\left(K_{12} - \omega^2 M_{12}\right)\lambda Y_2 = F_1, \quad (18)$$

which provides λ. So it is possible to determine the amplitudes of the forced oscillations of the other *n-1* masses.

3. RESULTS AND DISCUSSIONS

The work aims to show that, through a judicious dimensioning of the system, from the design phase, it can become a dynamic absorber for certain frequencies. The main advantage of such a system is the fact that it is no longer necessary to attach one (or several) dynamic shock absorbers, which would suppress the vibrations of certain excitation frequencies.

A simple example can illustrate this (Figure 1). The system is made by masses, linked together with elastic elements having known stiffness. Flywheels have translational vibration along the vertical direction. An exciting force acts on the flywheel 1. The aim is to determine the conditions in which the mass 1 will be at rest if an excitation force acts on the mass1. This system has 6 DOFs.

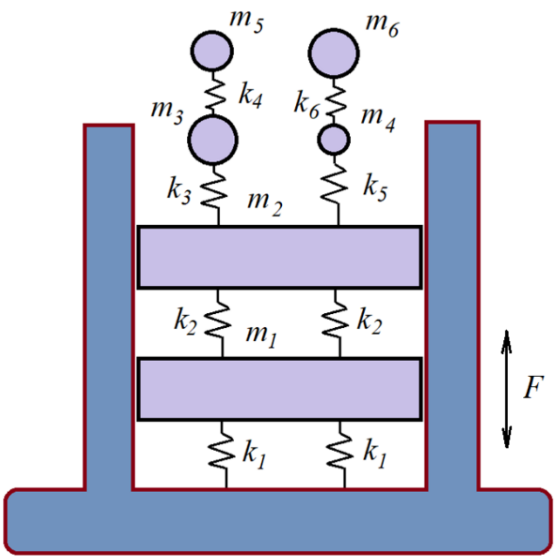

Figure 1. Elastic system

$$
\begin{aligned}
&m_1\ddot{x}_1+2k_1x_1+2k_2x_1-2k_2x_2=0 \quad ;\\
&m_2\ddot{x}_2+(2k_2+k_3+k_5)x_2-k_3x_3-k_5x_4=0 \quad ;\\
&m_3\ddot{x}_3+(k_3+k_4)x_3-k_3x_2-k_4x_5=0 \quad ;\\
&m_4\ddot{x}_4-k_5x_2+(k_5+k_6)x_4-k_6x_6=0 \quad ;\\
&m_5\ddot{x}_5-k_4x_3+k_4x_5=0 \quad ;\\
&m_6\ddot{x}_6-k_6x_4+k_6x_6=0 \quad ;
\end{aligned}
\tag{19}
$$

The displacements x_i are absolute value of displacement. For the system presented in Figure 1, the equations of motion can be written in the form:

$$
\begin{bmatrix} m_1 & 0 & 0 & 0 & 0 & 0\\ 0 & m_2 & 0 & 0 & 0 & 0\\ 0 & 0 & m_3 & 0 & 0 & 0\\ 0 & 0 & 0 & m_4 & 0 & 0\\ 0 & 0 & 0 & 0 & m_5 & 0\\ 0 & 0 & 0 & 0 & 0 & m_6 \end{bmatrix}
\begin{Bmatrix} \ddot{x}_1\\ \ddot{x}_2\\ \ddot{x}_3\\ \ddot{x}_4\\ \ddot{x}_5\\ \ddot{x}_6 \end{Bmatrix}
+
\begin{bmatrix} 2k_1+2k_2 & -2k_2 & 0 & 0 & 0 & 0\\ -2k_2 & 2k_2+k_3+k_5 & -k_3 & -k_5 & 0 & 0\\ 0 & -k_3 & k_3+k_4 & 0 & -k_4 & 0\\ 0 & -k_5 & 0 & k_5+k_6 & 0 & -k_6\\ 0 & 0 & -k_4 & 0 & k_4 & 0\\ 0 & 0 & 0 & -k_6 & 0 & k_6 \end{bmatrix}
\begin{Bmatrix} x_1\\ x_2\\ x_3\\ x_4\\ x_5\\ x_6 \end{Bmatrix}
=
\begin{Bmatrix} F_o\cos\omega t\\ 0\\ 0\\ 0\\ 0\\ 0 \end{Bmatrix}
\tag{20}
$$

It is denoted:

$$
M=\begin{bmatrix} m_1 & 0 & 0 & 0 & 0 & 0\\ 0 & m_2 & 0 & 0 & 0 & 0\\ 0 & 0 & m_3 & 0 & 0 & 0\\ 0 & 0 & 0 & m_4 & 0 & 0\\ 0 & 0 & 0 & 0 & m_5 & 0\\ 0 & 0 & 0 & 0 & 0 & m_6 \end{bmatrix}
$$

$$
K=\begin{bmatrix} 2k_1+2k_2 & -2k_2 & 0 & 0 & 0 & 0\\ -2k_2 & 2k_2+k_3+k_5 & -k_3 & -k_5 & 0 & 0\\ 0 & -k_3 & k_3+k_4 & 0 & -k_4 & 0\\ 0 & -k_5 & 0 & k_5+k_6 & 0 & -k_6\\ 0 & 0 & -k_4 & 0 & k_4 & 0\\ 0 & 0 & 0 & -k_6 & 0 & k_6 \end{bmatrix}
$$

$$
\boldsymbol{F}=\begin{Bmatrix} F_o\cos\omega t\\ 0\\ 0\\ 0\\ 0\\ 0 \end{Bmatrix}. \tag{21}
$$

Introducing the harmonic solution in the motion equations, it obtains:

$$
\left(
\begin{bmatrix} 2k_1+2k_2 & -2k_2 & 0 & 0 & 0 & 0\\ -2k_2 & 2k_2+k_3+k_5 & -k_3 & -k_5 & 0 & 0\\ 0 & -k_3 & k_3+k_4 & 0 & -k_4 & 0\\ 0 & -k_5 & 0 & k_5+k_6 & 0 & -k_6\\ 0 & 0 & -k_4 & 0 & k_4 & 0\\ 0 & 0 & 0 & -k_6 & 0 & k_6 \end{bmatrix}
-\omega^2
\begin{bmatrix} m_1 & 0 & 0 & 0 & 0 & 0\\ 0 & m_2 & 0 & 0 & 0 & 0\\ 0 & 0 & m_3 & 0 & 0 & 0\\ 0 & 0 & 0 & m_4 & 0 & 0\\ 0 & 0 & 0 & 0 & m_5 & 0\\ 0 & 0 & 0 & 0 & 0 & m_6 \end{bmatrix}
\right)
\begin{Bmatrix} e_1\\ e_2\\ e_3\\ e_4\\ e_5\\ e_6 \end{Bmatrix}
=
\begin{Bmatrix} F_1\\ 0\\ 0\\ 0\\ 0\\ 0 \end{Bmatrix}
\tag{22}
$$

where the component of the harmonic solution (3) there are in the vector:

$$\{X\}=[e_1 \quad e_2 \quad e_3 \quad e_4 \quad e_5 \quad e_6]^T.$$

$[C_{22}]$ is in our problem:

$$[C_{22}]=\begin{bmatrix} 2k_2+k_3+k_5-\omega^2 m_2 & -k_3 & -k_5 & 0 & 0 \\ -k_3 & k_3+k_4-\omega^2 m_3 & 0 & -k_4 & 0 \\ -k_5 & 0 & k_5+k_6-\omega^2 m_4 & 0 & -k_6 \\ 0 & -k_4 & 0 & k_4-\omega^2 m_5 & 0 \\ 0 & 0 & -k_6 & 0 & k_6-\omega^2 m_6 \end{bmatrix} \quad (23)$$

The determinant of the system is:

$$\begin{vmatrix} 2k_1+2k_2-\omega^2 m_1 & -2k_2 & 0 & 0 & 0 & 0 \\ -2k_2 & 2k_2+k_3+k_5-\omega^2 m_2 & -k_3 & -k_5 & 0 & 0 \\ 0 & -k_3 & k_3+k_4-\omega^2 m_3 & 0 & -k_4 & 0 \\ 0 & -k_5 & 0 & k_5+k_6-\omega^2 m_4 & 0 & -k_6 \\ 0 & 0 & -k_4 & 0 & k_4-\omega^2 m_5 & 0 \\ 0 & 0 & 0 & -k_6 & 0 & k_6-\omega^2 m_6 \end{vmatrix}. \quad (24)$$

In the example the values of the mechanical constants are: $k_1=10000$ N/m; $k_2=20000$ N/m; $k_3=30000$ N/m; $k_4=40000$ N/m; $k_5=50000$ N/m; $k_6=10000$ N/m; $m_1=10$ kg; $m_2=10$ kg; $m_3=1$ kg; $m_4=2$ kg; $m_5=3$ kg; $m_6=5$ kg.

The eigenfrequencies of the system with six degrees of freedom are 42.352 rad/s; 64.124 rad/s; 102.117 rad/s; 181.796 rad/s; 277.559 rad/s; 311.764 rad/s.

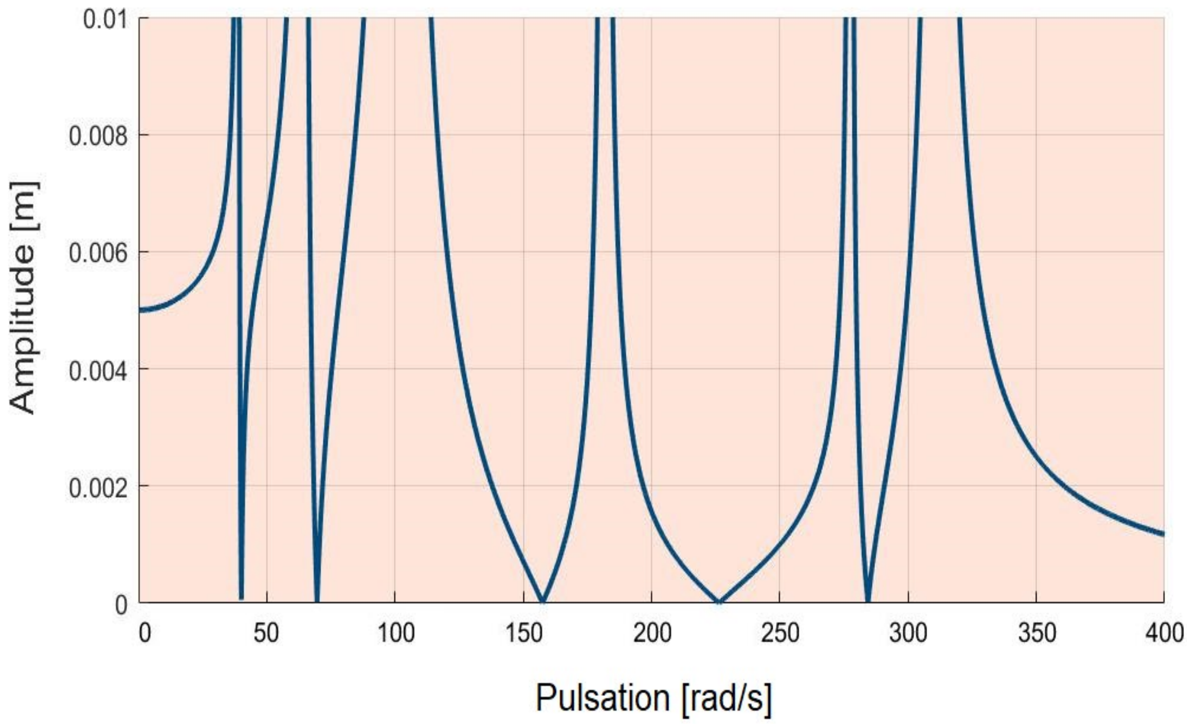

Figure 2. Amplitude of the mass 1 if an excitation force acts on the mass 1

It can be seen that there are five excitation frequencies for which mass 1 absorbs the excitation vibrations. If we inspect the other masses, we also notice that they can become dynamic absorbers for certain frequencies applied to mass 1 (Figures 3-4).

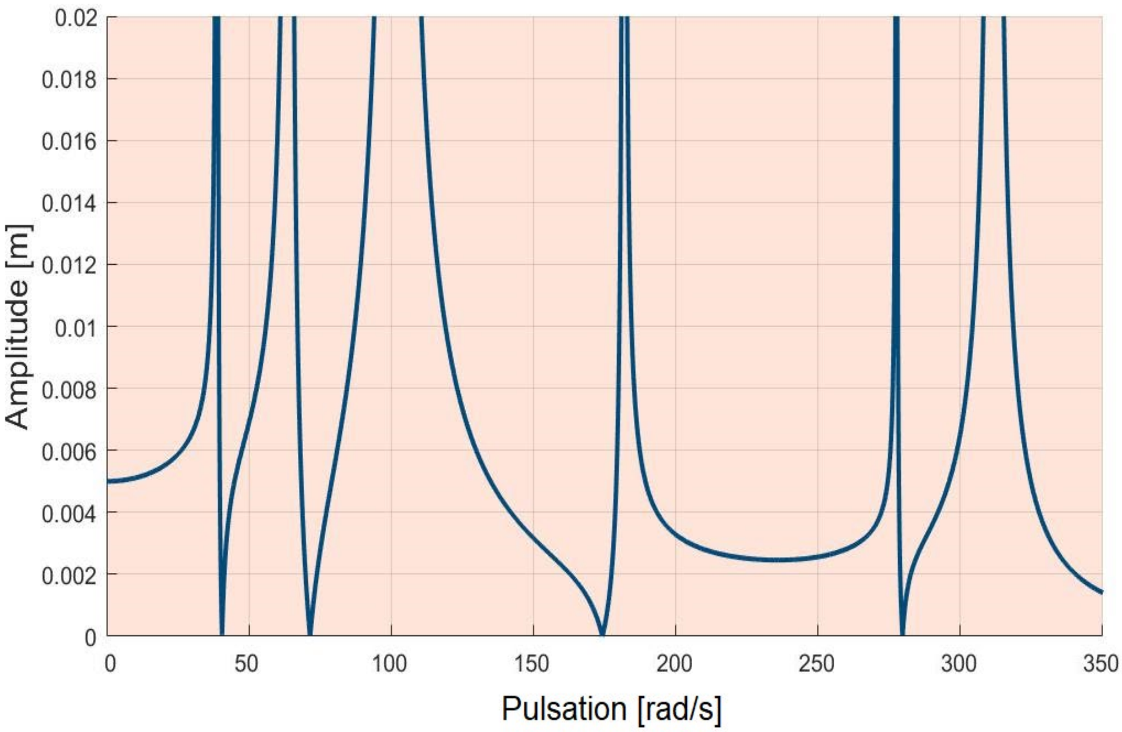

Figure 3. Amplitude of the mass 2 if an excitation force acts on the mass 1

Thus it can be found that certain elements of a mechanical system can become mechanical absorbers of that system. Through a judicious design, a project can be realized in which, at certain excitation frequencies, some elements of the system, or parts of it, act as dynamic absorbers.

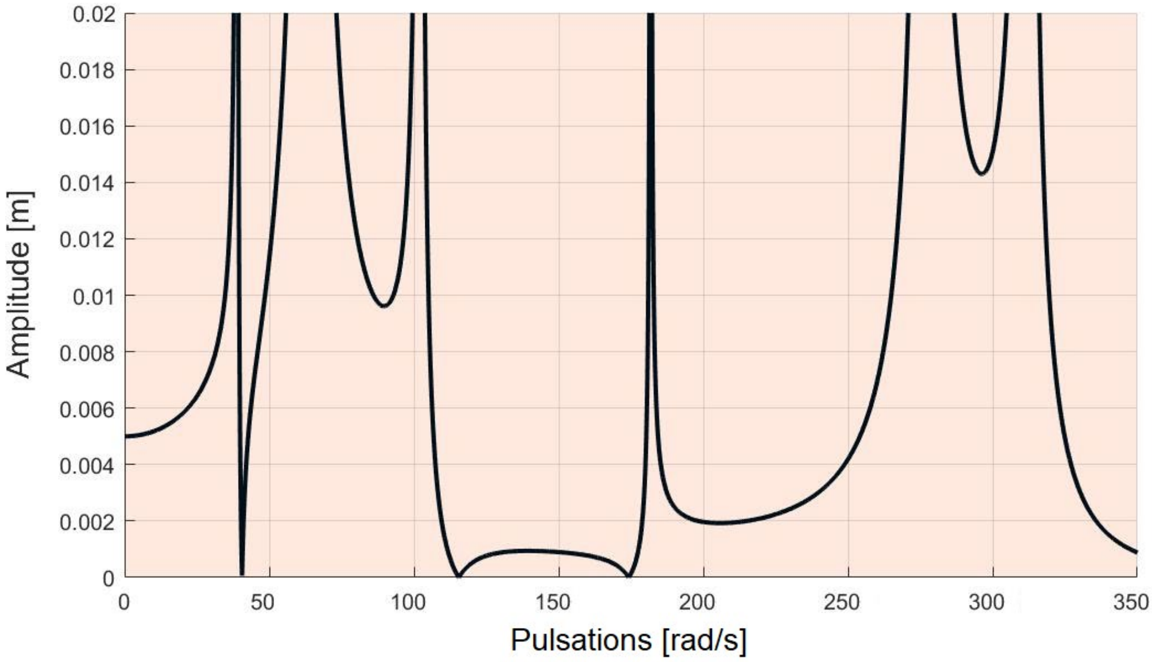

Figure 4. Amplitude of the mass 3 if an excitation force acts on the mass 1

4. CONCLUSIONS

In the analysis of mechanical vibrations, dynamic absorbers are considered the most spectacular means that allow vibration reduction. Simplicity and low price represent the advantages that these devices. This can be done by inserting dedicated absorbers, but not using structural parts of the system (that have a constructive-functional role) as absorbers.

It is no longer necessary to insert special elements, an operation that requires time and money.

An absorbtion of vibration can be ensured by a correct design of the system. The more parameters the system is characterized by, the greater the number of frequencies for which vibration suppression can be ensured. In a practical application this becomes very important. But it is difficult to formulate general recommendations; for each individual case, the designer must do this study and determine the optimal parameters of the system to ensure the conditions required by the project.

In other words, a judicious design of an elastic system allows handling the problem of resonant frequencies without using special devices, such as DVA, which requires a redesign and the attachment of an expensive device. In the work, these properties are exemplified for a dynamic system with concentrated masses. The subject can be developed by generating algorithms that provide the optimal solution for various practical applications. The obtained results show us that a mechanical system can act as a dynamic absorber for certain eigenpulsations. They can be a starting point for further developments in the field of vibration absorption.

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