Improved Mathematical Relation of The Modal Shapes of Thin Rectangular Plates

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Abstract: - In this paper a new analytical solution for solving the issues that arise in determining the correct form of the eigenmodes, for the case of simply supported on two opposite edges and clamped on the other two of a rectangular plate, is described. The case of a homogenous plate, isotropic, with constant thickness and uniformly distributed weight, and dynamically driven in the direction normal to the plane of the plate with a median surface in the xOy plane is considered. The deformed shape of the middle area Z(x, y) is analyzed. The theory of elasticity states that this phenomenon can be described by a bi-harmonic differential equation of smooth plates having constant thickness. However, the eigenfunctions obtained for this scenario, lead to eigenmodes representations which are accurate only for modes n=1 and m=1. In this paper it is shown that the proposed analytical solution accurately describes the eigenmodes for the above mentioned plate, with the given limit constraints, for every m and n modes. Moreover, this new solution stands out for its simplicity. MatLab environment was used for simulations. The results obtained using the proposed solution were compared with the results obtained by means of classical method. From the analysis we carried out, we observed a symmetrical distribution of the eigenmodes, on both sides of the equilibrium position.

Keywords: - free vibration, eigenmodes, eigenfunctions, thin plates, bi-harmonic differential equation

1. INTRODUCTION

Plates are extremely important in engineering, as they are found, in different shapes and sizes, as components of numerous metallic structures. The analysis and calculus regarding plates and metallic structures of plates behavior encounters difficulties because, most cases, the methods lead to obtaining systems of equations having partial derivatives, which are difficult to integrate and solve. The problems become more complicated with the increase in complexity of boundary conditions.

The theory of plates describes a series of simplified hypotheses, some formulated as principles, and others derived from calculus in which various terms of the equations were neglected. Along time, various approaches, many of them based on analytical methods, were studied in the attempt to find reliable and simpler solutions for various plate types with different boundary conditions.

The first mathematical approach for the plates issue was given by Euler, who performed an analysis of a plate's free vibration. After that, Chladni discovered several modes for the free vibration of plates. In the experiments on horizontal plates, he used powder, uniformly spread on the upper surface of the plate, which created regular shapes after the induction of a vibration. Modal lines appeared on the plate surface [1,2].

The differential equation of the plate was developed by Germain, but the latter lacks the term of deformation. The term of deformation was introduced into the differential equation of plates by Lagrange [3]. He was the first who correctly presented the general equation of plates.

Starting from the Germain-Lagrange equation, Poisson found the solution for a static-loaded plate. In this solution, the rigidity of the plate was considered constant, and three boundary conditions were introduced. These limit conditions and the discussions about their number and nature were the subject of many controversies and were subjected to ulterior research [4].

Kirchhoff had established that there are only two boundary conditions for retainer plate. Plate frequency equation was also an important discovery. This theory has been successfully applied in practice [5]. An analytical method was developed for the analysis of vibrations in rectangular plates with diverse elastic non-uniform constraints on the edges, that can be extended also for the triangular and L-shaped plates [6].

A solution for the case of the plate with boundary conditions simply supported on two opposite edges and clamped on the other two is given in [7]. It is derived from the static study, for determining the first eigenmode, for which m=1 and n=1.

In [8] is proposed a method based on Bessel functions which lead to obtaining the exact solutions for the free vibration analysis of thin rectangular plates with various edge conditions, among them being considered also the case of two opposite edges simply supported and the other two edges clamped. The results obtained with the proposed solutions were accurate but only for the eigenmodes for which n is an even number and m=n/2.

Sh. Hosseini-Hashemi et. all presented in 2009 a new approach for free vibration analysis of moderately thick rectangular plates composed of functionally graded materials and supported by either Winkler or Pasternak elastic foundations. The study was made for thick rectangular plates having two opposite edges simply supported and all possible combinations of free, simply supported, and clamped boundary conditions for the other two edges. The research was based on FSDT to derive and solve the equations of motion. For the shear correction factors authors proposed a new formula based on Mindlin plate theory. The solution proved to be highly efficient [9].

Y.F. Xu, W.D. Zhu used in [10] the operational modal analysis, and the experimental modal analysis for determining the eigenmodes of a rectangular plate, resulting an error of maximum 1.53% between determined data and values obtained from FEM analysis. Even more, these methods are quite sophisticated and laborious.

A different approach for thin plates with non-classical boundary conditions is proposed in [11]. The proposed solution was viable only for eigenmodes for which m and n are odd numbers.

This paper presents a new approach for obtaining a precise solution for the eigenfunctions which lead to correct representations of the eigenmodes, for every m and n value, for the case of a rectangular plate simply supported on two opposite edges and clamped on the other two edges. It is shown that the distribution of these eigenmodes is symmetrical, on both sides, with respect to the equilibrium position, for all the analyzed cases.

It is well known that for a homogeneous, isotropic plate of constant thickness h, with uniformly distributed mass and dynamically driven in the direction normal to the plane of the plate, with the median surface in the xOy plane, the deformed shape of the median surface Z(x,y) satisfies the bi-harmonic differential equation of plane plates of constant thickness, established in the theory of elasticity.

Considering the form of the quadratic bi-harmonic equation and imposing the boundary conditions, the eigenfunctions are obtained which constitute a generalized solution for determining the eigenmodes of a plate. For the case in discussion the boundary conditions are: two opposite edges simply supported, and the other two edges clamped. It will be shown that, for the generalized form of the eigenfunctions, the resulted eigenmodes are not accurate, with the increase of n and m values.

The case of plates with discontinuities is intensively studied, aiming to present solutions that can be used in damage detection [12-15]. Different types of plates [16-17] and different damage [18] is targeted.

The new approach proposed and presented in this paper implies modifying the eigenfunction equation by adding the sign function, thus, obtaining a better solution which describe eigenmodes correctly, for every n and m values.

MATLAB environment is used, in order to compare the eigenmodes for the proposed solution with the eigenmodes for the classical solution, for the above mentioned plate and boundary conditions.

2. PROBLEM STATEMENT AND NEW APPROACH FORMULATION

A homogeneous, isotropic plate of constant thickness h, with uniformly distributed mass and dynamically actuated in the direction normal to the plane of the plate, with the median surface in the xOy plane is considered. The deformed shape of the median surface Z(x,y) satisfies the bi-harmonic differential equation of flat plates of constant thickness, established in the theory of elasticity:

$$\frac{\partial^4 Z}{\partial x^4}+2\frac{\partial^4 Z}{\partial x^2\partial y^2}+\frac{\partial^4 Z}{\partial y^4}=\frac{p(x,y)}{D} \quad (1)$$

In the case of free vibrations, when the disturbing force p(x,y,t)=0, the equation with partial derivatives of the deformed surface is obtained, of the form:

$$\frac{\partial^4 Z}{\partial x^4}+2\frac{\partial^4 Z}{\partial x^2\partial y^2}+\frac{\partial^4 Z}{\partial y^4}-\lambda^4 Z(x,y)=0 \quad (2)$$

where:

$$\lambda^4=\omega^2\frac{\rho\cdot h}{D} \quad (2`)$$

- $D=\frac{Eh^3}{12(1-\upsilon^2)}$ is the plate bending rigidity;
- E is elasticity modulus;
- ρ is material density;
- Z=Z(x,y) is the shift of the plate median surface along the normal direction;
- h is the plate thickness;
- υ is the transverse contraction coefficient (Poisson coefficient).

In the case of a rectangular plate simply supported on two parallel edges x = 0 and x = a (Figure. 1), function Z(x,y) has the form [14]:

$$Z(x,y)=Y(y)\sin\left(\frac{n\cdot\pi\cdot x}{a}\right),\ \ where\ n\in N^* \quad (3)$$

which satisfies the following limit conditions:

$$Z(0,y)=0;\ Z(a,y)=Y(y)\sin n\pi=0,(n=1,2,3,\ldots) \quad (4)$$

$$\left.\left(\frac{\partial^2 Z}{\partial x^2}+\frac{\partial^2 Z}{\partial y^2}\right)\right|_{\substack{x=0\\x=a}}=0 \quad (5)$$

Introducing the Eq. (3) into the bi-harmonic Eq. (1), we obtained a fourth-order linear differential equation with constant coefficients, having the following form [13]:

$$\frac{d^4Y}{dy^4}-2\frac{n^2\pi^4}{a^2}\cdot\frac{d^2Y}{dy^2}+\left(\frac{n^4\pi^4}{a^4}-\lambda^4\right)Y=0 \quad (6)$$

The characteristic equation for the fourth-order differential Eq. (6) is:

$$r^4-2\frac{n^2\pi^2}{a^2}r^2+\left(\frac{n^4\pi}{a^4}-\lambda^4\right)=0 \quad (7)$$

and the roots of the characteristic Eq. (7) are:

$$r^2=\frac{n^2\pi^2}{a^2}\pm\lambda^2 \quad (8)$$

By using the notations:

$$\beta^2=\lambda^2-\frac{n^2\pi^2}{a^2},\quad \alpha^2=\lambda^2+\frac{n^2\pi^2}{a^2} \quad (9)$$

the roots of the characteristic Eq. (7) become:

$$r_{1,2}=\pm i\beta \qquad r_{3,4}=\pm\alpha \quad (10)$$

A solution for the differential equation of the form is adopted:

$$Y(y)=A\cosh(\alpha y)+B\sinh(\alpha y)+C\cos(\beta y)+D\sin(\beta y) \quad (11)$$

The constants are determined from the limit conditions that must be satisfied on the two simply supported edges, parallel with the Ox axis and from the limit conditions that must be satisfied on the two clamped edges, parallel with the Oy axis.

If the thin plate is clamped on the other two edges (Figure 1), to which the arrow and the rotation in a perpendicular plane on the edge are null, then the limit conditions are the following:

a) $Z(x,0)=Z(x,b)=0$

From this condition it results:

$$Y(0)\sin\left(\frac{n\pi x}{a}\right)=Y(b)\sin\left(\frac{n\pi x}{a}\right)=0, \qquad namely\ \ Y(0)=Y(b)=0 \quad (12)$$

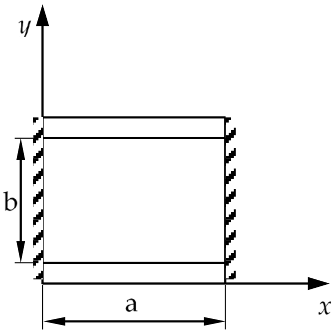

Figure 1. Plate clamped on the edges on *y* direction and simply supported on the edges on *x* direction

b) $Z'_y(x, y) = Y'(y) \sin\left(\frac{n\pi x}{a}\right)$,

$Z'_y(x,0) = Z'_y(x,b) = 0$

from this condition it results:

$$Y'(0) = Y'(b) = 0 \tag{13}$$

The first-order derivative of Y from equation (11), in relation to y, is:

$$Y'(y) = \alpha A \sinh(\alpha y) + \alpha B \cosh(\alpha y) - \beta C \sin(\beta y) + \beta D \cos(\beta y) \tag{14}$$

Using the limit condition (12) into the Eq. (11), and the limit condition (13) into the Eq. (14), the homogenous linear system with the unknown parameters A, B, C, D is obtained, having the form below:

$$\begin{cases} A + C = 0 \\ A\cosh(\alpha b) + B\sinh(\alpha b) + C\cos(\beta b) + D\sin(\beta b) = 0 \\ \alpha B + \beta D = 0 \\ \alpha A\sinh(\alpha b) + \alpha B\cosh(\alpha b) - \beta C\sin(\beta b) + \beta D\cos(\beta b) = 0 \end{cases} \tag{15}$$

From the first Eq. of system (15), results:

$$A = -C \tag{16}$$

A particular solution of the form α = 0 was considered. Further, from the third equation of the system (15) results:

$$D = 0 \tag{17}$$

By replacing the results (16) and (17) in the second and fourth equation of the system (15), is obtained:

$$\begin{cases} -C + C\cos(\beta b) = 0 \\ -\beta C\sin(\beta b) = 0 \end{cases} \tag{18}$$

From these equations of the system results:

$$\cos(\beta b) = 1 \ or \ \sin(\beta b) = 0 \tag{19}$$

and the solution of these equations:

$$\beta_m = \frac{2m\pi}{b} \tag{20}$$

By squaring the expression (20), is obtained the following relation:

$$\beta_m^2 = \frac{4m^2\pi^2}{b^2} \tag{21}$$

When introducing relation (21) into relation (10), the non-dimensional frequency parameter of the below form is obtained:

$$\lambda_{mn}^2 = \beta_{mn}^2 + \frac{n^2\pi^2}{a^2} = \pi^2\left(\frac{4m^2}{b^2} + \frac{n^2}{a^2}\right); m, n \in N^* \tag{22}$$

By replacing Eq. (22) into relation (2`) the eigenpulsation of the movement result in the form:

$$\omega_{mn} = \lambda_{mn}^2\sqrt{\frac{D}{\rho h}} = \pi^2\left(\frac{4m^2}{b^2} + \frac{n^2}{a^2}\right)\sqrt{\frac{D}{\rho h}} \tag{23}$$

From the relation (23) it can be concluded that there is infinity of eigenpulsations, depending on the eigenmodes m and n, with which the plate may execute free harmonic vibrations.

The case $\alpha \neq 0$ is considered.

From the third equation of system (15), results:

$$B = -\frac{\beta \cdot D}{\alpha} \tag{24}$$

By replacing relation (16) and (24) in the second and fourth equation of system (15), is obtained:

$$\begin{cases} -C\cosh(\alpha b) - \frac{\beta D}{\alpha}\sinh(\alpha b) + C\cos(\beta b) + D\sin(\beta b) = 0 \\ -\alpha C\sinh(\alpha b) - \beta D\cosh(\alpha b) - \beta C\sin(\beta b) + \beta D\cos(\beta b) = 0 \end{cases} \tag{25}$$

Setting the condition that the system (25) should also admit solutions different from the common solution, the equation of the eigenpulsations is obtained:

$$\Delta = \begin{vmatrix} -\cosh(\alpha b) + \cos(\beta b) & -\frac{\beta}{\alpha}\sinh(\alpha b) + \sin(\beta b) \\ -\alpha\sinh(\alpha b) - \beta\sin(\beta b) & -\beta\cosh(\alpha b) + \beta\cos(\beta b) \end{vmatrix} = 0 \tag{26}$$

By solving the equation of eigenpulsations (26) the transcendent equation results, having the form:

$$2\alpha - 2\alpha\cos(\beta b)\cosh(\alpha b) + \frac{1}{\beta}\left(\alpha^2 - \beta^2\right)\sin(\beta)\sinh(\alpha b) = 0 \quad (27)$$

A particular solution of Eq. (27) is:

$$sin(\beta b) = 0 \;\; or \;\; cos(\beta b) = 1 \qquad (28)$$

From Eq. (27) results that: $1 - cosh(\alpha b) = 0$, i.e. $\alpha = 0$, which is in contradiction with the forwarded hypothesis. Consequently, according to the analysis conducted, one solution of the transcendental equation (27) is the particular solution $\alpha = 0$.

For this case the system (15) solutions are:

$$B = D = 0 \;\text{ and }\; -A = C \neq 0 \qquad (29)$$

Therefore, the solution of the differential Eq. (14) becomes [22]:

$$Y_m(y) = A_m\left[1 - cos\left(\frac{2m\pi y}{b}\right)\right] \qquad (30)$$

The eigenfunctions of the motion, $Z_{mn}(x,y)$, for the bi-harmonic equation, from which rectangular plate eigenmodes are obtained, have the form:

$$Z_{mn}(x, y) = A_{mn}\, sin\frac{n\pi x}{a}\left[1 - cos\left(\frac{2m\pi y}{b}\right)\right] \quad (31)$$

The A_{mn} coefficients are calculated using the Fourier expansion:

$$A_{mn} = \frac{4}{a \cdot b}\int_0^a\int_0^b sin\left(\frac{n\cdot\pi\cdot x}{a}\right)\left[1 - cos\left(\frac{2m\pi y}{b}\right)\right]dxdy \quad (32)$$

where:

$$\int_0^a \sin\frac{n\cdot\pi\cdot x}{a}dx = -\frac{a}{n\cdot\pi}\left[(-1)^n - 1\right] = \begin{cases} 0; & for\, n - even \\ \dfrac{2a}{n\cdot\pi}; & for\, n - odd \end{cases} \quad (33)$$

$$\int_0^b \left[1 - cos\left(\frac{2m\pi y}{b}\right)\right]dy = b \qquad (34)$$

It results that the A_{mn} coefficients are:

$$A_{mn} = \begin{cases} 0; & for\, m,n - even \\ \dfrac{8}{n\cdot\pi}; & for\, m,n - odd \end{cases} \qquad (35)$$

By replacing results (35) in Eq. (31), eigenfunction equations become:

$$Z_{mn}(x, y) = \frac{8}{n\pi}\, sin\frac{n\pi x}{a}\left[1 - cos\left(\frac{2m\pi y}{b}\right)\right] \quad (36)$$

Relation (36) is a generalized form of the eigenmodes for simply supported on two opposite edges and clamped on the two edges rectangular plate. Once the eigenpulsations and eigenfunctions are determined, the expression of the shift in vibration according to the eigenmode can be written, of the form:

$$w_{mn}(x, y, t) = A_{mn}\sin\frac{n\pi x}{a}\left[1 - \cos\left(\frac{2m\pi y}{b}\right)\right]\sin(\omega_{mn}t + \theta_{mn}) \quad (37)$$

As the function $1 - cos\left(\dfrac{2m\pi\cdot y}{b}\right)$ is positive, the eigenmodes won't contain both the transversal and longitudinal components of the vibrational mode, as it will be seen, later, in simulation results. Therefore, this issue can be solved by introducing "sign" function in relation (36), which becomes:

$$Z_{mn}(x, y) = \frac{8}{n\pi} sin\frac{n\pi x}{a}\cdot sign\left\{cos\left[\frac{\pi}{2}\left(\frac{2my}{b} - 1\right)\right]\right\}\left[1 - cos\left(\frac{2m\pi y}{b}\right)\right] \quad (38)$$

3. TESTS AND RESULTS

In order to certify the validity of the new solution, as stated in relation (38), a steel rectangular plate, having the edges b = 500 mm and a = 1000 mm, simply supported on two parallel edges (length) and clamped on the other two edges (width), with the elasticity modulus $E = 2.1\times10^{11}$ N/m^2, Poisson coefficient $\upsilon = 0.29$, density $\rho = 7850$ kg/m^3 is considered.

For this plate, simply supported on two parallel edges and clamped on the other two edges, simulations were made, using MATLAB software. A step of 0.001 value was chosen and both relations (36) and (38) were applied.

The results for the determined eigenmodes are depicted in figure 2.a), c) for relation (36) and 2.b), d) for relation (38), in which, for comparison, have been chosen two cases: first for n=2 and m=2, second for n=3 and m=4.

Analyzing the results from figure 2.a) and 2.c) it can be noticed that, when using relation (36), for n and m values above 1, the transversal component is not present, which leads to the conclusion that the classical analytical solution, for the studied case of the plate with the boundary conditions previously mentioned is not accurate when n and m are greater than 1.

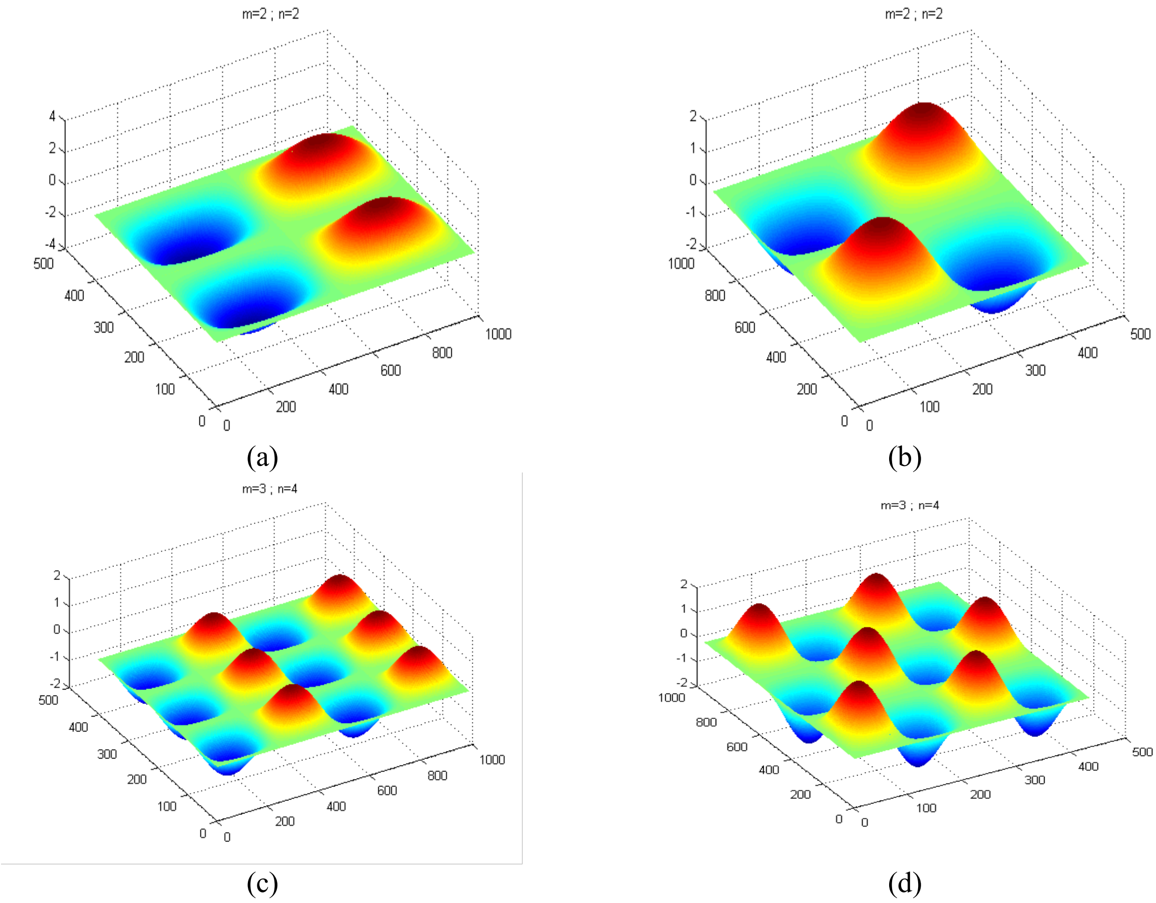

Figure 2. Comparison of eigenmodes for the plate simply supported on two parallel edges and clamped on the other two edges: (a), (c) with relation (36); (b), (d) with relation (38)

Moreover, analyzing the results from figure 2.b) and 2.d) it can be noticed that, when using the new proposed solution given by relation (38), the transversal component is present, for both n and m case values, and for any n and m value, odd or even. Thus, it can be concluded that relation (38) responds accurate and correct, showing both transversal and longitudinal components of the eigenmode, for every n and m value.

In Figure 3 is shown the normalized amplitude of the beam (blue) embedded at the ends as well as the normalized amplitude of the cross-section of the considered plate (magenta), embedded at the opposite ends and simply supported on two others. Using relation (39) from below, deduced in the case of the beam embedded at the ends, a more physically credible curve is obtained, represented in figure 3.

$$Z(x)=\frac{\cosh\lambda_j-\cos\lambda_j}{\sin\lambda_j-\sinh\lambda_j}\left(-\sin\lambda_j\cdot x+\sinh\lambda_j\cdot x\right) \quad (39)$$
$$-\cos\lambda_j\cdot x+\cosh\lambda_j\cdot x$$

Comparing the results obtained analytically for the considered plate case, with relation (38), represented in figure 2.b), with the results obtained for the clamped beam at both ends determined with relation (39), as it is depicted in graphic figure 3, it is observed that, the shape of the normalized amplitude both for the case of the studied plate and for the case of the beam are similar.

Though, when the amplitude value reaches 0 point, the plate clamped on the edges imitates the clamping phenomenon.

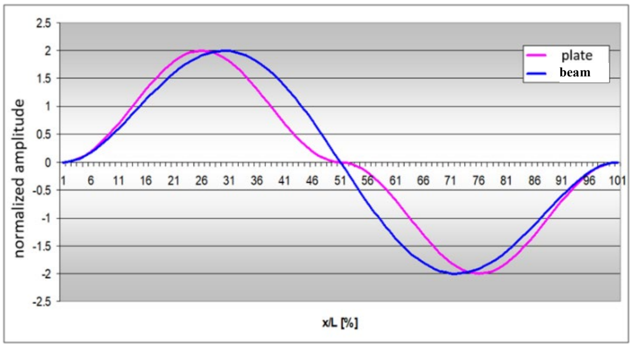

Figure 3. Normalized amplitude for beam and plate-comparison for mode shape 2

Figure 3 shows a very good correlation when using the relations involving hyperbolic and trigonometric functions. This is also demonstrated in [19], where the Python language is used for calculus and visualization.

4. CONCLUSIONS

The study conducted on thin elastic plates with the same thickness, but with different boundary conditions revealed the usefulness of diverse theories in the theoretical modelling of thin plates. Classical analytical method for determining the eigenmodes for a thin plate simply supported on two parallel edges and clamped on the other two edges is responds well only for value 1 of n and m.

Late research results, using various analytical methods have been proven to be accurate, but only partially, for odd or even values of n and m, or other specific values, solutions which can be used for specific case situations.

The simulations made for a real plate case, with above mentioned boundary conditions, and with relation (38) led to obtaining credible results. Using only the known relations from the literature, represented by trigonometric functions, does not lead to exact solutions for all eigenmodes and it is necessary to use more precise functions.

The research described in the paper led to obtaining a new generalized solution for determining the eigenmodes of the free vibration in thin plates simply supported on two parallel edges and clamped on the other two edges. It has been shown that the new solution, represented by relation (38), gives more accurate results in describing the eigenmodes of vibration for the considered plate and has a faster convergence for every n and m value.

In all analyzed cases, we observed that the eigenmodes are distributed symmetrically, on both sides of the equilibrium position.

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