Formulation of Statistical Model to Determine Natural Frequencies of the Cantilever Beam for Linear Variation of Circular Perforation Along the Length

Akhil EAGA
BITS Pilani K K Birla Goa campus, India, akhil.bitsian2013@gmail.com

Sanjyot BHOSALE
BITS Pilani K K Birla Goa campus, India, sanjyot51095@gmail.com

Kiran MALI
BITS Pilani K K Birla Goa campus, India, kiranm@goa.bits-pilani.ac.in

Abstract: - In the present work effect of circular perforation on free vibrations of a cantilever beam is studied. The arrangement considered in this study is the linear variation of single circular cut-out starting from the support end of the cantilever. This study focuses on the dependence of the natural frequency of beam on various perforation parameters. Perforation parameters considered are the diameter of perforation and distance of perforation from the support end and geometry parameters of the beam like length, breadth, and thickness. An expression for natural frequency relating to above parameters was found out and formulated into a polynomial equation of fourth order using curve fitting techniques. The main aim of this research is to non-dimensionalize the dependency relation so that the results are valid for a broad range of dimensions of cantilever without the need for modal analysis to find natural frequencies of perforated cantilever beams. These non-dimensionalized expressions are further validated by predicting frequencies of test cases and gave error percentages in the range of 1% - 3%. The natural frequencies of perforated beams were predicted as the effective resonant frequency which is the ratio of the natural frequency of perforated beam to that of the solid beam.

Keywords: - Natural frequency, Effective resonance frequency, Perforated beams, Hole-Support distance

1. INTRODUCTION

Cantilever beams have vast applications in the engineering world. Straight and horizontal cantilever beams under no load, bend due to self weight. Under no external excitation, the beam vibrates at its natural frequencies. These frequencies are the result of characteristics of the cantilever beam namely stiffness and mass properties. Resonance occurs when external frequency becomes equal to a body's natural frequency causing the body to oscillate at higher amplitudes. Resonance creates vibration, noise problems and can lead to major catastrophes. However, since cantilever beams are used in major structures, natural frequency determination is crucial.

Burgemeister and Hansen introduced the term 'effective resonant frequencies' [1]. They determined the effective resonant frequencies of perforated panels using Finite Element Analysis by varying perforation geometries. The effect of perforations in the panel on the density, Young's modulus and Poisson's ratio is inspected to compute the effective resonant frequencies. Torabi and Azadi have applied the Rayleigh-Ritz method to obtain natural frequencies of rectangular plate with a circular hole [2]. The plate was provided with a point support and effect of increasing diameter of holes and number of point supports were analysed. The impact of hole was examined by subtracting hole energy from the whole plate energy.

Mali and Singru have developed an analytical model to predict fundamental frequencies for vibrations of perforated rectangular plates with circular holes in a rectangular perforation pattern. Concentrated negative mass has been used to factor in the effect of perforations [3]. Ghonasgi et. al. analysed the first three natural frequencies of multi perforated rectangular plates [4]. Different parameters like perforations, pattern of the perforations, aspect ratio of the plate, dimensions of the plate, ligament efficiency and the Mass Remnant Ratio (MRR) are examined so as to determine the most influencing factors on natural frequencies. Mali and Singru developed an expression for modal constant of the fundamental frequency for perforated plate experimentally using Rayleigh's method [5]. The fundamental frequency can be calculated for any combination of ligament efficiency and perforation diameter.

Perforated cantilever beams have numerous applications like castellated beams. In this study, natural frequencies are calculated using modal analysis. Modal analysis is used profoundly for analysing dynamic properties of engineering structures [6,7]. A mathematical equation is developed using modal analysis to determine the natural frequency of perforated beams [8]. This equation encompasses physical and material parameters namely length, width and thickness of the cantilever beam, diameter of holes, hole to beam-support distance, Young's modulus E, Poisson's ratio ν and density ρ.

2. BEAM SPECIFICATIONS

Material used for modal analysis is Mild Steel (AISI 1018). All the materials and their properties referenced in this paper are cited from AZoM (*https://www.azom.com*), leading online publication for Materials Science Community. Material properties for Mild Steel (AISI 1018) are specified in the Table 1.

Table 1. Material properties

Material properties	Mild Steel (AISI 1018)
Young's Modulus (*E*)	205 GPa
Poisson's Ratio (ν)	0.29
Density (ρ)	7870 Kg/m^3

Dimensions of the beam are specified in the Table 2.

Table 2. Dimensions of the beam

Length (*L*)	500 mm
Width (*W*)	50 mm
Thickness (*t*)	6 mm

Perforated cantilever beam is shown in the Figure 1. It has a hole at distance '*x*' from support of the beam to the centre of the hole. This is termed as 'Hole-Support' distance (abbreviated as HS distance). '*d*' is the diameter of the hole. The hole is placed at the centre line of the top face of the beam. Other dimensions are similar to the solid beam i.e. length, width and thickness.

Linear variation of circular perforation has one hole starting from support and varied till the end of beam given the limiting conditions. Different hole diameters (*d*) used for linear variation are 10 mm, 15 mm, 20 mm, 25 mm, 30 mm. For different iterations, Hole-Support distance (HS) was varied from 20 mm to 480 mm with an increment of 20 mm per iteration. Therefore, for the beam of 500 mm length, 24 iterations were needed.

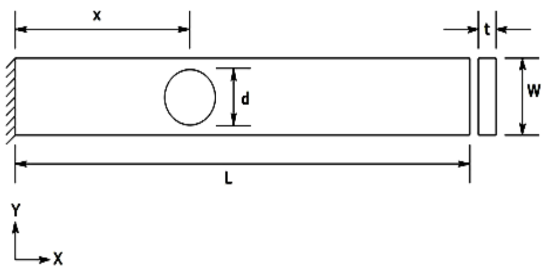

Figure 1. Perforated cantilever beam

The limiting conditions on the perforations are:

a) Hole-Support distance i.e., '*x*' has to be greater than radius of the perforation.

$$\frac{d}{2} < x < \left(L - \frac{d}{2} \right), \tag{1}$$

b) Diameter of the hole i.e., '*d*' has to be less than width of the beam '*W*'.

$$0 < d < W \, , \tag{2}$$

3. MODAL ANALYSIS

Finite Element Method (FEM), Modal analysis was conducted using ANSYS Mechanical APDL (ANSYS Parametric Design Language) 15.0 student version. 'Block Lanczos' mode extraction method was used to extract frequencies. Material is assumed to be linear, elastic and isotropic. Element type used was 'SOLID186' which is a quadratic hexahedron. Quadratic hexahedron gave more precise and accurate results than quadratic tetrahedron. Linear tetrahedron and linear hexahedron gave results which are higher than analytical results of solid cantilever beam by a significant amount.

The mesh size used for the analysis was validated through convergence study (details of the study are provided in the validation section). Thus finest mesh size possible was used for the simulation. Solid beam was meshed with an element size of 0.00385m resulting in 26117 nodes and 12668 elements.

Results of modal analysis for solid beam are given in Table 3.

Table 3. Modal analysis results

Mode	Frequency, Hz	Mode Shape
1	19.903	Transverse
2	124.62	Transverse
3	163.88	In plane
4	348.85	Transverse
5	373.60	Twisting
6	683.45	Transverse
7	983.28	In plane
8	1126.5	Twisting
9	1129.5	Transverse
10	1686.5	Transverse

Mesh size, number of nodes and elements for holes of different diameters is specified below in the Table 4.

Table 4. Meshing Parameters

Diameter, mm	Mesh Size, m	Nodes	Elements
10	0.003850	27000+	13000+
15	0.003850	27000+	13000+
20	0.003850	28000+	14000+
25	0.003850	28000+	14000+
30	0.003850	27000+	13000+

Natural frequencies for all Hole-Support (HS) distances and diameters are listed in Table 5.

Table 5. Natural frequencies (in Hz) for different HS distance

HS, mm	Diameter, mm				
	10	15	20	25	30
20	19.662	19.362	18.922	18.320	17.530
40	19.693	19.443	19.046	18.500	17.752
60	19.716	19.486	19.142	18.652	17.975
80	19.739	19.536	19.232	18.797	18.189
100	19.761	19.585	19.320	18.936	18.397
120	19.782	19.631	19.402	19.070	18.595
140	19.801	19.673	19.480	19.195	18.785
160	19.819	19.714	19.553	19.313	18.965
180	19.836	19.751	19.621	19.425	19.136
200	19.851	19.786	19.684	19.529	19.296
220	19.866	19.819	19.743	19.627	19.447
240	19.879	19.849	19.798	19.718	19.589
260	19.892	19.877	19.850	19.804	19.722
280	19.904	19.904	19.899	19.884	19.846
300	19.915	19.929	19.946	19.961	19.965
320	19.926	19.954	19.992	20.035	20.078
340	19.936	19.978	20.035	20.106	20.187
360	19.947	20.002	20.079	20.176	20.294
380	19.957	20.026	20.122	20.247	20.400
400	19.968	20.050	20.167	20.318	20.508
420	19.979	20.076	20.213	20.393	20.619
440	19.991	20.102	20.261	20.470	20.736
460	20.003	20.130	20.312	20.553	20.859
480	20.016	20.160	20.366	20.640	20.991

4. METHODOLOGY

Using modal analysis, natural frequencies were found for each specimen in which diameter of hole was fixed and Hole-Support distance was varied. These natural frequencies of each specimen (with fixed hole diameter) were further plotted against the varied HS distance. Equations obtained from these plots were 4th order polynomial which gave natural frequencies in terms of HS distance for a particular value of hole diameter.

To make the equation valid for all diameters, coefficients of each powers of the equation i.e., power varying from 0, 1, 2, 3, 4 were plotted against the hole diameters. The curve fitting equations obtained gave coefficient values for each power in terms of any value of diameter given. These equations obtained for coefficients were substituted as the coefficients in the original 4th order polynomial obtained. Thus the 4th order polynomial now gives natural frequency for any value of hole diameter and the HS distance. The equation obtained is given below:

$$
\begin{aligned}
f = {} & x^4\begin{bmatrix} -2.0583e-16d^4+2.6475e-14d^3-8.6433e-13d^2 \\ +1.0888e-11d-4.4977e-11 \end{bmatrix}+ \\
& x^3\begin{bmatrix} 1.7026e-13d^4-2.5441e-11d^3+8.864e-10d^2 \\ -1.0997e-08d+4.5754e-08 \end{bmatrix}+ \\
& x^2\begin{bmatrix} -3.2567e-11d^4+7.2509e-09d^3-2.9082e-07d^2 \\ +3.4636e-06d+1.4658e-05 \end{bmatrix}+ \\
& x^1\begin{bmatrix} 5e-10d^4-5.7127e-07d^3+3.9261e-05d^2 \\ -0.00034091d+0.0015241 \end{bmatrix}+ \\
& x^0\begin{bmatrix} -5.3333e-07d^4+1.4667e-05d^3-0.0030267d^2 \\ +0.0064333d+19.864 \end{bmatrix}
\end{aligned} \tag{3}
$$

5. NON-DIMENSIONALISATION

However, the equation found only allows the hole diameter and HS distance to be varied. It does not take into account other material and geometric properties. The aim of this study is to include all parameters to predict the frequency without the need for FEA.

Parameters left to be involved are: Hole-Support distance (x), Diameter of hole (d), Length of beam (L), Width of beam (W), Thickness of beam (t), Young's modulus (E), Density (ρ), Poisson's Ratio (ν, Optional for Transverse Vibrations)

Since only transverse vibrations are to be considered, Poisson's Ratio is not important. It has significance when longitudinal vibration frequencies are to be predicted or when transverse vibrations with shape functions are to be analysed.

Non-Dimensionalising [9,10] helps to simplify and parameterize the equation so that it can accommodate any values of parameters listed above. Thus to decrease the number of variables and to involve the effect of all the above parameters, some non-dimensional parameters are defined. They are as follows:

Non-dimensional HS distance: $\left(\dfrac{x}{L}\right)$ hereafter referred to as $\bar{x}$

where x = HS distance and L = length of beam

Non-dimensional Hole diameter: $\left(\frac{d}{W}\right)$ hereafter referred to as $\bar{d}$

where d = hole diameter and W = width of beam.

Non-dimensional Frequency: $\left(\frac{f}{f^*}\right)$ hereafter referred to as $\bar{f}$

where f = frequency of perforated cantilever beam and f^* = frequency of solid beam i.e. reference cantilever beam

Burgemeister and Hansen have termed the Non-dimensional frequency as 'effective resonance frequency' [1]. It is defined as the ratio of modal resonance frequency of perforated beam to the resonance frequency of the corresponding solid beam.

Revisiting the analytical equation of natural frequency for solid cantilever beam tells that,

$$Frequency: \omega_n = \sqrt{\frac{K_{eff}}{M_{eff}}}\ rad/s \tag{4}$$

where K_{eff} = effective stiffness of cantilever beam,

M_{eff} = effective mass of beam

$$K_{eff} = \left(\frac{3EI}{L^3}\right) \tag{5}$$

where E = Young's modulus, I = Area moment of inertia, L = Length of Beam

$$M_{eff} = \frac{33}{140}m \tag{6}$$

where m = self mass of the beam

This depicts that Young's Modulus and thickness are taken care by K_{eff} and I (Area moment of Inertia). The HS distance is divided by 500 to get $\bar{x}$ and all the frequencies are divided by 19.903 which is f^* for solid cantilever beam to get $\bar{f}$.

The above same method is followed to obtain the equation for predicting frequency. Graph of $\bar{f}$ against $\bar{x}$ is plotted in Figure 2.

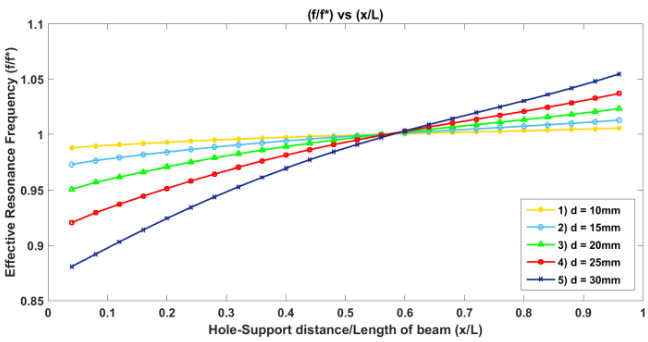

Figure 2. Effective resonance frequency vs. $\bar{x}$

Rewriting the data in Table 5. with the above modifications.

Table 6. Effective resonance frequencies ($\bar{f}$)

$\bar{x}$	Perforation diameter, mm				
	10	15	20	25	30
0.04	0.9879	0.9728	0.9507	0.9205	0.8808
0.08	0.9894	0.9764	0.9569	0.9295	0.8919
0.12	0.9906	0.9790	0.9618	0.9371	0.9031
0.16	0.9918	0.9816	0.9663	0.9444	0.9139
0.20	0.9929	0.9840	0.9707	0.9514	0.9243
0.24	0.9939	0.9863	0.9748	0.9581	0.9343
0.28	0.9949	0.9884	0.9787	0.9644	0.9438
0.32	0.9958	0.9905	0.9824	0.9704	0.9529
0.36	0.9966	0.9924	0.9858	0.9760	0.9615
0.40	0.9974	0.9941	0.9890	0.9812	0.9695
0.44	0.9981	0.9958	0.9920	0.9861	0.9771
0.48	0.9988	0.9973	0.9947	0.9907	0.9842
0.52	0.9994	0.9987	0.9973	0.9950	0.9909
0.56	1.0001	1.0001	0.9998	0.9990	0.9971
0.60	1.0006	1.0013	1.0022	1.0029	1.0031
0.64	1.0012	1.0026	1.0045	1.0066	1.0088
0.68	1.0017	1.0038	1.0066	1.0102	1.0143
0.72	1.0022	1.0050	1.0088	1.0137	1.0196
0.76	1.0027	1.0062	1.0110	1.0173	1.0250
0.80	1.0033	1.0074	1.0133	1.0209	1.0304
0.84	1.0038	1.0087	1.0156	1.0246	1.0360
0.88	1.0044	1.0100	1.0180	1.0285	1.0419
0.92	1.0050	1.0114	1.0205	1.0327	1.0480
0.96	1.0057	1.0129	1.0233	1.0370	1.0547

Basic curve fitting is done using MATLAB. All the plots are curve fitted with 4th degree polynomials to mitigate error as much as possible. The equations obtained at different hole diameters are given below:

$$1)\bar{f}_{@d=10mm} = 0.0059134(\bar{x})^4 + 0.0042747(\bar{x})^3 - 0.02737(\bar{x})^2 + 0.037052(\bar{x})^1 + 0.98653$$

$$2)\bar{f}_{@d=15mm} = 0.008781(\bar{x})^4 + 0.018756(\bar{x})^3 - 0.066773(\bar{x})^2 + 0.083947(\bar{x})^1 + 0.96979$$

$$3)\bar{f}_{@d=20mm} = 0.018561(\bar{x})^4 + 0.025589(\bar{x})^3 - 0.11203(\bar{x})^2 + 0.14873(\bar{x})^1 + 0.9453$$

$$4)\bar{f}_{@d=25mm} = 0.063672(\bar{x})^4 - 0.038927(\bar{x})^3 - 0.1163(\bar{x})^2 + 0.2213(\bar{x})^1 + 0.91214$$

$$5)\bar{f}_{@d=30mm} = 0.16284(\bar{x})^4 - 0.22246(\bar{x})^3 - 0.038889(\bar{x})^2 + 0.29175(\bar{x})^1 + 0.86906 \tag{7}$$

Table 7. Coefficients of $(\bar{x})^i$ (where i = 0, 1, 2, 3, 4)

$\bar{d}$	$i=4$	$i=3$	$i=2$	$i=1$	$i=0$
0.2	0.0059	0.0043	-0.0274	0.0371	0.9865
0.3	0.0088	0.0188	-0.0668	0.0839	0.9698
0.4	0.0186	0.0256	-0.1120	0.1487	0.9453
0.5	0.0637	-0.0389	-0.1163	0.2213	0.9121
0.6	0.1628	-0.2225	-0.0389	0.2918	0.8691

Now the $(\bar{x})^i$ (where i = 0, 1, 2, 3 and 4) coefficients are plotted against $\bar{d}$

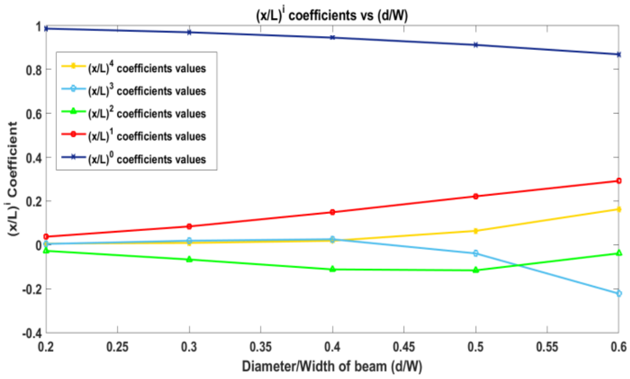

Figure 3. $(\bar{x})^i$ coefficients vs. $\bar{d}$

$$
\begin{aligned}
C_4 &= -4.0386(\bar{d})^4 + 10.39(\bar{d})^3 \\
&\quad - 6.7846(\bar{d})^2 + 1.7093(\bar{d})^1 - 0.14122 \\
C_3 &= 6.6803(\bar{d})^4 - 19.969(\bar{d})^3 \\
&\quad + 13.916(\bar{d})^2 - 3.4531(\bar{d})^1 + 0.28733 \\
C_2 &= -2.5612(\bar{d})^4 + 11.393(\bar{d})^3 \\
&\quad - 9.1373(\bar{d})^2 + 2.1765(\bar{d})^1 - 0.18422 \\
C_1 &= 0.080833(\bar{d})^4 - 1.7967(\bar{d})^3 \\
&\quad + 2.4669(\bar{d})^2 - 0.42841(\bar{d})^1 + 0.0383 \\
C_0 &= -0.1375(\bar{d})^4 + 0.039167(\bar{d})^3 \\
&\quad - 0.34713(\bar{d})^2 + 0.0076583(\bar{d})^1 + 0.99879
\end{aligned}
\tag{8}
$$

All the above graphs are curve fitted with 4th degree polynomial to mitigate error as much as possible.

$$\bar{f} = C_4(\bar{x})^4 + C_3(\bar{x})^3 + C_2(\bar{x})^2 + C_1(\bar{x})^1 + C_0 \tag{9}$$

Substituting the above equations in place of $(\bar{x})^i$ Coefficients in the $\bar{f}$ vs. $\bar{x}$ graph, the equation obtained is:

Proposed equation:

$$
\begin{aligned}
\bar{f} = \\
&(\bar{x})^4 \begin{bmatrix} -4.0386(\bar{d})^4 + 10.39(\bar{d})^3 \\ -6.7846(\bar{d})^2 + 1.7093(\bar{d})^1 - 0.14122 \end{bmatrix} + \\
&(\bar{x})^3 \begin{bmatrix} 6.6803(\bar{d})^4 - 19.969(\bar{d})^3 \\ +13.916(\bar{d})^2 - 3.4531(\bar{d})^1 + 0.28733 \end{bmatrix} + \\
&(\bar{x})^2 \begin{bmatrix} -2.5612(\bar{d})^4 + 11.393(\bar{d})^3 \\ -9.1373(\bar{d})^2 + 2.1765(\bar{d})^1 - 0.18422 \end{bmatrix} + \\
&(\bar{x})^1 \begin{bmatrix} 0.080833(\bar{d})^4 - 1.7967(\bar{d})^3 \\ +2.4669(\bar{d})^2 - 0.42841(\bar{d})^1 + 0.0383 \end{bmatrix} + \\
&(\bar{x})^0 \begin{bmatrix} -0.1375(\bar{d})^4 + 0.039167(\bar{d})^3 \\ -0.34713(\bar{d})^2 + 0.0076583(\bar{d})^1 + 0.99879 \end{bmatrix}
\end{aligned}
\tag{10}
$$

Matrix form of above equation is given below:

$$
\bar{f} = \begin{bmatrix} (\bar{d})^4 \\ (\bar{d})^3 \\ (\bar{d})^2 \\ (\bar{d})^1 \\ (\bar{d})^0 \end{bmatrix}^T
\begin{bmatrix}
-4.0386 & 6.6803 & -2.5612 & 0.080833 & -0.1375 \\
10.39 & -19.969 & 11.393 & -1.7967 & 0.039167 \\
-6.7846 & 13.916 & -9.1373 & 2.4669 & -0.34713 \\
1.7093 & -3.4531 & 2.1765 & -0.42841 & 0.0076583 \\
-0.14122 & 0.28733 & -0.18422 & 0.0383 & 0.99879
\end{bmatrix}
\begin{bmatrix} (\bar{x})^4 \\ (\bar{x})^3 \\ (\bar{x})^2 \\ (\bar{x})^1 \\ (\bar{x})^0 \end{bmatrix}
\tag{11}
$$

This equation was validated using MATLAB. The above equation predicts mode 1 frequencies (Fundamental Natural Frequency) of beam with any dimensions and material properties given that the limiting conditions are followed. Surface plots for the above equation are given in Figure 4 and 5.

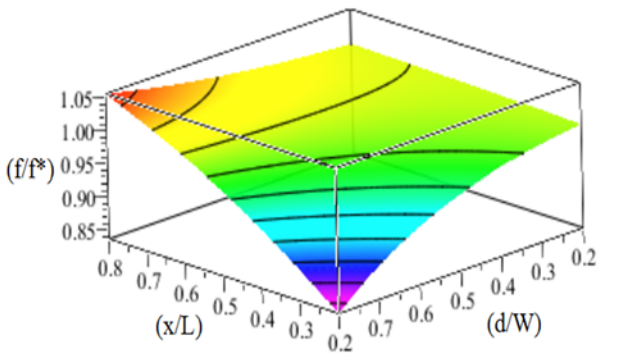

Figure 4. Surface plot 1

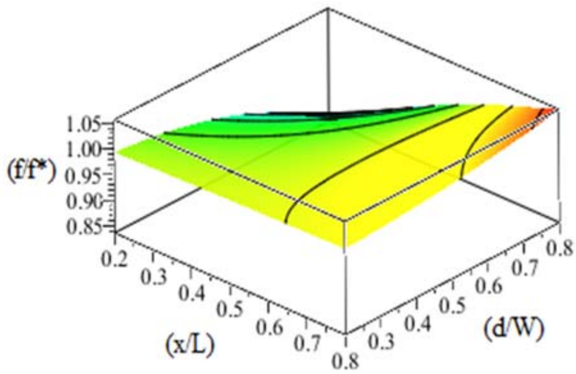

Figure 5. Surface plot 2

The black lines in the surface plots represent all points which have same $\bar{f}$ for different $\bar{x}$ and $\bar{d}$ values, which means to achieve a particular effective resonance frequency multiple sets of $\bar{x}$ and $\bar{d}$ exist.

6. VALIDATION

Validation of the results obtained from proposed approach i.e., equation no. 10, is given in this section. Validation is carried out by considering two scenarios, first within the test scope and second outside the test scope and also convergence study is discussed in this section. The summary of the results obtained from validation study is given in Table 8.

'Within the test scope' refers to taking values of material properties used for formulating the proposed equation i.e. AISI 1018 Mild Steel. 'Outside the test scope' refers to the values of material properties other than AISI 1018 Mild Steel.

- *Example 1:* (*Within the test scope*)

Mild Steel (*AISI 1018)*

$x = 350$ mm, $L = 1000$ mm, $d = 35$ mm, $W = 50$ mm

Equation no. 10 gives $\bar{f} = 0.9371 \Rightarrow f = 0.9371 \times f^*$ $= 0.9371*4.9627 =$ **4.6505617 Hz**

APDL gives **4.8003 Hz**

Error = 3.11%

- *Example 2:* (*Within the test scope*)

Mild Steel (*AISI 1018*)

$x = 100$ mm, $L = 250$ mm, $d = 30$ mm, $W = 50$ mm

Equation no. 10 gives $\bar{f} = 0.9695 \Rightarrow f = 0.9695 \times f^*$ $= 0.9695*79.988 =$ **77.548366 Hz**

APDL = **75.204 Hz**

Error = 3.11734%

- *Example 3:* (*Outside the test scope*)

Nickel Aluminium Bronze (*UNS C95800*)

$x = 500$ mm, $L = 680$ mm, $d = 40$ mm, $W = 50$ mm

Density = 7640 kg/m^3

Young's modulus = 117 GPa

Poisson's Ratio = 0.34

Equation no. 10 gives $\bar{f} = 1.0387 \Rightarrow f = 1.0387 \times f^*$ $= 1.0387*8.2256 =$ **8.57509172 Hz**

APDL gives **8.4760 Hz**

Error = 1.169%

- *Example 4:* (*Outside the test scope*)

AISI 1055 Carbon Steel (*UNS G10550*)

$x = 500$ mm, $L = 680$ mm, $d = 40$ mm, $W = 50$ mm

Density = 7850 kg/m^3

Young's modulus = 200 GPa

Poisson's Ratio = 0.285

Equation no. 10 gives $\bar{f} = 1.0387 \Rightarrow f = 1.0387 \times f^*$ $= 1.0387*10.627 =$ **11.0382649 Hz**

APDL gives **10.913 Hz**

Error = 1.148%

Note: f values in the following cases are found using ANSYS APDL for accuracy*

Table 8. Percentage error calculation*

Test cases	Within the test scope		Outside the test scope	
x, mm	350	100	500	500
d, mm	35	30	40	40
W, mm	50	50	50	50
L, mm	1000	250	680	680
E, GPa	205	205	117	200
ρ, kg/m^3	7870	7870	7640	7850
FEM (Hz)	4.8003	75.204	8.476	10.913
Proposed Equation (10) (Hz)	4.6505	77.548	8.575	11.038
% Error	3.120	3.116	1.168	1.145

All the FEM simulations were based on the results of the convergence study which yielded the finest mesh size. Meshing element used was SOLID 186 which is a 20 node quadratic hexahedron. Convergence can be seen in Figure 6.

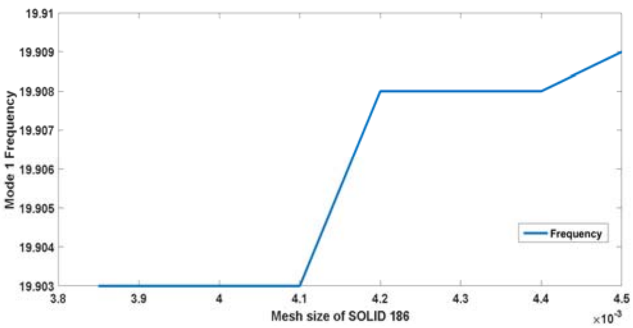

Figure 6. Convergence Study

Finest mesh was taken as 0.00385 m beyond which, the number of nodes crossed 32000. In the student version of ANSYS 15.0, maximum number of allowable nodes is 32000. When 0.00385 m was used as mesh size it resulted in 26117 nodes and 12668 elements.

7. RESULTS AND DISCUSSIONS

Effective resonance frequency is plotted against $\bar{x}$ for different hole diameters in Figure 2. The intersection points of the curves are shown below in Figure 7.

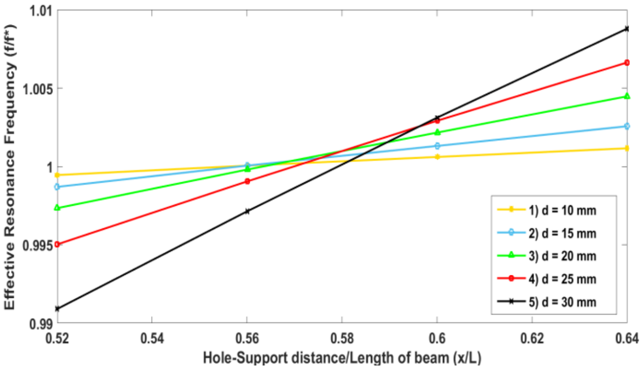

Figure 7. Intersection of curves

For the range of $\bar{x}$ from 0.55 to 0.6, the effective resonance frequency varies from 0.996 to 1.003 which is approximately equal to 1. Therefore, for the range of $\bar{x}$ from 0.55 to 0.6, frequency of perforated beam is approximately equal to the frequency of corresponding solid beam.

From the surface plots (Figure 4) it can be observed that for a particular value of $\bar{d}$ the effective resonance frequency $\bar{f}$ increases with $\bar{x}$. At lower values of $\bar{d}$, the change of $\bar{x}$ does not influence $\bar{f}$ as much as it influences when $\bar{d}$ value is high. At lower values of $\bar{x}$ i.e., 0.2 < $\bar{x}$ < 0.6, the change of $\bar{d}$ has more influence on $\bar{f}$. Also, this change in $\bar{d}$ has negative influence on $\bar{f}$. But at higher values of $\bar{x}$ i.e., 0.6 < $\bar{x}$ < 0.8, the change of $\bar{d}$ has comparatively less influence on $\bar{f}$. Besides this, in the range of 0.6 < $\bar{x}$ < 0.8, $\bar{d}$ affects $\bar{f}$ positively. It can also be seen that the absolute variation in effective resonance frequency is higher with change in $\bar{x}$ as compared to the change in $\bar{d}$.

8. CONCLUSIONS

This study deals with proposing an equation for predicting the natural frequency of perforated cantilever beams given the material properties and geometric parameters. It is valid for a single perforation placed at any distance on the centre line of the beam. Comparing the results of the proposed equation with ANSYS APDL gave percentage errors in the range of 1% - 3%.

For a particular range of $\bar{x}$, the effective resonance frequency is approximately equal to one. $\bar{d}$ has a positive impact on $\bar{f}$ when $\bar{x}$ is kept constant whereas $\bar{x}$ has a combined effect i.e., both positive and negative on $\bar{f}$ when $\bar{d}$ is kept constant. Also $\bar{x}$ has more influence on $\bar{f}$ than $\bar{d}$.

ACKNOWLEDGMENTS

Authors are thankful for the facility support from BITS Pilani University. This work was supported under Research Initiation Grant (RIG) provided by BITS-Pilani University to Dr. Kiran D. Mali, Assistant Professor, Dept. of Mech. Engg., BITS-Pilani, K.K. Birla Goa Campus.

REFERENCES

[1] Burgemeister, K. A., and C. H. Hansen., Calculating resonance frequencies of perforated panels, *Journal of Sound and Vibration*, 196.4, 387-399, 1996.

[2] Torabi, K., and A. R. Azadi., Vibration analysis for rectangular plate having a circular central hole with point support by Rayleigh-Ritz method, *Journal of Solid Mechanics*, 6.1, 28-42, 2014.

[3] Mali, Kiran D., and Pravin M. Singru., Determination of the fundamental frequency of perforated rectangular plates: Concentrated negative mass approach for the perforation, *Advances in Acoustics and Vibration*, 2013.

[4] Ghonasgi, Keya, Kalpit Bakal, and Kiran D. Mali., A Parametric Study on Free Vibration of Multi-Perforated Rectangular Plates, *Procedia Engineering*, 144, 60-67, 2016.

[5] Mali, Kiran D., and Pravin M. Singru., Determination of modal constant for fundamental frequency of perforated plate by Rayleigh's method using experimental values of natural frequency, *International Journal of Acoustics and Vibrations*, 20.3, 177-184, 2015.

[6] Gilbert-Rainer Gillich, Emilian Stanciu, Zoltan Iosif Korka, Zeno-Iosif Praisach, Codruta Hamat, Assessing Corrosion Damage from the Natural Frequency Changes, *Romanian Journal of Acoustics and Vibration*, vol. 14, no. 2, 2017, pag. 63-68.

[7] Cristian Tufisi, Gilbert-Rainer Gillich, Codruta Oana Hamat, Nicoleta Gillich and Zeno-losif Praisach, 2018. Numerical Study of the Stiffness Degradation Caused by Branched Cracks and its Influence on the Natural Frequency Drop. *Romanian Journal of Acoustics and Vibration*. 15, 1 (Aug. 2018), 53-57.

[8] Amit Sharma, Ashish Kumar Sharma, Mathematical Modeling of Vibration on Parallelogram Plate with Non Homogeneity Effect, *Romanian Journal of Acoustics and Vibration*, vol. 13, no. 1, 2016, pag. 53-57.

[9] Kanak Kalita, Ishwer Shivakoti, Ranjan Kumar Ghadai, Salil Haldar, Rotary Inertia Effect in Isotropic Plates Part I: Uniform Thickness, *Romanian Journal of Acoustics and Vibration*, vol. 13, no. 2, 2016, pag. 68-74.

[10] Kanak Kalita, Ishwer Shivakoti, Ranjan Kumar Ghadai, Salil Haldar, Rotary Inertia Effect in Isotropic Plates Part II: Tapper Thickness, *Romanian Journal of Acoustics and Vibration*, vol. 13, no. 2, 2016, pag. 75-80.