
Statistical Method for Damage Severity and Frequency Drop Estimation for a Cracked Beam using Static Test Data

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Abstract: - We present in this paper a statistical method to estimate the severity of a transverse crack from the free-end deflections of a cantilever beam. To this aim, we carry out simulations involving the ANSYS software and register the free-end deflections resulted for different crack positions near the fastened end. By involving a statistical method, we find the damage severity which reflects the crack effect on the frequency drop if the crack is located exactly at the fixture. The accuracy of the achieved damage severity values is verified by comparing the calculated frequency drops for several crack locations with those obtained from simulations. We found that the damage severity contrived by involving the statistical method is accurate and can be successfully used to predict the dynamic behavior of a cracked beam.

Keywords: - Statistical methods, Severity estimation, Transverse crack, Frequency shift, Cantilever beam.

1. INTRODUCTION

Transverse cracks locally modify the geometry of a structure due to the existence of discontinuities. As a consequence, the global stiffness of the structure is altered and a mass change can take place in some cases as well. The stiffness alteration generates the change of the modal parameters, among which the most affected are the eigenfrequencies, modal shapes and curvatures respectively damping ratios [1]. This happens because the stiffness loss causes a decrease in the quantity of energy that a structure can store [2]. If dynamic analysis is performed, the effect of a crack is best observable by the eigenfrequency drop, whose value depends on the position [3-5] and the dimensions [6-8] of the crack. The literature is rich in works that make a link between the crack location and dimensions (depth and width) with the eigenfrequency drop [9-12]. By applying inverse methods, these relations can be used to assess damages in structures.

Each crack can be assigned a parameter that uniquely characterizes that crack referred herein as damage severity. Quantification of this parameter is

made by some researchers involving the fracture mechanics theory [13-15], whilst other researchers use energy methods [16-19].

A problem that researchers face when they evaluate the damage severity is that its value apparently changes when the crack is moved along the beam. This is why the damage severity is considered for the crack located at the slice which is subjected to the highest bending moment. We have shown in prior research [12] that the severity is the same irrespective to the restraints a beam is subjected. Thus, it is convenient to analyze a cantilever beam because it has a single location with the highest bending moment. But, if taking the crack at this location, i.e. the fastened beam end, the frequency drop is smaller than the drop if the crack is located at a certain distance from this end. This is in disagreement with the relationship we developed for the frequency shift curves [20].

Another limitation if considering the crack at the fastened end is the impossibility to determine the damage severity for complex-shaped cracks, which may have a longitudinal extent [21].

In this paper, we investigate by means of a statistical method of how the damage severity can be estimated if the deflection increases due to several cracks located at the fastened end are known. It is shown that the values obtained in this way concord with those obtained from the theory.

2. PROBLEM FORMULATION

In prior research [12] we found following mathematical relation for the frequency of a beam with a breathing crack that has depth d and is positioned at distance c from a beam's end:

$$f_i(c, d) = f_{iU} \left\{ 1 - \gamma(0, d) \left[\bar{\phi}_i''(c) \right]^2 \right\}, \quad (1)$$

where f_{iU} is the frequency of the healthy beam. In this equation, i is the number of the weak-axis bending vibration mode, $\bar{\phi}_i''(c)$ is the normalized modal curvature at location c and $\gamma(0, d)$ is the severity of the damage. We found also a simple mathematical relation, deduced involving an energetic method, to calculate the damage severity, that is:

$$\gamma(0, d) = \frac{\sqrt{\delta_D(0, d)} - \sqrt{\delta_U}}{\sqrt{\delta_D(0, d)}}. \quad (2)$$

In the above equation δ_U is the beam's free-end deflection under dead mass and $\delta_D(0, d)$ is the same deflection for the beam having a crack with depth d at the fastened end, i.e. $c=0$. For all other crack locations $c \neq 0$ the pseudo-severity is expressed as:

$$\gamma(c, d) = \frac{\sqrt{\delta_D(c, d)} - \sqrt{\delta_U}}{\sqrt{\delta_D(c, d)}}. \quad (3)$$

Hence, from equations (1) to (3) result the relation of the pseudo-severity:

$$\gamma(c, d) = \frac{\sqrt{\delta_D(0, d)} - \sqrt{\delta_U}}{\sqrt{\delta_D(0, d)}} \left[\bar{\phi}_i''(c) \right]^2. \quad (4)$$

We plot in Figure 1 the eigenfrequency of the first vibration mode for the healthy cantilever beam (dotted line).

Next, we plot the evolution for of the frequency for the beam having a crack with depth $d=1\text{mm}$ which is replaced along the beam (blue continuous line). The frequencies $f_1(c, d)$, which are calculated from equation (1), are in good agreement with those

obtained by simulation. An exception is found for the crack located at the fastened end $c=0$. Here, the frequency derived with both methods, analytically and by simulation, does not fit the trend depicted by the blue line but is significantly bigger as expected. However, the values, represented with a violet square in Figure 1, are very similar.

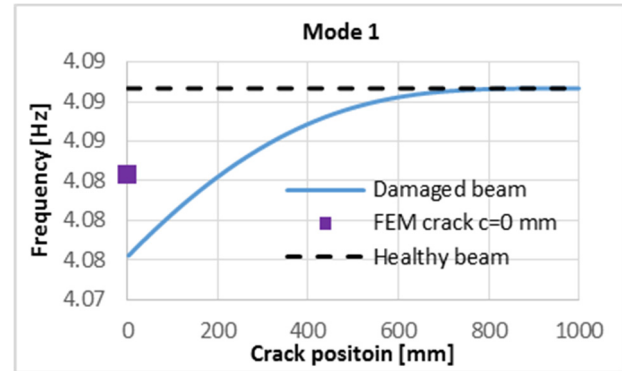


Figure 1. Eigenfrequency evolution with the damage location compared with the frequency of the healthy beam and that having a crack at the fastened end

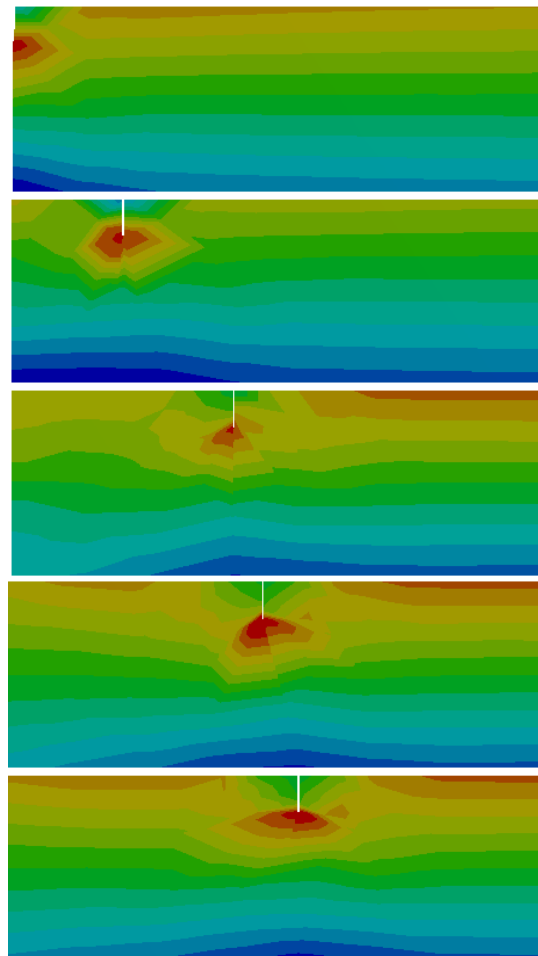


Figure 2. Frequency evolution with the damage location

Figure 2 illustrates the stress distribution around the cracks located at several locations near the

fastened end. As it can be observed, the crack located exactly at the fixing has stress, and consequently permit deformation, just on its right side. At the left side no deformation is permitted, dissimilar to all other cases. This causes a lower deflection and frequency drop, even if theoretically the highest values of these features should be obtained.

3. MATERIALS AND METHODS

The study is made by involving a beam which is fixed at the left and free at the right end (i.e. a cantilever beam). It has the length $L = 1000$ mm, the width $B = 50$ mm and a thickness $H = 5$ mm. The beam geometry, see Figure 3, is modeled using the *SolidWorks* design software, while the static and modal analysis is made by involving the *ANSYS* simulation software. The properties for the material assigned for the cantilever (structural steel) are indicated in Table 1. We use hexahedral mesh elements with the maximum edge 2 mm.

Table 1. Physical and mechanical properties of the material assigned to the cantilever beam

| Yield strength [MPa] | Ultimate strength [MPa] | Mass density ρ [kg/mm ³] | Young modulus E [N/m ²] | Poisson ratio ν [-] |
|----------------------|-------------------------|---|---------------------------------------|-------------------------|
| 250 | 460 | 7850 | $2 \cdot 10^{11}$ | 0.3 |

In the first simulation, we consider the healthy beam and find the free-end deflection and eigenfrequencies for the first four out-of-plane vibration modes. Afterward, we create transverse open cracks with depth $d = 1$ mm. The first crack is located exactly at the fastened end, the position being denoted $x_0 = 0$ mm and the following are replaced with a step $s = 3$ mm to the free end, until the location $x_{12} = 36$ mm is reached. A schematic of the beam, with a detailed view on the fixed end, and several crack positions is presented in Figure 3.

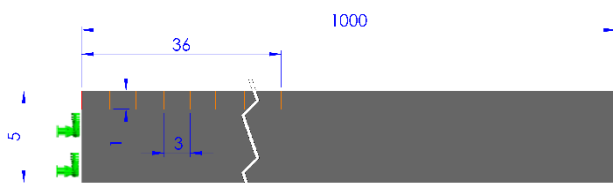


Figure 3. The beam with several transversal cracks

Simulations are made for the beam with all described cracks, considered one by one. Again, the free-end deflection and eigenfrequencies are found.

The aim of this study is to find the deflection and the eigenfrequencies for the crack located at $x_0 = 0$ mm from the trendline constructed on the results achieved for all other cracks, and to show that these values must be taken into consideration to estimate the damage severity. To this aim we use the regression analysis, which is a statistical method that allows examining the relationship between two or more variables of interest. In this way, it is possible to examine the influence of the independent variables on a dependent variable.

In fact, the regression determines which factors is most important in the study, which factors can be ignored, and how can these factors influence each other. The start point is to define for the study case the dependent variables as the main factor to predict and the independent variables that in our hypothesis have an impact on dependent variable.

The dependent variable (in our case the deflection δ respectively the eigenfrequency f) is plotted on the y -axis, and the independent variable (the crack location x) is plotted on the x -axis. The regression line can be precisely calculated using a standard statistics program like MS Excel. The formula for linear regression curve is:

$$y = ax + b + \varepsilon \quad (5)$$

or for the parabolic case, as:

$$y = ax^2 + bx + c + \varepsilon \quad (6)$$

In the above relations a , b and c are constants to be determined and ε is the error. To find out how the regression curve fit the points used to be constructed, one can calculate the R-squared value R^2 ; the closer R^2 is to one, the better curve fit the measurement results.

We extended the curve with the identified constants a , b and c is until achieve $x_0 = 0$ mm and the theoretical deflection is calculated for this value of x . Now, it is possible to estimate the crack severity and predict the frequency drop for different crack. These last values are compared with the simulation results to prove the correctness of the obtained severities.

4. RESULTS AND DISCUSSIONS

The simulation results in terms of free-end deflections, for the healthy beam and the beam with the described cracks, are registered and represented in graphical form in Figure 4. In this diagram one can observe that the smallest deflection is obtained, as expected, for the healthy beam.

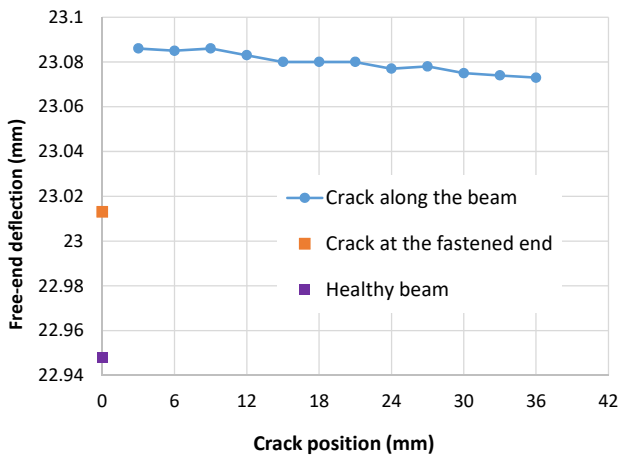


Figure 4. Deflection for the healthy bam and vs. crack position

From Figure 4 one can also observe that the deflection of the beam with a crack at the fastened end is significantly smaller as these for all other damage scenarios. To find the “theoretical” deflection, which should be used to define the crack severity, we plot the diagram in Figure 5 and use the MS Excel facility to find the mathematical relation for the trendline for the linear and parabolic regression curves.

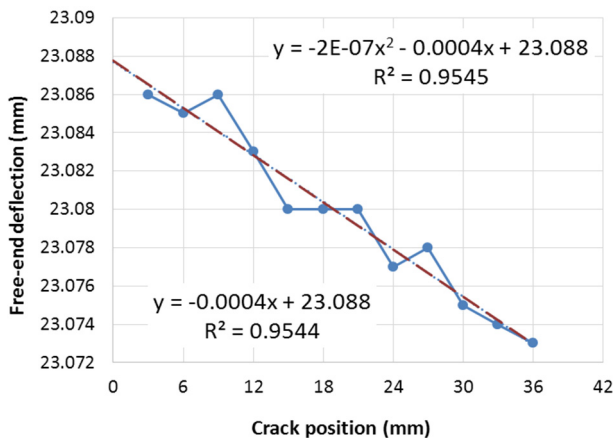


Figure 5. Trendlines (linear and parabolic regression) constructed using the deflections of the cracks located near the fixed end, and the associated relations

As one can observe in Figure 5, the points used for interpolation have a distribution that makes irrelevant the degree of the mathematical function, both curves taking the value 23.088 for the crack position $x_0 = 0$. So, we can assume the theoretical deflection as being $\delta_D(0,1) = 23.088$ mm. On the other hand, the deflection for the healthy beam is found to be $\delta_U = 22.948$ mm. From relation (2), we obtain the damage severity $\gamma(0,1) = 0.003036$.

Knowing the values for the severity and for the normalized squared modal curvatures (NSMC), we can now calculate the bracket in equation (1) for any damage position. For exemplification, we use the locations presented in Table 2, where also the values for the NSMC for the first four out-of-plane vibration modes are presented.

Table 2. The normalized squared modal curvatures for the first four modes in respect to the chosen locations

| Crack location | | 110 (mm) | 350 (mm) | 480 (mm) | 650 (mm) |
|----------------|--------|----------|----------|----------|----------|
| NSMC | Mode 1 | 0.72165 | 0.27641 | 0.13206 | 0.04007 |
| | Mode 2 | 0.22887 | 0.22260 | 0.49539 | 0.42214 |
| | Mode 3 | 0.02459 | 0.38638 | 0.00835 | 0.40572 |
| | Mode 4 | 0.02042 | 0.00208 | 0.48500 | 0.00856 |

Using equation (1) we calculate the frequencies for the four damage cases. The results are provided in Table 3, along with the results obtained from simulation. The comparison is made by calculating the errors for all crack locations and vibration modes, and extremely small errors are found.

Table 3. Frequencies and errors for the cracked beams

| Crack location: 110 mm from the fastened end | | | | |
|--|----------------|----------|----------|----------|
| Calculation method | Frequency (Hz) | | | |
| | Mode 1 | Mode 2 | Mode 3 | Mode 4 |
| Analytic | 4.081042 | 25.60947 | 71.75133 | 140.6229 |
| FEM | 4.081131 | 25.60937 | 71.75084 | 140.6244 |
| Error | 0.002178 | -0.00041 | -0.00068 | 0.001077 |
| Crack location: 350 mm from the fastened end | | | | |
| Calculation method | Frequency (Hz) | | | |
| | Mode 1 | Mode 2 | Mode 3 | Mode 4 |
| Analytic | 4.086571 | 25.60996 | 71.67252 | 140.6307 |
| FEM | 4.086592 | 25.61064 | 71.67478 | 140.6303 |
| Error | 0.000526 | 0.002657 | 0.003162 | -0.00031 |
| Crack location: 480 mm from the fastened end | | | | |
| Calculation method | Frequency (Hz) | | | |
| | Mode 1 | Mode 2 | Mode 3 | Mode 4 |
| Analytic | 4.088331 | 25.58863 | 71.75467 | 140.4262 |
| FEM | 4.088363 | 25.58874 | 71.75487 | 140.4246 |
| Error | -0.00078 | -0.00043 | -0.00027 | 0.00119 |
| Crack location: 630 mm from the fastened end | | | | |
| Calculation method | Frequency (Hz) | | | |
| | Mode 1 | Mode 2 | Mode 3 | Mode 4 |
| Analytic | 4.089517 | 25.59496 | 71.67093 | 140.6276 |
| FEM | 4.089505 | 25.59444 | 71.66683 | 140.628 |
| Error | 0.000283 | 0.002032 | 0.003662 | -0.00028 |

Because the eigenfrequencies of the cracked beams for the crack locations chosen randomly are found with accuracy, the errors being less than 0.004%, we conclude that the severity is correctly estimated from the trendline derived involving the linear regression method. This qualifies the severity deduced in this way to be used in vibration-based damage detection.

5. CONCLUSIONS

In this paper we show how the fastening of a beam end affects the stress distribution if a crack arises at that location. We show in the first part that the eigenfrequency for beams with a transverse crack located at the fastened end, if calculated with the severity deduced from an energy method, does not frame in the frequency shift curve. This happens neither if the eigenfrequency is obtained from simulations or measurements. Because the frequency should attain the lowest value for the crack located at the fastened end, misinterpreting in crack evaluation is a certainty. To overcome this inconvenience, we designed a methodology to assess the damage severity using statistical analysis of static test data. The severity calculated in this way was used to predict the frequency for beams having randomly located cracks. Values that accurately fit the simulation results were obtained for all analyzed damage case.

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