

---

---

# The Effects of Number and Location of Braced Bays on Redundancy of Eccentrically-Braced Steel Moment Frames

**Alireza FAROUGHI**

*Assistant Professor, Department of civil engineering, East Tehran Branch, Islamic Azad University, Tehran, Iran, faroughi@gmail.com*

**Abdolreza S. MOGHADAM**

*Associate Professor of International Institute of Earthquake Engineering and Seismology (IIEES), Tehran, Iran, Moghdam@iiees.ac.ir*

**Mohammad GHANOONIBAGHA**

*Assistant Professor, Department of civil engineering, East Tehran Branch, Islamic Azad University, Tehran, Iran, ghanoonibagha@alumni.iust.ac.ir*

*Abstract:* Steel braced frames are among the most effective lateral load resisting systems. The eccentrically braced frame has a higher ductility, less stiffness, and a better serviceability (limited retrofitting needed after the seismic excitations) compared to concentrically braced ones. Static indeterminacy is generally considered as the number of extra constraints or members in a structural system. In other words, redundancy is the ability of a structure to keep its stability after the onset of the failure or removal of its lateral load resisting elements, preventing it from collapse in the initial stages of loading. The redundancy of a system depends on the “dynamic” characteristics of its structure and properties of the applied earthquake excitations as well as its structural geometric features and details. This indeterminacy is called dynamic redundancy.

In this study, the effects of the number and location of braced bays on redundancy were investigated to obtain the best pattern for regular 5 and 8-story buildings using nonlinear analysis. It also develops a relationship for the redundancy factor of the dual structural systems. Results show that position of braced bays is more effective on redundancy than the number of bays. It is also concluded that the building’s height affects the required number of bracing bays. The study also showed that the dynamic redundancy is different from the over-strength. According to the results of this research, for dual structural systems, it is recommended not to arrange the braces only in the perimeter of the plan.

*Keywords:* Indeterminate, redundancy, eccentrically braced moment frames, time history analysis, earthquake specifications

---

## 1. INTRODUCTION

Generally, indeterminacy is the number of members or additional constraints that exist in a structural system, known as "static indeterminacy." One of the advantages of this type of indeterminacy is that higher static indeterminacy results in higher reliability to prevent the collapse of the structure after the first failure or destruction of the members of the lateral load resisting system. This feature is called redundancy. The redundancy factor of a structure results from the potential of providing an alternative path for load transferring after the failure of the first structural member. Some of the effective parameters on the redundancy of a system subjected to static [1] and dynamic loads [2] have been introduced in the literature including the number of lateral load resisting elements, ductility capacity of the members, deterioration characteristics of the members,

structural configuration and diversity of demands caused by an earthquake. Many studies since the 1994 Northridge earthquake have studied the effect of the capacity of the beam-column connections on the reliability and redundancy of the structures [2][3]. However, it is difficult to quantify the redundancy, especially for complex structures. This difficulty is caused by the explanation of the nonlinear and response behavior of the structural system in addition to evaluating the probability of local (in one or more members) and global collapse. The redundancy factor has suggested being considered based on the first failure in the member, and the total collapse for the simple regular structures [4]. Since the determination of the redundancy is more complicated especially for dual systems with eccentrically braced frames and depends on the numbers and properties of the braced frames in addition to the moment frames properties, further investigations are needed for this issue. This topic has been studied in the present research.

Eccentrically braced frames were introduced by Popov for the first time [5]. This system is developed to withstand lateral forces and control large deformations and displacements, especially in high-rise buildings. Extensive researches have shown an eccentrically braced system is capable of combining high stiffness especially in the inelastic phase of the material. Eccentrically braced frames are hybrid systems. The primary objective for using eccentrically braced frames is to locate the shear yielding in a short part of the main beam known as the link-beam [6][7][8][9][10]. In this paper, Static Indeterminacy and Dynamic Redundancy are introduced and calculated for 28 structural models with different numbers and locations of bracing bays.

## 2. STATIC INDETERMINACY

Indeterminacy is typically defined as the number of extra members and constraints (or restraints) and thus extra unknowns in a system to increase the safety level of a structure's stability. Higher degrees of indeterminacy results in a higher safety factor to prevent the immediate collapse of the structure after the first failure of the lateral load resisting members. In other words, indeterminacy includes the lateral load resisting elements or extra constraints which prevent the structure from collapse after the initial damage. The importance of this issue is indicated when this parameter is included in the fourth edition of the 2800 standard (Iranian code for seismic design of the buildings). The most important effect of the static indeterminacy is the redundancy or capacity of providing an extra (alternative) load transfer path after the first failure in the member. As can be seen in Figure (3), redundancy leads to redistribution of the load to other parts of the structure in the case of collapse or overstress of the members [14].

## 3. DYNAMIC REDUNDANCY

The initial significant studies on this dynamic redundancy and some other investigations on the frames resistant against the wind were conducted by Moses [12]. His efforts showed that the frame's behavior depends on the strength and loading. Therefore, the reliability of the single-bay frames is less than multi-bay frames [13]. The study by Bertero et al. is one of the important references about dynamic redundancy [13]. In this study Redundancy is defined as follows: The indeterminacy degree of a system stated by "n" is the minimum number of critical regions or plastic hinges of a structural system that yield or collapse under the earthquake excitations. Therefore, the degree of redundancy in a system is not

just influenced by the geometrical characteristics and details of the structure but also depends on the dynamical characteristics of the structure and earthquake record [14].

### 3.1. Active and Inactive Dynamic Redundancies (collapse order)

The redundancy degree of a system can be either active or inactive, as some redundancy combinations are inactive and they become active when some active redundancies collapse. For a structural system, redundancy is usually active. In special cases, the structural system with a high redundancy may have a soft story. This may lead to form a mechanism in the soft story before the creation of the plastic hinges and make the structure collapse. In this case, the redundancy degree will not be activated so the inactive redundancy has no effect on the dynamical redundancy degree.

### 3.2. Dynamic Redundancy Coefficient

Quantifying the redundancy in a structure is generally difficult particularly in sophisticated structures. The equation of the response modification factor is introduced in In ATC-19(1995). This factor includes a  $R_R$  coefficient in order to consider the effects of redundancy on structure's behavior [4].

The response modification factor equation is as follows [16]:

$$R = R_{\mu} R_S R_R , \quad (1)$$

The overall effects of redundancy on the ultimate capacity of a structure can be completely explained by the ratio of the ultimate strength of a structural system to the ultimate strength of a similar system without redundancy. Therefore, the redundancy factor  $R_R$  indicating the redundancy, could be obtained as follows: [17]

$$R_R = \frac{S_u}{S_{nr}} , \quad (2)$$

where  $S_u$  and  $S_{nr}$  are the strengths of redundant and non-redundant structural systems, respectively. Assuming that the strength of a structure is distributed normally, the design or characteristic strength of a structural system can be expressed as a function of its mean and standard deviation [17].

Hence  $S_u$  and  $S_{nr}$  parameters can be written as follows.

$$S_u = \bar{S}_u - k\sigma_f , \quad (3)$$

$$S_{nr} = \bar{S}_{nr} - k\sigma_{nr}, \quad (4)$$

Where  $\bar{S}_u$  and  $\bar{S}_{nr}$  are the mean values of the ultimate strength of the redundant and non-redundant frames, respectively.  $\sigma_f$  and  $\sigma_{nr}$  are the standard deviation values of the strength of the redundant and non-redundant frames, respectively. By dividing standard deviation of the frame strength (equation 6) by the average strength of the frame (equation 5), the strength variation coefficient of the frame (equation 7) can be obtained:

$$\bar{S} = nc\bar{V}_e, \quad (5)$$

$$\sigma_f = c\sigma_f\sqrt{n + n(n-1)\rho_e}, \quad (6)$$

$$v_f = v_e\sqrt{\frac{1+(n-1)\rho_e}{n}}, \quad (7)$$

Where  $\bar{V}_e$  is the average shear strength of the structural member,  $n$  is the number of the hinges according to the failure or collapse mechanism,  $c$  is coefficient,  $\rho_e$  is the correlation coefficient between the strength of the members (assuming that every two pairs of members are the same).  $v_e$  and  $v_f$  are the variation coefficient of elements strengths and the strength variation coefficient of the frame, respectively. The redundancy variation index equals to the ratio of these two coefficients.

$$r_v = \frac{v_f}{v_e} = \frac{\sigma_f/\bar{S}_u}{v_e}, \quad (8)$$

$$r_v = \frac{\sigma_f}{v_e r_s \bar{S}_{nr}}, \quad (9)$$

$$\sigma_f = r_v r_s v_e \bar{S}_{nr}, \quad (10)$$

In equations above,  $r_s$  is the redundancy resistance index that can be obtained as follows: [17], [18]

$$r_s = \frac{\bar{S}_u}{\bar{S}_{nr}} = \frac{\bar{S}_u}{\bar{S}_y}, \quad (11)$$

In equation (11) in order to calculate yield strength and ultimate strength of the structural system, the mean value of the strength (not nominal strength that is used for design based on nominal values of the material properties and other variables) is considered. Therefore, the following variables are used for computing the redundancy resistance index:

- The base shear in the outset of the system yielding ( $S_y$ ).

- The ultimate base shear ( $S_u$ )

In the non-linear static analysis method, these two parameters can be easily obtained from a non-linear load-displacement curve (Figure 1).

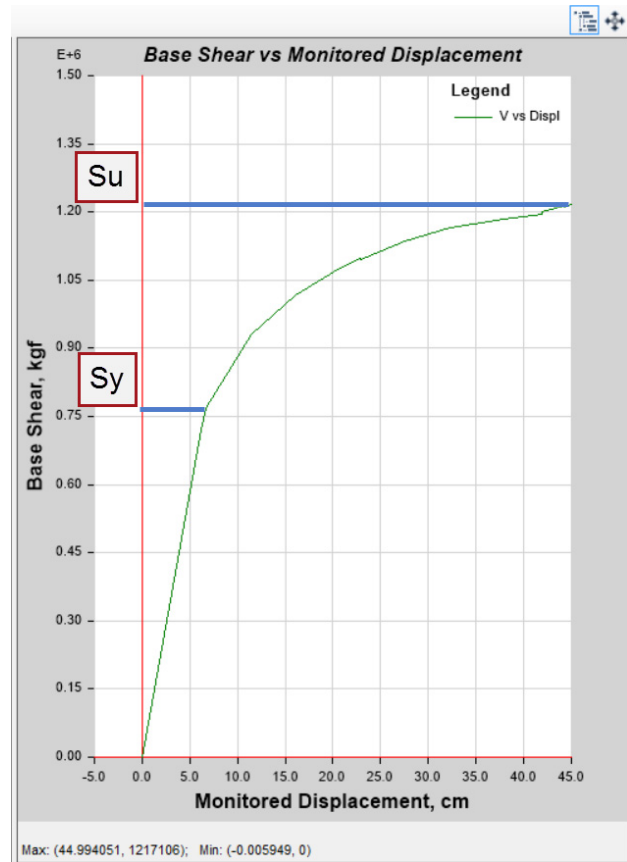


Figure 1. Sample of structural behavior diagram related to the equation 11 from which the elastic and final strengths can be obtained

Combining equations 10 and 11 with equation 3 will result in the following equation:

$$S_u = r_s \bar{S}_{nr} - k r_v r_s v_e \bar{S}_{nr} = r_s (1 - k r_v v_e) \bar{S}_{nr}, \quad (12)$$

Finally, combining equations 4 and 12 with equation 3 will result in:

$$R_R = r_s \left( \frac{1 - k v_e r_v}{1 - k v_{nr}} \right), \quad (13)$$

As a result,  $v_{nr} = \frac{\sigma_{nr}}{\bar{S}_{nr}}$  is the strength variation coefficient of a non-redundant frame.

A non-redundant frame should be modeled as series systems in which failure of a single member leads to the overall failure of the system and safety factor of the system would be equal to the average safety factor of the member. Hence for a non-redundant system  $v_{nr} = v_e (n = 1)$  and as a result:

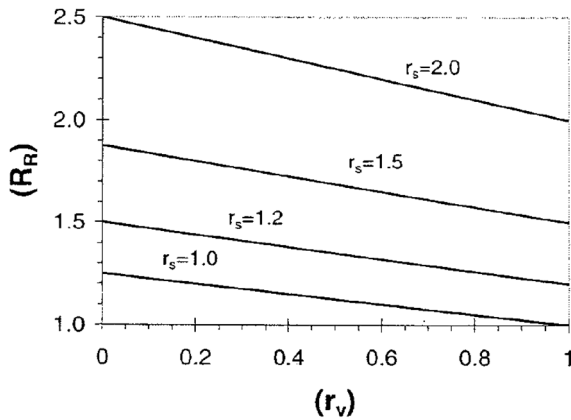
$$R_R = r_s \left( \frac{1 - kv_e r_v}{1 - kv_e} \right), \quad (14)$$

For the strength with a normal distribution an occurrence probability between 85% to 95%, the  $k$  factor varies between 1.5 to 2.5 and the variation coefficient of member,  $v_e$ , will vary in the range of 0.08 to 0.14 [18].

Therefore, a value of  $kv_e=0.2$  with acceptable an accuracy for computing the effects of  $r_s$  and  $r_v$  on  $R_R$  may be used. [16].

According to equation (14) and the considered value for  $kv_e$ ,  $R_R$  expression will be simplified as Figure 2 and the following equation:

$$R_R = r_s \left( \frac{1 - 0.2r_v}{0.8} \right) = 1.25r_s(1 - 0.2r_v), \quad (15)$$



**Figure 2.**  $R_R$  variations over  $r_s$  and  $r_v$  coefficients, considering  $kv_e = 0.2$  [18]

According to the equation (15) and figure (6), if a system is infinitely redundant ( $r_v=0$ ),  $R_R$  value will be equal to  $1.25r_s$  ( $R_R = 1.25r_s$ ) which means that the redundancy has a maximum probable effect of 25% on the redundancy factor. If a system is non-redundant ( $r_v=1$ ),  $R_R$  value will be equal to  $r_s$  ( $R_R = r_s$ ) and it means that probable effect of redundancy is zero [17], [18].

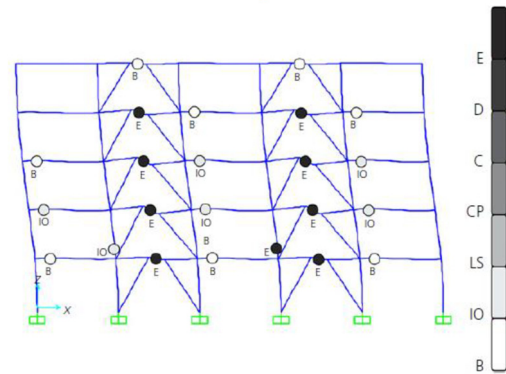
On the other hand, if the strength range of the system does not increase from yielding limit to the ultimate limit ( $r_s=1$ ),  $R_R$  variations will range between 1 and 1.25 and if the increase in the strength is more than 1 ( $r_s=1.4$ ),  $R_R$  variations will range between 1.4 and 1.75 [18].

### 3.2.1. Redundancy of the dual structural systems

Generalizing equation (15) to the dual structural system was one of the main challenges in this research, especially for the steel moment frame with

eccentrically braces. First assumption is that the redundancies of the two subsystems are added together. As redundancy can be inactive, according to possibility of the collapse of the system before using the whole capacity of all members summation of the system's redundancy is incorrect. Instead, considering the summation of the active dynamic redundancy using the effective resistances during the collapse seems to be a more logical assumption.

Since the redundancy coefficient is computed in the stage of strength limit state and before entering the structural ductility range, in this stage each of the subsystems reaches a percentage of its capacity. For the braced frames this percentage is more than other parts of the structure due to their high stiffness. The dual frame view in this stage can be seen in Figure 3.



**Figure 3.** View of collapse moment and beginning of strength reduction in braced frame

Considering dual system in equation (2) and sum of the strengths we will have the following equation:

$$R_R = \frac{\alpha S_{u1} + \beta S_{u2}}{\gamma S_{nr1} + \eta S_{nr2}}, \quad (16)$$

Relation (16) shows that system resistance in collapse step is equal to the sum of final resistances (capacities) of the subsystem ( $\alpha S_{u1} + \beta S_{u2}$ ). by replacing relation (3) and (4) into equation (16) we can write:

$$R_R = \frac{\alpha[\bar{S}_{u1} - k\sigma_{f1}] + \beta[\bar{S}_{u2} - k\sigma_{f2}]}{\gamma[\bar{S}_{nr1} - k\sigma_{nr1}] + \eta[\bar{S}_{nr2} - k\sigma_{nr2}]}, \quad (17)$$

$\alpha$ : force to capacity ratio of the first subsystem in the strength limit state.

$\beta$ : Force to capacity ratio of the second subsystem in the stage of strength limit state

$\gamma$ : Force to capacity ratio of the first system in the non-linear stage

$\eta$ : Force to capacity ratio of the second system in the non-linear stage.

Therefore, the average of final resistance for redundant system,  $\bar{S}_u$ , and non-redundant system,  $\bar{S}_{nr}$ , for dual frame will be:

$$\alpha\bar{S}_{u1} + \beta\bar{S}_{u2} = \bar{S}_u, \quad (18)$$

$$\gamma\bar{S}_{nr1} + \eta\bar{S}_{nr2} = \bar{S}_{nr}, \quad (19)$$

Replacing equations (18) and (19) in equation (17) results in:

$$R_R = \frac{\bar{S}_u - k(\alpha\sigma_{f1} + \beta\sigma_{f2})}{\bar{S}_{nr} - k(\gamma\sigma_{nr1} + \eta\sigma_{nr2})}, \quad (20)$$

$$R_R = \frac{\bar{S}_u - k(\alpha r_{v1} v_{e1} \bar{S}_{u1} + \beta r_{v2} v_{e2} \bar{S}_{u2})}{\bar{S}_{nr} - k(\gamma v_{nr1} \bar{S}_{nr1} + \eta v_{nr2} \bar{S}_{nr2})}, \quad (21)$$

Considering a normal distribution for redundancy, the variation coefficients of dual system of the braced and moment frames are obtained as follows:

$$v_{nr1} = v_{nr2} = v_e, \quad (22)$$

$$v_{e1} = v_{e2} = v_e, \quad (23)$$

Therefore:

$$R_R = \frac{\bar{S}_u - k v_e (\alpha r_{v1} \bar{S}_{u1} + \beta r_{v2} \bar{S}_{u2})}{\bar{S}_{nr} - k v_e (\gamma \bar{S}_{nr1} + \eta \bar{S}_{nr2})}, \quad (24)$$

$$R_R = \frac{\bar{S}_u - 0.2(\alpha r_{v1} \bar{S}_{u1} + \beta r_{v2} \bar{S}_{u2})}{\bar{S}_{nr} - 0.2(\bar{S}_{nr})}, \quad (25)$$

$$R_R = \frac{\bar{S}_u (1 - 0.2 \frac{(\alpha r_{v1} \bar{S}_{u1} + \beta r_{v2} \bar{S}_{u2})}{\bar{S}_u})}{\bar{S}_{nr} (1 - 0.2)}, \quad (26)$$

$$R_R = 1.25 r_s \left( 1 - 0.2 \frac{\alpha r_{v1} \bar{S}_{u1} + \beta r_{v2} \bar{S}_{u2}}{\bar{S}_u} \right), \quad (27)$$

$\alpha\bar{S}_{u1}$  and  $\beta\bar{S}_{u2}$  are contributions of the two subsystems in the total base shear capacity  $\bar{S}_u$ . Therefore, the term inside parentheses of relation (27) is the sum of percentages of subsystems' resistance in dual system with their redundancy changes index effect.

In a 3D structural system with "m" similar parallel plane frames, it is assumed that after collapse of "m-1" plane frames the system loses its torsional strength completely and becomes unstable. In such a case, redundancy variation index equals to [16]:

$$r_v = \frac{v_s}{v_e} = \sqrt{\frac{1+(n-1)\bar{\rho} + (m-2)\bar{\rho}_f}{n(m-1)}}, \quad (28)$$

If the structure is composed of a series of frames and completely uncorrelated members then:

$$\bar{\rho}_f = 0, \bar{\rho} = 0, \quad (29)$$

Therefore,  $r_v$  changes to:

$$r_v = \sqrt{\frac{1}{n(m-1)}}, \quad (30)$$

Where  $m$  and  $n$  are the number of seismic frames and hinges, respectively.

Therefore, the redundancy coefficient is sensitive to the redundancy variation index which depends on the number of seismic frames and hinges. This is the definition of 'dynamic (active) redundancy'.

## 4. METHODOLOGY

### 4.1. The statistical population and sample size

1. The statistical population: ordinary offices or residential structures

2. Sampling size: review, analysis and design of 28 different structural systems, including 5-baysquare plan buildings with bay size of 5 m and number of stories of 5 and 8 with 3 m story height. The length of link beams is 1m and they have basically shear behavior [19-22].

3. Calculation method: inelastic behavior modeling and analysis as per ASCE [20] and instruction for seismic rehabilitation (MPO 360, 2014) have been applied.

### 4.2. Design Considerations

The required number and length of the braces where selected according to the past experiences assuming 10 to 30 percent of the structure's area. 28 models where used according to the importance of that models.

Box Sections were used in models with the sizes varying from 8 to 40 according to their function. Beams are mainly I-shaped (No. 16 to 27). The material used in all cases is steel ST37. The analyzed models have been respectively numbered 1 to 28 from left to right.

### 4.3. Methods of Analysis and Design

#### 4.3.1. Design of the Structure

The linear and elastic design of the structures is based on Iranian National Codes and 2800 standard (The Iranian code for the seismic design of structures) [15].

#### 4.3.2. Non-linear static push-over analysis

Non-linear model of sections is assigned to those locations of members which have the highest

probability of inelastic behavior. For example, for beams with rigid connection, bending hinges is assigned at both ends of member and also in columns, at both ends (in the absence of concentrated load). In these cases, the non-linear behavior of materials is considered by introducing the plastic hinge parameters. In this method in order to evaluate a structure, it is analyzed under the effect of uniform loading pattern and this analysis is carried out by considering the displacement of the roof center of masses the control point and checking whether it reaches the level that has previously been calculated.

In the present study this displacement value, so-called the target displacement, has been calculated with regard to the particular purpose of performance and have been used as the basis for evaluating members of a structure. When deformation, rotation, and forces of members reach to this stage, evaluation will be carried out [15]. So based on the results of a nonlinear static analysis mentioned above, the variables required to calculate the Redundancy coefficient are calculated in accordance with equation 27 and finally, it is summarized.

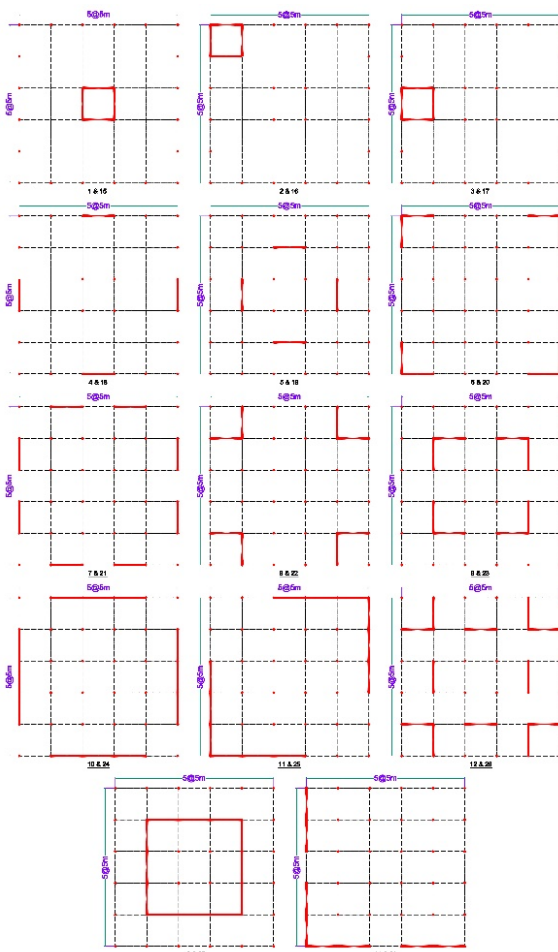


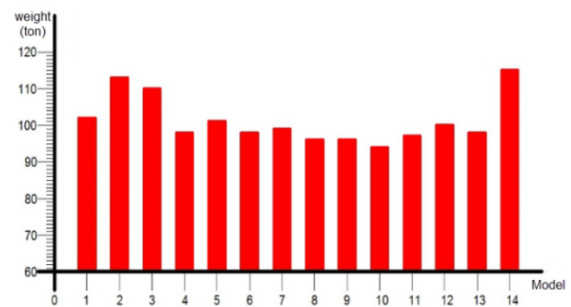
Figure 4. Model Plans.

## 5. ANALYSIS AND RESULTS

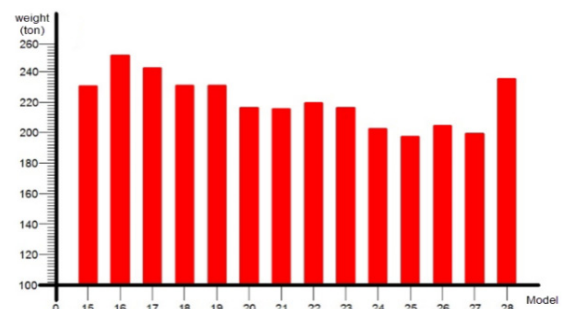
In this part the results are discussed. The models are divided into two categories that include 5-story low-rise and 8-story mid-rise buildings.

### 5.1. Weights of Models

Weights of models diagram as an economical index and table of number of hinges are as follows:



a)



b)

Chart 1. Weights of models to the number of model: a) 5-story models, b) 8-story models

Two categories of diagrams of models weights versus number of models in 5 and 8-story buildings are shown in Figure 5. This figure shows that increase of the number of bracing bays and their regular distributions result in a considerable reduction in weights of the models. Irregular layout of the bracings in models 14 and 28 despite the increase in the number of braced bays result in an increase in the weight of the structure.

### 5.2. Categorization and Analysis of Data

Based on the aforementioned analysis method in section 3-3-2 (non-linear static), 28 models were investigated. Details of the analyses are presented in Table 1, in which Coefficients, mentioned in equations 11, 27 and 30, have been calculated in stage of maximum strength (beginning of the strength degradation stage).

**Table 1.** Results of non-linear static analyses of models and plastic hinges

Model	No stories	N.B.C.C.B		N.B.C.C.M.F		N.B.C.B		N.B.C.BR	
		Push X	Push Y	Push X	Push Y	Push X	Push Y	Push X	Push Y
1	5	0	0	0	0	8	8	0	0
2	5	0	0	3	0	6	10	0	0
3	5	0	0	0	0	10	10	0	0
4	5	0	0	0	0	10	10	0	0
5	5	0	0	0	0	8	8	0	0
6	5	2	4	0	0	12	12	0	0
7	5	4	4	0	0	12	12	0	0
8	5	3	4	0	0	12	12	0	0
9	5	4	4	0	0	12	12	0	0
10	5	0	0	0	0	12	12	0	0
11	5	0	0	0	0	12	12	0	0
12	5	0	0	0	0	12	12	0	0
13	5	0	0	0	0	12	12	0	0
14	5	0	0	3	0	20	20	0	0
15	8	0	0	0	2	16	16	0	0
16	8	4	0	43	12	22	16	0	0
17	8	0	0	0	1	14	10	0	1
18	8	0	0	2	1	14	16	0	0
19	8	0	0	0	0	14	14	0	0
20	8	0	0	6	2	28	28	0	0
21	8	4	0	0	1	24	24	0	0
22	8	0	0	4	8	24	24	0	0
23	8	2	2	0	0	24	24	0	0
24	8	2	3	0	2	36	36	0	0
25	8	0	1	1	1	30	30	0	0
26	8	0	0	0	0	36	36	0	0
27	8	2	0	2	2	30	30	0	0
28	8	0	0	3	8	32	32	0	0

N.B.C.C.M.F: Number of beyond CP hinges in columns connected to moment frame  
 N.B.C.C.B: Number of beyond CP hinges in bracings  
 N.B.C.B: Number of beyond CP hinges in beams  
 N.B.C.BR: Number of beyond CP hinges in bracings

According to Table 1, in models that columns are connected to the braces in two directions, and columns are imposed to the seismic forces in two directions, number of hinges beyond the CP performance level has significantly increased, compared to the models with disperse braces. This effect appears to reduce the columns' flexural capacity by increasing the axial force. This case has not been addressed in codes.

It is also concluded that the effect of the position of the braces (irregular or present in the perimeter) on the number of moment frame's failure hinges is much more effective than their small number.

### 5.3. Calculate of Redundancy

At the end, based on the analysis method in section 3-2-1 (eq. 27) and Table 1 for calculating  $r_v$  (eq. 30), Redundancy of 28 models are calculated and mentioned in Table 2.

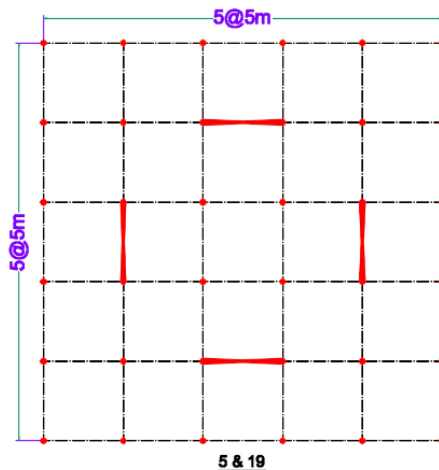
Among the 5-story models, No. 3, 5, and 14 have the most redundancy. Model No. 3 significantly uses

the moment frame capacity (in the Y direction), so its over-strength and redundancy are high, and Model no.5 is absent due to the lack of a common column and the arrangement braces inside the plan has the most dynamic redundancy.

**Table 2.** Results of non-linear static analysis of models and redundancy coefficients

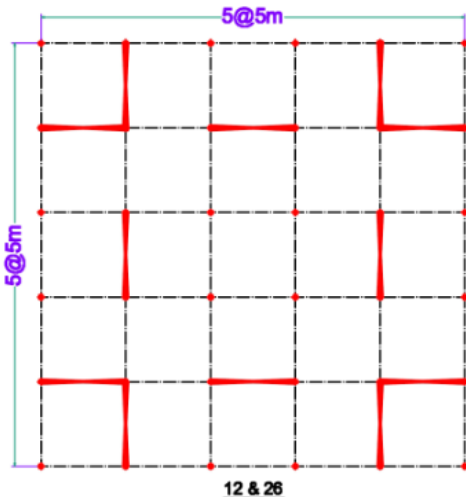
model	story	over strength		MR redundancy variation index		EB redundancy variation index		RR	
		rsx	rsy	rvx	rvy	rvx	rvy	X	Y
1	5	1.74	1.85	0.01	0.01	0.11	0.11	2.13	2.28
2	5	1.90	1.85	0.00	0.00	0.11	0.10	2.36	2.30
3	5	1.83	2.40	0.01	0.01	0.09	0.08	2.27	2.97
4	5	1.76	1.77	0.01	0.01	0.08	0.09	2.18	2.19
5	5	2.02	2.07	0.01	0.01	0.09	0.09	2.50	2.56
6	5	1.70	1.67	0.01	0.01	0.04	0.05	2.11	2.08
7	5	1.66	1.73	0.01	0.01	0.05	0.04	2.06	2.15
8	5	1.66	1.70	0.01	0.01	0.04	0.04	2.06	2.10
9	5	1.66	1.68	0.01	0.01	0.04	0.04	2.05	2.09
10	5	1.50	1.57	0.01	0.01	0.03	0.03	1.86	1.95
11	5	1.63	1.69	0.01	0.01	0.02	0.02	2.03	2.10
12	5	1.58	1.59	0.01	0.01	0.03	0.03	1.96	1.98
13	5	1.60	1.65	0.01	0.01	0.03	0.03	1.99	2.05
14	5	2.13	2.17	0.01	0.00	0.02	0.02	2.65	2.70
15	8	1.83	1.85	0.00	0.00	0.06	0.05	2.27	2.30
16	8	1.72	2.22	0.00	0.00	0.03	0.05	2.14	2.77
17	8	1.93	1.90	0.00	0.00	0.07	0.09	2.38	2.36
18	8	1.93	1.94	0.00	0.00	0.07	0.07	2.39	2.40
19	8	2.21	1.79	0.00	0.00	0.07	0.07	2.74	2.22
20	8	1.68	1.66	0.00	0.00	0.03	0.03	2.10	2.07
21	8	1.78	1.82	0.00	0.00	0.03	0.03	2.21	2.27
22	8	1.67	1.66	0.00	0.00	0.03	0.03	2.08	2.07
23	8	1.61	1.62	0.00	0.00	0.03	0.03	2.00	2.02
24	8	1.60	1.61	0.00	0.00	0.02	0.02	1.99	2.00
25	8	1.70	1.71	0.00	0.00	0.01	0.01	2.12	2.13
26	8	1.72	1.74	0.00	0.00	0.02	0.02	2.14	2.17
27	8	1.60	1.61	0.00	0.00	0.02	0.02	1.99	2.00
28	8	2.08	2.07	0.00	0.00	0.01	0.01	2.59	2.59

According to Table 2, redundancy variation index reduces by increase in the number of bracing spans (in general and according to eq. 15 and fig 2, it increases the redundancy), but dynamical redundancy coefficient does not increase. This also means that dynamical redundancy is different from over-strength. Dynamical redundancy does not necessarily increase by the increase in the over strength for the systems investigated in this study.



**Figure 5.** Best model according to the redundancy in 5-story structures with 2 bracing bays.

As with the rule of 5-story models, among the models in this category (two bays in each direction - models no.16 to 19), the model no.19 with an arrangement similar to the model no. 5 (Figure 5) and model no.18 have the highest redundancy. Model no.21 has the highest redundancy among 4-bays models. In models with 6 bays, model no.26 has the highest redundancy in each direction. Therefore, as the height and number increases, the symmetrical and scattered arrangement (Figure 6) has the most indeterminacy inside the plan.



**Figure 6.** Best model in 8-story structures with 3 bracing bays in each direction due to the redundancy

The average over strength is shown in Table 3 to classify the models (2, 4, 6 spans in 5 and 8 story models).

According to table (3), it is observed that increase in the number of bracing bays in dual system doesn't usually mean the increase of the over-strength in the system. In addition, less values are obtained compared to the international codes such as ASCE7 (about 45% less).

**Table3.** Average of the over strength of classifications

Classification	Over strength ( $\Omega$ ) in X direction	Over strength ( $\Omega$ ) in Y direction
First classification (models 1-5)	1.80	1.99
Second classification (models 6-9)	1.67	1.70
Third classification (models 10-14)	1.69	1.73
Fourth classification (models 15-19)	1.92	1.94
Fifth classification (models 20-23)	1.68	1.69
Sixth classification (models 24- 27)	1.66	1.67
Seventh classification (models 28)	2.08	2.07

It is also concluded from table (3) that models with fewer number of bracing bays have more over-strength unexpectedly. This shows that as the moment frames contribution in lateral loads bearing increases, the over-strength will also be increased.

## 6. CONCLUSIONS

The effects of the number and position of the braced bays on the redundancy of the whole system were investigated in this study. A relationship was presented for the dual systems (relationship 27) and attempts have been made to achieve the best pattern for the arrangement of the braced bays for the regular 5- and 8-story structures.

1-The results indicate that the dynamic redundancy is different from the over-strength and does not necessarily increase with the increase of the over-strength in the systems studied in this paper.

2-In spite of the static indeterminacy, bracing location is quite effective on the dynamic redundancy of the structures while increasing the number of bracing bays doesn't increase the dynamic redundancy of the models.

3-Bracing locations at the perimeter of the structure reduces dynamic redundancy (for about 6%).Therefore, it is suggested to put braces inside the building plan.

4-Dynamic redundancy will decrease (about 5%) by the increase in the height of the structure which means more braced bays are needed as the height of the structure increases. Therefore, it is suggested to consider the redundancy coefficient as a parameter which depends on the structure's height.

## REFERENCES

- [1] Gollwitzer S, Rackwitz R., On the reliability of Daniels systems, *Structural Safety*, 1990, 7(2-4):229-43.
- [2] Kang, Ying-jie, and Ling-yun Peng, Optimisation Design and Damping Effect Analysis of Large Mass Ratio Tuned Mass Dampers, *Shock and Vibration*, 2019.
- [3] Yun, S. Y., and Foutch, D. A., Performance prediction and evaluation of low ductility steel moment frames for seismic loads, *SAC Background Document SAC/BD-00/26, SAC Joint Venture*, Richmond, Calif, 2000.
- [4] De RS, Karamchandani A, Cornell CA., Study of redundancy in near-ideal parallel structural systems, *In Structural safety and reliability*, (1989), pp. 975-982.
- [5] Popov, E.P. and Roeder, C.W., Design of an eccentrically braced frame, *AISC Engineering Journal*. Third Quarter, 1978.
- [6] Merovich, A.T., Nicoletti, J.P. and Hartle, E., Eccentric Bracing in Tall Buildings, *ASCE Journal of the Structural Division*, 1982, 108(ST9), September, pp. 2066-2079.
- [7] Anajafi, H., & Medina, R. A., Comparison of the seismic performance of a partial mass isolation technique with conventional TMD and base-isolation systems under broadband and narrow-band excitations., *Engineering Structures*, 2018, 158, 110-123.

- 
- 
- [8] Azad, S. K., & Topkaya, C., A review of research on steel eccentrically braced frames., *Journal of constructional steel research*, 2017, 128, 53-73.
- [9] Wang, K., Lu, X., & He, X., Experimental Investigation of Vibration Characteristics in a Centrifugal Pump with Vaned Diffuser, *Shock and Vibration*, 2018.
- [10] Shen, Jay & Seker, Onur & Akbas, Bulent & Seker, Pinar & Momenzadeh, Seyedbabak & Faytarouni, Mahmoud, Seismic performance of concentrically braced frames with and without brace buckling, *Engineering Structures*. 141. 461-481. 10.1016/j.engstruct.,2017.03.043.
- [11] The Review Standing Committee, *Instruction for Seismic Rehabilitation of Existing Buildings* (Publication No 360), Vice President of Strategic Planning and Control, 2014.
- [12] Moses, F., Reliability of structural systems, *J. Struct. Div.*, ASCE, 1974,100(9), 1813–1820
- [13] Bertero RD, Bertero VV., Redundancy in earthquake-resistant design. *Journal of Structural Engineering*. 1999, 125(1):81-8
- [14] R. C. Hibbeler., *Engineering Mechanics Statics (12th Edition)*, Prentice Hall, New Jersey, USA, 2009.
- [15] Permanent Committee of Revising, *The Iranian Code of Practice for Seismic Resistant Design of Buildings (Standard 2800-4rd edition)*, Building and Housing Research Center, 2014
- [16] ATC, *Structural response modification factors. ATC-19*, Applied Technology Council, Redwood City, California, 1995.
- [17] Husain M, Tsopelas P., Measures of structural redundancy in reinforced concrete buildings. I: redundancy indices., *Journal of Structural Engineering*, 2004, 130(11):1651-8,
- [18] Husain, M. and P. Tsoplas., Measures of structural redundancy in reinforced concrete buildings. II: Redundancy indices. *J. Struct. Eng.*, 2004, 130: 1659-1666
- [19] M.A. Rezvani; M. Esmacili; M.M. Feizi., Discrete modeling for dynamic response of buildings in the vicinity of railway tracks due to train-induced ground vibrations, *Scientia Iranica*, 2017, Volume 24, Issue 4, Page 1922-1939.
- [20] Richards PW., Estimating the stiffness of eccentrically braced frames. Practice Periodical, *Structural Design and Construction*, 2010, 15(1):91-5.
- [21] ASCE 41, American Society of Civil Engineers, *Seismic Evaluation and Retrofit of Existing Buildings*, Reston, VA, USA, 2017.
- [22] *FEMA P695*, Federal Emergency Management Agency, Quantification of Building Seismic Performance Factors, 2009.
- [23] Song SH, Wen YK., Redundancy of dual systems under earthquake loads, *8th ASCE Specialty Conference on Probabilistic Mechanics and Structural Reliability*, 2000.