A Numerical Model for Computing the Geometrical Errors Using Transfer Matrices in Case of a 5R Robot

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Abstract: - The present paper aims to present a mathematical model used for computing the geometrical errors, based on transfer matrices, which is applied on a 5R robot structure, FANUC LR Mate 100iB. The robot movement was analyzed by considering the position and orientation data from every driving joint provided from the robot teach - pendant. During the work process were collected a set of data containing the orientation angles and the generalized coordinates from each robot joint corresponding to each point from the working space reached by the end-effector and considered in the analysis. The mechanical structure for Fanuc robot as well as for the mobile platform were modeled using the Solid Work application. Thus, it was created a virtual working space similar to the real one. Thus, it was possible to gather accurate information regarding the nominal values of the generalized coordinates from each robot joint corresponding to the points the robot had to go through during the work process. Using the transfer matrices for the locating errors are finally obtained the column vectors for the geometrical errors, from every robot joint, by considering that the basic geometrical errors are known.

Keywords: - geometrical errors, transfer matrix, robotics

1. INTRODUCTION

The working accuracy of industrial robots can be evaluated by means of errors. Errors are defined as the difference between the targeted value (nominal value) and the actual reached value (actual value).

The term *error* is used in the form of a couple $\pm u$ for which it can be asserted with a certain probability that the position of the point or the orientation of the characteristic line is within a range $\left(\overline{x}-u,\ \overline{x}+u\right)$, where x represents the average of the results for the measurements performed in order to determine a certain parameter. To eliminate the influence of the error characterizing the measuring device and for computing the accuracy characteristics, the specific repeatability conditions are defined. These conditions correspond to some repeated measurements, carried out on the same robot, using the same measuring device (sufficiently precise for the error of the measuring instrument to be negligible in relation to the robot error) or by applying the same method, by the action of the same quantities that are practically kept constant all throughout the measurements. In robot mechanics the transfer matrix of errors plays an essential role in both, forward and inverse modeling of position and orientation errors. It should also be noted that the transfer matrices are used to determine by means of statistical methods, the variation range for position and orientation errors.

2. THEORETICAL APPROACH

The geometrical errors are affecting the position and orientation accuracy of the robot end-effector, thus causing a diminishing in the quality of the handled or machined parts. Geometric errors are mainly influenced by the thermal effects, the effects of the accelerations and of the gravity force, the accuracy of the components in motion, and so on.

2.1. The Geometrical Errors. Transfer Matrices

In order to determine the matrix of geometrical errors (Fig.1), the selected robot mechanical structure must be modeled using one of the algorithms from the geometrical modeling. In this case, the Algorithm of Locating Matrix was selected.

By applying this algorithm on the mechanical structure of the robot, are finally obtained the homogenous transformation matrices between the mobile frames $\{i\}$ and $\{i-1\}$, attached in the geometrical center of each driving joint, as well as between mobile frames $\{i\}$ and the fixed frame $\{0\}$, the latter being attached in the robot base. Both transformations are determined by considering the nominal configuration (error free) and the real configuration (affected by errors) of the robot.

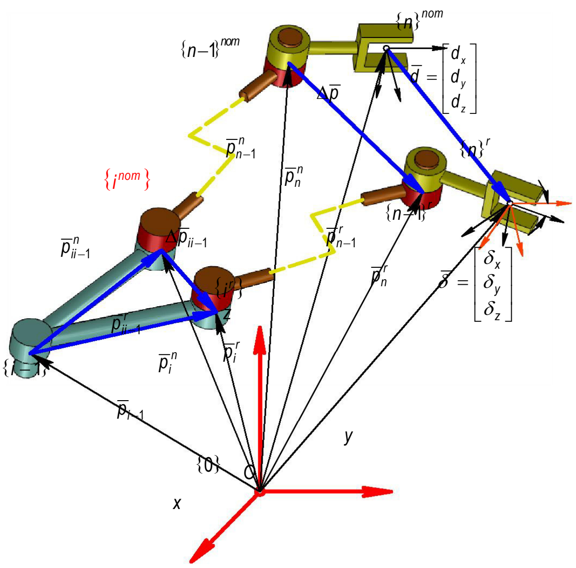

Figure 1. The Geometrical Errors

Based on the algorithms presented in [1], [5, [9], and for $i = 1 \rightarrow n$, are established the homogenous transformation matrices between the moving frames $\{i\}$ and $\{i-1\}$ for both, nominal (T^n_{ii-1}) and real (T^r_{ii-1}) configurations.

$$T^n_{ii-1} = \left[\begin{array}{ccc|c} {}^{i-1}\overline{x}_i^n & {}^{i-1}\overline{y}_i^n & {}^{i-1}\overline{z}_i^n & {}^{i-1}\overline{p}_{ii-1}^n \\ \hline 0 & 0 & 0 & 1 \end{array}\right]; \quad (1)$$

$$T^r_{ii-1} = \left[\begin{array}{ccc|c} {}^{i-1}\overline{x}_i^r & {}^{i-1}\overline{y}_i^r & {}^{i-1}\overline{z}_i^r & {}^{i-1}\overline{p}_{ii-1}^r \\ \hline 0 & 0 & 0 & 1 \end{array}\right]; \quad (2)$$

Similarly, with the properties of the time derivative of the locating matrix, which was presented in [2], it becomes obvious that the following relationship established between the homogeneous transformation matrix characterizing the robot nominal structure and the real transformation matrix, can be expressed in the form presented below, namely:

$$T^r_{ii-1} = T^n_{ii-1} + T^n_{ii-1} \cdot \delta T_{ii-1} \Rightarrow \Delta T_{ii-1} = T^n_{ii-1} \cdot \delta T_{ii-1}; \quad (3)$$

where ΔT_{ii-1} represents the first order differential error for the homogenous transformation between $\{i\}$ and $\{i-1\}$ moving frames.

The expression for the matrix operator of the errors δT_{ii-1} is determined according to the mathematical model developed in [5]. The starting equation in defining the matrix operator of errors is:

$$T^n_{ii-1} \cdot \delta T_{ii-1} = T^r_{ii-1} - T^n_{ii-1}; \quad (4)$$

On the relation (4) are applied some matrix transformations which lead to the following expression for the matrix operator of errors:

$$\delta T_{ii-1} = \left(T^n_{ii-1}\right)^{-1} \cdot T^r_{ii-1} - I_4; \quad (5)$$

For any robot joint $i = 1 \rightarrow n$, the matrix product $\left(T^n_{ii-1}\right)^{-1} \cdot T^r_{ii-1}$ is determined according to:

$$\left(T^n_{ii-1}\right)^{-1} \cdot T^r_{ii-1} = \left[\begin{array}{ccc|c} & R^{nT}_{ii-1} \cdot R^r_{ii-1} & & \begin{array}{c} R^{nT}_{ii-1} \cdot {}^{i-1}\overline{p}^r_{ii-1} - \\ -R^{nT}_{ii-1} \cdot {}^{i-1}\overline{p}^n_{ii-1} \end{array} \\ \hline 0 & 0 & 0 & 1 \end{array}\right]; \quad (6)$$

where the homogenous transformation matrix T^r_{ii-1} is defined according to the expression (2).

The matrix operator of errors δT_{ii-1} from (5) can be also defined, according to [5], by means of the relative errors from the robot joints:

$$\delta T_{ii-1} = \left[\begin{array}{ccc|c} & \{\Delta\overline{\psi}_i \times\} & & \Delta\overline{p}_{ii-1} \\ \hline 0 & 0 & 0 & 1 \end{array}\right]; \quad (7)$$

In the expression above, $\{\Delta\overline{\psi}_i \times\}$ is an antisymmetric matrix whose components are represented by the relative orientation errors $\Delta\alpha$, $\Delta\beta$ and $\Delta\gamma$, while $\Delta\overline{p}_{ii-1}$ is the column vector of relative positional errors .

$$\Delta\overline{p}_{ii-1} = \overline{p}^r_{ii-1} - \overline{p}^n_{ii-1}; \quad (8)$$

In order to define the matrix operator of errors, the following notations are introduced:

$$\overline{\varepsilon}_{yQ} = \left\{\overline{\varepsilon}_{yS}\right\}, y = \left\{p_Q;\ e;\ g\right\}; \quad (9)$$

$$p_Q = p_S = \left\{p_{xi};\ p_{yi};\ p_{zi};\ \alpha_i;\ \beta_i;\ \gamma_i\right\}; \quad (10)$$

where y is an index which considers the corresponding locating errors (p_Q –each parameter; e – each element; g – to the entire structure).

The basic errors of first order are defined by means of the following matrix expressions:

$$\delta^0 T_n^1 = \sum_{i=1}^{n} {}_i^0[T] \cdot \delta^i T_{ii-1} \cdot {}_i^0[T]^{-1} ; \qquad (11)$$

where,
$$\delta^i T_{ii-1} = \left[\begin{array}{ccc|c} \multicolumn{3}{c|}{\{\Delta\overline{\psi}_i \times\}} & \Delta\overline{p}_{ii-1} \\ \hline 0 & 0 & 0 & 1 \end{array} \right]; \qquad (12)$$

By substituting the relation (12) in(11), it results:

$$\delta^0 T_n = \left[\begin{array}{ccc|c} \multicolumn{3}{c|}{{}_i^0[R] \cdot \{\Delta\overline{\psi}_i \times\} \cdot {}_i^0[R]^{-1}} & \begin{array}{c} {}_i^0[R] \cdot \Delta\overline{p}_{ii-1} - \\ -{}_i^0[R] \cdot \{\Delta\overline{\psi}_i \times\} \cdot {}_i^0[R]^{-1} \cdot \overline{p}_i \end{array} \\ \hline 0 & 0 & 0 & 0 \end{array} \right]; \quad (13)$$

The vector of basic errors for position and orientation is defined as follows:

$$\overline{\varepsilon} = \left[\Delta\overline{p}_{ii-1} \;\; \Delta\overline{\psi}_{ii-1} \right]; \qquad (14)$$

By separation of the basic errors for position $\Delta\overline{p}_{ii-1}$ and orientation $\Delta\overline{\psi}_{ii-1}$ the result is a matrix expression which highlights the mathematical connection between a $(6 \times 6n)$ matrix, denoted $E_{d\delta}$ and known as the *transfer matrix*, and the vector of basic errors $\left[\overline{d}^T \;\; \overline{\delta}^T \right]^T$. According to (13), the general expression of the transfer matrix is determined starting from the following expressions:

$${}_i^0[R] \cdot \{\Delta\overline{\psi} \times\} \cdot {}_i^0[R]^{-1} = \begin{bmatrix} 0 & -\delta_{iz} & \delta_{iy} \\ \delta_{iz} & 0 & -\delta_{ix} \\ -\delta_{iy} & \delta_{ix} & 0 \end{bmatrix}; \quad (15)$$

$$\left\{ \begin{array}{l} \{\overline{\delta}_i \times\} = \begin{bmatrix} 0 & -\delta_{iz} & \delta_{iy} \\ \delta_{iz} & 0 & -\delta_{ix} \\ -\delta_{iy} & \delta_{ix} & 0 \end{bmatrix} = \\ = \begin{bmatrix} \alpha_{ix} & \alpha_{iy} & \alpha_{iz} \\ \beta_{ix} & \beta_{iy} & \beta_{iz} \\ \gamma_{ix} & \gamma_{iy} & \gamma_{iz} \end{bmatrix} \cdot \begin{bmatrix} 0 & -\Delta\gamma_i & \Delta\beta_i \\ \Delta\gamma_i & 0 & -\Delta\alpha_i \\ -\Delta\beta_i & \Delta\alpha_i & 0 \end{bmatrix} \cdot \begin{bmatrix} \alpha_{ix} & \beta_{ix} & \gamma_{ix} \\ \alpha_{iy} & \beta_{iy} & \gamma_{iy} \\ \alpha_{iz} & \beta_{iz} & \gamma_{iz} \end{bmatrix} \end{array} \right\} (16)$$

Performing the matrix product and by identifying the components of the resultant matrix obtained [3], [7]:

$$\left\{ \begin{array}{c} \delta_{ix} = \Delta\alpha_i \cdot \left(\gamma_{iz} \cdot \beta_{iy} - \gamma_{iy} \cdot \beta_{iz} \right) + \\ +\Delta\beta_i \left(\gamma_{ix} \cdot \beta_{iz} - \gamma_{iz} \cdot \beta_{ix} \right) + \\ +\Delta\gamma_i \left(\gamma_{iy} \cdot \beta_{ix} - \gamma_{ix} \cdot \beta_{iy} \right) \end{array} \right\}; \quad (17)$$

$$\left\{ \begin{array}{c} \delta_{iy} = \Delta\alpha_i \cdot \left(\alpha_{iz} \cdot \gamma_{iy} - \alpha_{iy} \cdot \gamma_{iz} \right) + \\ +\Delta\beta_i \left(\alpha_{ix} \cdot \gamma_{iz} - \alpha_{iz} \cdot \gamma_{ix} \right) + \\ +\Delta\gamma_i \left(\alpha_{iy} \cdot \gamma_{ix} - \alpha_{ix} \cdot \gamma_{iy} \right) \end{array} \right\}; \quad (18)$$

$$\left\{ \begin{array}{c} \delta_{iz} = \Delta\alpha_i \cdot \left(\alpha_{iy} \cdot \beta_{iz} - \beta_{iy} \cdot \alpha_{iz} \right) + \\ +\Delta\beta_i \left(\beta_{ix} \cdot \alpha_{iz} - \beta_{iz} \cdot \alpha_{ix} \right) + \\ +\Delta\gamma_i \left(\alpha_{ix} \cdot \beta_{iz} - \alpha_{iz} \cdot \beta_{ix} \right) \end{array} \right\}; \quad (19)$$

The components defined with (17)- (19) are included in the term of orientation errors, corresponding to line i. The position errors are computed by performing the matrix product:

$$\left\{ \begin{array}{c} \overline{d}_i = \left[d_x \quad d_y \quad d_z \right]^T = \\ = {}_i^0[R] \cdot \Delta\overline{p}_i - {}_i^0[R] \cdot \left\{ \Delta^i\overline{\psi}_i \times \right\} \cdot \overline{p}_i \end{array} \right\}; \quad (20)$$

Performing the matrix product and by separating the basic errors for position and orientation are obtained the expressions:

$$\left\{ \begin{array}{c} d_{ix} = \alpha_{ix} \cdot \Delta p_{xi} + \alpha_{iy} \cdot \Delta p_{yi} + \\ +\alpha_{iz} \cdot \Delta p_{zi} + \Delta\alpha_i \cdot \left(\alpha_{iy} \cdot p_{zi} - \alpha_{iz} \cdot p_{yi} \right) + \\ +\Delta\beta_i \cdot \left(\alpha_{iz} \cdot p_{xi} - \alpha_{ix} \cdot p_{zi} \right) + \\ +\Delta\gamma_i \cdot \left(\alpha_{ix} \cdot p_{yi} - \alpha_{iy} \cdot p_{xi} \right) \end{array} \right\}; \quad (21)$$

$$\left\{ \begin{array}{c} d_{iy} = \beta_{ix} \cdot \Delta p_{xi} + \beta_{iy} \cdot \Delta p_{yi} + \beta_{iz} \cdot \Delta p_{zi} + \\ +\Delta\alpha_i \cdot \left(\beta_{iy} \cdot p_{zi} - \beta_{iz} \cdot p_{yi} \right) + \\ +\Delta\beta_i \cdot \left(\beta_{iz} \cdot p_{xi} - \beta_{ix} \cdot p_{zi} \right) + \\ +\Delta\gamma_i \cdot \left(\beta_{ix} \cdot p_{yi} - \beta_{iy} \cdot p_{xi} \right) \end{array} \right\}; \quad (22)$$

$$\left\{ \begin{array}{c} d_{iz} = \gamma_{ix} \cdot \Delta p_{xi} + \gamma_{iy} \cdot \Delta p_{yi} + \gamma_{iz} \cdot \Delta p_{zi} + \\ +\Delta\alpha_i \cdot \left(\gamma_{iy} \cdot p_{zi} - \gamma_{iz} \cdot p_{yi} \right) + \\ +\Delta\beta_i \cdot \left(\gamma_{iz} \cdot p_{xi} - \gamma_{ix} \cdot p_{zi} \right) + \\ +\Delta\gamma_i \cdot \left(\gamma_{ix} \cdot p_{yi} - \gamma_{iy} \cdot p_{xi} \right) \end{array} \right\}; \quad (23)$$

In the expressions (21) – (23) are defined the components of vector $\overline{d}_i$ projected on the axes of the Cartesian reference frame.

Thus, the following expressions can be written:

$$\begin{bmatrix} d_i & \delta_i \end{bmatrix}^T = \begin{bmatrix} d_{ix} & d_{iy} & d_{iz} & \delta_{ix} & \delta_{iy} & \delta_{iz} \end{bmatrix}^T ; \quad (24)$$

$$\varepsilon_{iS} = \begin{bmatrix} \Delta p_{xi} & \Delta p_{yi} & \Delta p_{zi} & \Delta\alpha_i & \Delta\beta_i & \Delta\gamma_i \end{bmatrix}^T ; \quad (25)$$

where expression (25) defines the vector of locating errors, corresponding to element (i).

The transfer matrix corresponding to element (i) it is a (6×6) matrix which represents one element from the $(6\times6n)$ resultant transfer matrix.

Thus the resultant transfer matrix is comprising matrix blocks each of them having the size (6×6).

$$E_{d,\delta} = \left[\boxed{E^1_{d,\delta}} \;\vdots\; \boxed{E^2_{d,\delta}} \cdots \;\vdots\; \boxed{E^i_{d,\delta}} \cdots \;\vdots\; \boxed{E^n_{d,\delta}} \right] ;(26)$$

The column vector of the basic errors which characterizes the (i) joint, is obtained by performing the matrix product between the transfer matrix corresponding to (i) joint and the column vector of the locating parameters errors, from same (i) joint.

$$\left\{ \begin{aligned} & \begin{bmatrix} \overline{d}_i^T & \overline{\delta}_i^T \end{bmatrix}^T = E^i_{d,\delta} \cdot \overline{\varepsilon}_{iS} = \\ & = \begin{bmatrix} d_{ix} & d_{iy} & d_{iz} & \delta_{ix} & \delta_{iy} & \delta_{iz} \end{bmatrix}^T \end{aligned} \right\} ; \quad (27)$$

The evaluation of a robot performance is achieved by taking into account the differential error pattern having the x order, where $x = \{1,\ 2,\ 3\}$. The differential errors model is applied on the homogenous transformations between the frames $\{0\} \to \{n\}$.

In this paper is presented the differential errors of first order. To improve the accuracy performances of a serial robot structure is recommended to compute the differential errors of higher order [4], [5].

The matrix equation which defines the differential model for position and orientation is written as:

$$\delta^0 T_n^2 = \begin{bmatrix} -\frac{1}{2}\cdot\left(\delta y^2 + \delta z^2\right) & -\delta z & \delta y & d_x \\ \delta y \cdot \delta x + \delta z & -\frac{1}{2}\cdot\left(\delta x^2 + \delta z^2\right) & -\delta x & d_y \\ \delta z \cdot \delta x - \delta y & \delta z \cdot \delta y + \delta x & -\frac{1}{2}\cdot\left(\delta y^2 + \delta x^2\right) & d_z \\ \hline 0 & 0 & 0 & 0 \end{bmatrix}$$

where, $$\left\{ \begin{aligned} & \delta^0 T_n^2 = \delta^0 T_n^{(1)} + \delta^0 T_n^{(2)} = \\ & \sum_{i=1}^{n} \left({}^{0}_{i-1}[T] \cdot \delta^{i-1} T_{ii-1} \cdot {}^{0}_{i-1}[T]^{-1} \right) + \\ & + \sum_{i=1}^{n-1} \sum_{j=1}^{n} \left(\delta^0 T_{ii-1} \cdot \delta^0 T_{jj-1} \right) \end{aligned} \right\} ; \quad (28)$$

$$\delta^0 T_n^{(1)} = \sum_{i=1}^{n} \left({}^{0}_{i}[T] \cdot \delta^{i-1} T_{ii-1} \cdot {}^{0}_{i}[T]^{-1} \right) ; \quad (29)$$

$$\delta^0 T_n^{(2)} = \sum_{i=1}^{n-1} \sum_{j=1}^{n} \left(\delta^0 T_{ii-1} \cdot \delta^0 T_{jj-1} \right) ; \quad (30)$$

Based on the third order linear approximation it can be obtained the matrix equation which defines the third order model of locating errors [5].

As shown in the above analysis, the error transfer matrices play an essential role in both forward and inverse modeling of position and orientation errors.

It should also be noted that the transfer matrices are used to determine by statistical methods the variation range for the locating errors.

In the following paragraph it will be presented a numerical application for computing the transfer matrices that characterize each joint and the transfer matrix for the entire structure of a five-degree robot structure, model Fanuc LR Mate 100iB.

3. NUMERICAL MODELING

3.1 The implementation of the 5R robot in a working process

The 5 R robot structure which is subjected to analysis in this section, is represented by Fanuc LR Mate 100iB, an industrial robot which has been designed to ensure operating in the industrial environment but that can be also implemented in different applications for educational purposes.

The mechanical structure of the Fanuc robot, consisting of four base modules connected between them by five rotational joints driven by DC motors, allows it to perform loading / unloading operations of the machine tools, accurate handling operations and in case a work tool is attached, it can be used for complex and of high accuracy welding operations.

In Fig. 2 is presented the mechanical structure of the Fanuc LR Mate 100iB robot, in which the nominal values for the position and orientation as well as the working space described by the robot during the work process are highlighted.

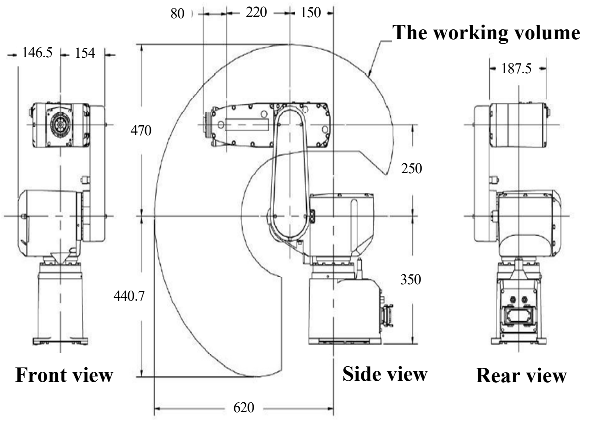

Figure 2. The Mechanical Structure of 5R Robot

The Fanuc LR Mate 100iB has been implemented in a technological process of cooperation with a mobile robot [10] – [12].

This paper presents the second phase of the working process. In the first phase of the process the robot is programmed to start from an initial configuration and then it handles the part in a position and orientation that allows it to be machined. In the next phase, the robot manipulates the processed part from the manufacturing center and position it on a mobile platform from where it is transported to a storage.

After the process is completed, another blank is brought in by the mobile robot and the whole work process is reinitiated.

The movement of the robot in the working space was analyzed by considering the position and orientation data from every driving joint provided from the robot teach - pendant.

Figure 3a. Sequence 5 previous configuration)

Figure 3b. Sequence 6 (current configuration)

During the process of robot programming were collected a set of data containing the orientation angles and the generalized coordinates from each robot joint corresponding to each point from the working space reached by the end-effector and included in the analysis.

Using the Solid Work application, the mechanical structure for Fanuc robot as well as for the mobile platform were modeled.

Thus, it was created a virtual working space similar to the real one and it was possible to gather accurate information regarding the nominal values of the generalized coordinates from each robot joint corresponding to the points the robot had to go through during the work process.

Within Figures 3a, and Figure 3b, are presented the work sequences which were considered in the analysis of the geometrical errors [8].

Based on the information collected from the robot teach-pendant, the real values (affected by errors) as well as the nominal values of the generalized coordinates from every driving joint, corresponding to all 3 robot configurations subjected to analysis, are presented in the tables bellow. The input geometrical errors (Table 1) for the considered configurations are the following:

Table 1. Input Geometrical Errors

[mm]			[°]		
$\Delta\bar{p}_x$	$\Delta\bar{p}_y$	$\Delta\bar{p}_z$	$\Delta\alpha_z$	$\Delta\beta_y$	$\Delta\gamma_x$
0	-0.045	0.034	0	0	0
0	0.016	0.05	0	-0.013	0
0	0.014	0.08	0	0.02	0
0	0.028	0.036	0	0	0

The generalized coordinates in nominal and real (affected by errors) values, for the analyzed sequence according to Table 2, 3, are the following:

Table 2. Nominal values for generalized coordinates

$k \to 15-18$	q_1	q_2	q_3	q_4	q_5
15	90	59.33	-80.39	111.06	90
16	90	57.3	-74.7	107.39	90
17	90	55.27	-67.67	102.4	90
18	90	53.25	-53.12	89.86	90

Table 3. Real values for generalized coordinates

$k \to 15-18$	q_1	q_2	q_3	q_4	q_5
15	89.96	59.42	-80.25	111.19	90.12
16	89.96	57.36	-74.81	107.24	90.12
17	89.96	55.40	-67.72	102.33	90.12
18	89.96	53.31	-53.31	89.83	90.12

Applying the mathematical model presented above, for $k = 16$ are obtained the transfer matrices which are the elements of the transfer matrix $E_{d,\delta}$ defined with (26) based on which it results the column vectors of the basic errors. The matrix $E_{d,\delta}$ is a $(6 \times 6n)$ matrix and $E_{d,\delta}^{i}$ a (6×6) matrix.

$$E^1_{d,\delta}=\begin{bmatrix} 0.000698 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0.000698 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.000698 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0.000698 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix};$$

$$E^2_{d,\delta}=\begin{bmatrix} 0.00037 & -1 & 0.00058 & -0.088 & 0 & 0.161 \\ 0.539 & 0.00069 & 0.842 & 0 & -0.00088 & -0.0809 \\ -0.842 & 0 & 0.539 & -0.0809 & -0.00056 & -0.126 \\ 0 & 0 & 0 & 0.00037 & -1 & 0.00058 \\ 0 & 0 & 0 & 0.539 & 0.00069 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.539 \end{bmatrix}$$

$$E^3_{d,\delta}=\begin{bmatrix} 0.00066 & -1 & -0.00020 & -0.134 & -0.0898 & 0.491 \\ 0.953 & 0.00069 & -0.299 & 0.108 & -0.128 & 0.343 \\ 0.299 & 0 & 0.953 & -0.343 & -0.0401 & 0.108 \\ 0 & 0 & 0 & 0.00066 & -1 & -0.0020 \\ 0 & 0 & 0 & 0.953 & 0.00069 & -0.299 \\ 0 & 0 & 0 & 0 & 0 & 0.953 \end{bmatrix}$$

$$E^4_{d,\delta}=\begin{bmatrix} 0 & -1 & 0.00069 & -0.201 & -0.00023 & 0.399 \\ 0.0036 & 0.00069 & 1 & -0.570 & -0.83 & 0.002 \\ -1 & 0 & 0.0036 & -0.0020 & -0.200 & -0.570 \\ 0 & 0 & 0 & 0 & -1 & 0.00069 \\ 0 & 0 & 0 & 0.0036 & 0.00069 & -0.299 \\ 0 & 0 & 0 & 0 & 0 & 0.0036 \end{bmatrix}$$

$$E^5_{d,\delta}=\begin{bmatrix} 0 & 0.00279 & 1 & -0.571 & 0.398 & 0.000347 \\ 0.0036 & 1 & -0.002796 & -0.0175 & 0.0692 & 0.00169 \\ -1 & 0.0036 & 0 & -0.070 & -0.0191 & -0.571 \\ 0 & 0 & 0 & 0 & 0.00279 & 1 \\ 0 & 0 & 0 & 0.0036 & 1 & -0.0027 \\ 0 & 0 & 0 & 0 & 0.0036 & 0 \end{bmatrix}$$

The column vector of the basic errors is obtained:

$$[d\ \ \delta]^T = [0.00062\ \ 0.833\ \ -0.187\ \ -0.0034\ \ 0.000102\ \ 0]^T.$$

The vector $[d\ \ \delta]^T$ contains the position and orientation errors at the end-effector for the Fanuc robot implemented in a work process, from which was considered sequence 16.

4. CONCLUSIONS

The objective of this paper was to present a mathematical model used for computing the geometrical errors, based on transfer matrices. The model was applied on a 5R robot structure, FANUC LR Mate 100iB. The robot movement in the configurations space, was analyzed by taking into account the position and orientation data from every driving joint provided by the robot teach - pendant.

During the work process were collected a set of data containing the orientation angles and the generalized coordinates from each robot joint corresponding to each point from the working space reached by the end-effector and considered in the analysis. The mechanical structure for Fanuc robot as well as for the mobile platform were modeled using the Solid Work application, Thus, it was created a virtual working space similar to the real one and also it was possible to gather accurate information regarding the nominal values of the generalized coordinates from each robot joint corresponding to the points the robot had to pass over during the work process. Using the transfer matrices for the locating errors, finally result the column vectors for geometrical errors, from every robot joint, considering that basic geometrical errors are known.

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