Vibration of Functionally Graded Beam Subjected to Moving Oscillator Using Caputo-Fabrizio Fractional Derivative Model

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Abstract: - In this paper, the vibration of an Euler-Bernoulli functionally graded beam under a moving oscillator is investigated. The beam is considered to be simply supported whereas its material composition is varying along the thickness according to a power law. Furthermore, the internal damping of the beam is modeled by viscoelastic fractional Kelvin-Voigt model which is described by Caputo-Fabrizio definition. However, the novelty in this paper is the utilizing of Caputo-Fabrizio definition which has two main advantages over other fractional derivative models; it does not contain fractional powers in the Laplace transformation domain and it is more convenient in describing material heterogeneities. The governing equations are solved by the decomposition method coupled with Laplace transforms. Three comparison studies were conducted and good agreements were obtained. The results clearly indicate the advantages of using Caputo-Fabrizio fractional derivative model. Also it is observed that the oscillator velocity, the material grading order, the damping ratio, and the fractional derivative order have significant effects on the dynamic response of the beam.

Keywords: Euler-Bernoulli beam, Fractional viscoelasticity, Functionally graded beam, Fractional damping, Caputo-Fabrizio.

1. INTRODUCTION

The moving loads on structures is a problem of great practical engineering interest and attracted the attention of many researchers due to its wide applications; like interaction between vehicles and bridges, pipes subjected to moving fluids, moving projectiles inside firing barrels and machine tools [1]. Many studies have been conducted to examine the dynamic behavior of different types of beams under various types of moving loads. Obtaining the dynamic response of structures carrying moving subsystems can be classified into three types of problems [2]. In the first problem the inertia effect is neglected resulting in a moving force problem. The second problem results from inclusion of inertia forces while the subsystem has stiffness approaches to infinity leading to a moving mass problem while in the third problem, the moving subsystem has finite stiffness rendering it as a moving oscillator problem. Homogeneous beams subjected to various dynamic loads have been studied intensively in literature by many authors, [3–6]. Ouyang and Mottershead [3] presented a numerical-analytical approach to describe the dynamic behavior of a beam under a moving flexible body which is modeled as a moving oscillator with a defined spring and damping coefficients. Stancioiu et al. [4] investigated the excitations of an Euler-Bernoulli (E-B) beam carrying oscillator by considering the separation and reattachment between the oscillator and the beam. Majka and Hartnett [5] presented a numerical solution to examine the effect of the speed of moving vehicles on the dynamic response of a railway bridge. Yin [6] employed E-B approach to develop semi-analytical solution to examine the interaction between vehicles and bridges.

In order to capture more realistic dynamic behavior of the beams, the fractional derivatives and integrals theory was utilized to get a better description for the material properties and generalized form of the law of deformation of viscoelastic materials [7]. Therefore, the fractional viscoelatic behavior of the beams was considered in many studies [8–15]. Sorrentino and Fasana [8] examined the effect of the fractional derivative order on the dynamic behavior of a cantilever beam. They [8] used the fractional Kelvin-Voigt model, where the fractional derivatives are defined according to Riemann–Liouville (R-L) and Caputo definitions. Liang and Tang [9] adopted R-L definition to describe the damping in continues beam with homogenous boundary conditions, where the Adomian decomposition method was used to obtain the dynamic response of the beam . Starting from the relation between the fractional axial stress and the fractional axial strain, Di Paola et al. [10] studied the vibration of viscoelastic E-B beam subjected to quasi static and dynamic loads using the fractional Kelvin-Voigt model, where R-L fractional integral and Caputo fractional derivative are employed. Demir et al. [11] presented a general linear model for fractionally damped beams and rods, where R-L definition for the fractional derivative. Di Lorenzo et al. [12] used R-L and Caputo fractional derivatives to derive a model for a fractionally damped E-B beam subjected to generic input loads. He et al. [13] proposed a solution for large amplitude vibrations of viscoelastic beams with different material composition, where Caputo fractional derivative definition is applied to describe the damping in the beam. Freundlich [14] employed R-L fractional derivatives to describe the damping in simply supported beam subjected to moving point load, where the beam response is attained by Green's function in fractional form. Martin [15] investigated the dynamic behavior of viscoelastic simply supported beam under two different types of loads using fractional Zener model, where the fractional derivative is defined according to R-L definition.

Since their inception in 1987 in Japan [16], the functionally graded (FG) materials became the field of interest of many researchers because of their wide applications. Therefore, many studies were presented to study of the effect of such materials on the dynamic behavior of the beams [17–25]. Aydogdu and Taskin [17] utilized different higher order shear deformation theories and classical beam theories to investigate the behavior of an FG beam under free vibration. By assuming that the material composition is varying according to the power law, Simsek and Kocaturk [18] investigated the dynamic behavior for an FG beam under moving harmonic load. Khalili, et al. [19] examined the effect of material inhomogeneity and the load parameters on the dynamic response of FG beams under moving loads. Using E-B, Timoshenko and third order shear deformation beam theories, Simsek [20] studied the vibration of simply supported FG beam subjected to moving mass, where the material is assumed to be graded continuously according to the power law distribution in the thickness direction. Dave et al. [21] studied the viscoelastic behavior of an FG asphalt concrete pavements. Rajabi et al. [22] employed Hamilton's principle to derive the equations of motion of an FG E-B beam under moving oscillator, where the beam is considered to be simply supported with properties vary continuously in the thickness direction according to the power law. Lei et al. [23] analyzed the dynamic behavior of FG nanobeams under moving force, where the significance of the material distribution and the velocity of the moving load is indicated in the numerical results. Wang et al. [24] analyzed the dynamic behavior of an FG beam with different boundary conditions subjected to moving point force. Using isogeometric analysis, Phung-Van et al. [25, 27, 29] inspected the transient dynamic response of different types of FG nanoplates.

Based on the foregoing literature review, it can be seen that the aforementioned studies focused on examining the effects of the velocity of the moving load, the material composition, the boundary conditions and the fractional derivative order on the distributed systems behavior. However, the problem of obtaining the dynamic behavior of an FG beam subjected to moving oscillator using Caputo-Fabrizio fractional viscoelastic models was not investigated in the literature. The Caputo-Fabrizio fractional derivative model [26] describes the internal damping of beams; the advantages of this models its suitability in applying Laplace transforms and its ability to describe material heterogeneities and structures with different scales, which cannot be well described by classical fractional derivative models [26]. Thus, this paper focus on studying the dynamic response of a simply supported FG beam and the factors that affect its dynamic behavior including the moving oscillator velocity, the grading order of the material, and the damping ratio whereas in all cases the Caputo-Fabrizio fractional derivative model is used.

2. THEORY AND FORMULATION

An E-B FG beam subjected to moving oscillator is shown in Figure 1. The beam is assumed to be simply supported with length L, while the cross-section has dimensions of thickness h and width b. The governing equation for an FG beam subjected to moving loads can be found in [18, 20, 22]. A fractional viscoelastic Kelvin-Voigt model with time-independent Poisson's ratio is used to describe the beam damping [28], where the equation of motion of a fractionally damped beam is obtained by similar procedure presented in reference [12]. Thus, the equations that describe the motion of an FG fractionally damped beam as well as the moving oscillator are given by [2, 22]

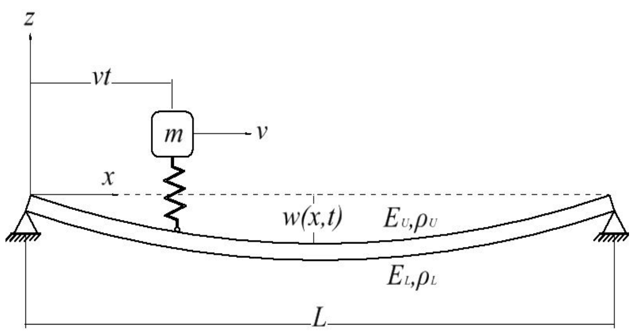

Figure 1. Functionally graded simply supported beam subjected to moving oscillator

$$C_\alpha I\frac{\partial^4}{\partial x^4}\left(D_t^{(\alpha)}w\left(x,t\right)\right)+\left(EI\right)_{eq}\frac{\partial^4 w\left(x,t\right)}{\partial x^4}+\rho_{eq}\frac{\partial^2 w\left(x,t\right)}{\partial t^2}=-m\left(\frac{d^2q\left(t\right)}{dt^2}+g\right)\delta\left(x-vt\right) \tag{1}$$

$$m\frac{d^2q\left(t\right)}{dt^2}+kq\left(t\right)=k\,w\left(vt,t\right)-mg \tag{2}$$

where $w\left(x,t\right)$ is the beam displacement, $w\left(vt,t\right)$ is the displacement under the oscillator, I is the moment of inertia, m is the oscillator mass, g is the gravity acceleration constant, $q\left(t\right)$ is the oscillator displacement, k is the oscillator spring stiffness, $\delta\left(\bullet\right)$ is the Dirac delta function, C_α is characteristic coefficient which depends on the material type of the FG beam [12], and α is the fractional derivative order. $D_t^{(\alpha)}w(x,t)$ is the fractional derivative operator which can be defined according to Caputo and Fabrizio as [26]

$$D_t^\alpha w\left(x,t\right)==\left(\frac{M\left(\alpha\right)}{\left(1-\alpha\right)}\right)\left(\int_0^t w'\left(x,\bar{t}\right)\exp\left[-\frac{\alpha\left(t-\bar{t}\right)}{1-\alpha}\right]d\bar{t}\right),\ (0<\alpha\le 1) \tag{3}$$

where $M\left(\alpha\right)$ is normalization function in which $M\left(0\right)=M\left(1\right)=1$ [26]. The equivalent values ($\left(EI\right)_{eq}$ and ρ_{eq}) of flexural rigidity, and mass per unit length for the FG beam can be defined respectively by

$$\rho_{eq}=b\int_{-h/2}^{h/2}\rho\left(z\right)dz \tag{4}$$

$$\left(EI\right)_{eq}=\int_{-h/2}^{h/2}E\left(z\right)z^2\,dz \tag{5}$$

where $\rho\left(z\right)$ is the mass density and $E\left(z\right)$ is Young's modulus of the FG beam, which vary along the thickness direction according to the following power law [18]

$$P\left(z\right)=\left(P_U-P_L\right)\left(\frac{z}{h}+0.5\right)^\lambda+P_L \tag{6}$$

where P_U and P_L are the properties of the upper and the lower surfaces of the beam respectively, and λ is the grading order. It is evident from Equation (6) that when $\lambda=0$ then $P=P_U$ and when $\lambda\to\infty$ then $P=P_L$. Also, the axial stress in an FG beam is given by [30]

$$\sigma\left(x,z,t\right)=-zE(z)\frac{\partial^2 w(x,t)}{\partial x^2} \tag{7}$$

Since the material composition is varying, the neutral axis position will not be any more at the mid-plane of the FG beam [30, 31], so the new position of the neutral axis is given as

$$h_0=\frac{\int_{-h/2}^{h/2}E(z)z\,\mathrm{d}z}{\int_{-h/2}^{h/2}E(z)\,\mathrm{d}z} \tag{8}$$

Now, Equation (1) can be solved by mode superposition principle, by setting

$$w(x,t)=\sum_{n=1}^{\infty}W_n(x)T_n(t) \tag{9}$$

where $W_n(x)$ represents the n[th] normal mode of the beam which is a function that depends only on *x*, and $T_n(t)$ is the generalized coordinate in the n[th] mode which is a function that depends only on *t*. For a uniformly simply supported beam, the mode shapes can be written as

$$W_n(x)=\sin\left(\beta_n x\right) \tag{10}$$

where β_n are the mode shapes for the simply supported beam which are defined by

$$\beta_n=\frac{n\pi}{L} \tag{11}$$

By substituting Equation (9) and its partial derivatives into Equation (1) and multiplying both sides by $W_m(x)$ leads that

$$\sum_{n=1}^{\infty}\left[\begin{array}{l}\dfrac{C_\alpha I\beta_n{}^4}{\rho_{eq}}\dfrac{d^\alpha T_n(t)}{dt^\alpha}W_n(x)W_m(x)\\+\dfrac{(EI)_{eq}\beta_n{}^4}{\rho_{eq}}W_n(x)W_m(x)T_n(t)\\+W_n(x)W_m(x)\dfrac{d^2T_n(t)}{dt^2}\end{array}\right]= \\ =\frac{-m}{\rho_{eq}}W_m(x)\left(\frac{d^2q(t)}{dt^2}+g\right)\delta(x-vt) \tag{12}$$

By integrating Equation (12) over the beam length and applying the orthogonality condition [32, 33] yields

$$\frac{d^2T_n(t)}{dt^2}+\omega_n^2T_n(t)=\frac{-m}{\rho_{eq}M_n}\frac{d^2q(t)}{dt^2}\sin(\beta_m vt) \\ -\frac{mg}{\rho_{eq}M_n}\sin(\beta_m vt)-2\omega_n\zeta_n\frac{d^\alpha T_n(t)}{dt^\alpha} \tag{13}$$

where

$$M_n=\int_0^L W_n(x)W_m(x)\,dx \tag{14}$$

$$\omega_n=\sqrt{\frac{(EI)_{eq}\beta_n{}^4}{\rho_{eq}}} \tag{15}$$

$$\zeta_n=\frac{C_\alpha I\omega_n}{2(EI)_{eq}} \tag{16}$$

Now, by substituting Equation (9) into Equation (2) gives

$$\frac{d^2q(t)}{dt^2}+\kappa^2q(t)=\kappa^2\sum_{n=1}^{\infty}\sin(\beta_n vt)T_n(t)-g \tag{17}$$

where κ is the natural frequency of the oscillator which is defined by

$$\kappa=\sqrt{\frac{k}{m}} \tag{18}$$

In order to find the generalized deflection functions $T_n(t)$ and the vertical displacement of the oscillator $q(t)$; the decomposition method coupled with Laplace transforms. Hence, $T_n(t)$ and $q(t)$ can be decomposed into infinite series in terms of their dependent variables as [32, 33]

$$T_n(t)=\sum_{i=0}^{\infty}T_n{}^i(t)=T_n{}^0(t)+T_n{}^1(t)+T_n{}^2(t)+\dots \tag{19}$$

$$q(t)=\sum_{i=0}^{\infty}q^i(t)=q^0(t)+q^1(t)+q^2(t)+\dots \tag{20}$$

Here, it is assumed that the weight of the oscillator has the maximum effect on the dynamic response of the beam at its fundamental-frequency. Substituting Equations (19) and (20) into Equation (13) and (17) respectively gives

$$\frac{d^2}{dt^2}\left(\sum_{i=0}^{\infty}T_n{}^i(t)\right)+\omega^2\left(\sum_{i=0}^{\infty}T_n{}^i(t)\right)= \\ =\frac{-m}{\rho_{eq}M_n}\sin(\beta vt)\frac{d^2}{dt^2}\left(\sum_{i=0}^{\infty}q^i(t)\right) \\ -\frac{mg}{\rho_{eq}M_n}\sin(\beta vt)-2\omega\zeta\frac{d^\alpha}{dt^\alpha}\left(\sum_{i=0}^{\infty}T_n{}^i(t)\right) \tag{21}$$

$$\frac{d^2}{dt^2}\left(\sum_{i=0}^{\infty}q^i(t)\right)+\kappa^2\left(\sum_{i=0}^{\infty}q^i(t)\right)= \\ =\kappa^2\left[\sin(\beta vt)\left(\sum_{i=0}^{\infty}T_n{}^i(t)\right)\right]-g \tag{22}$$

Equations (21) and (22) represent a system of equations with two dependent variables; $T(t)$ and $q(t)$. Each equation can be divided into two recursive formulas for $i=0$ and $i\geq 1$ as

$$\frac{d^2T^0(t)}{dt^2}+\omega^2T^0(t)= \\ =\frac{-mg}{\rho_{eq}M_n}\sin(\beta vt)\text{ , for } i=0 \tag{23}$$

$$\frac{d^2q^0(t)}{dt^2}+\kappa^2q^0(t)=-\mathrm{g}\quad\text{, for }\; i=0 \tag{24}$$

$$\frac{d^2T^i(t)}{dt^2}+\omega^2T^i(t)=\frac{-m}{\rho_{eq}M_n}\sin(\beta vt)\frac{d^2q^{i-1}(t)}{dt^2} \\ -2\omega\zeta\frac{d^\alpha T_n{}^{i-1}(t)}{dt^\alpha}\quad\text{, for } i\geq 1 \tag{25}$$

$$\frac{d^2 q^i(t)}{dt^2} + \kappa^2 q^i(t) = \\ = \kappa^2 \sin(\beta v t) T^{i-1}(t) \quad , \text{for} \quad i \geq 1 \tag{26}$$

The previous equations can be solved using the Laplace transform, in which the Laplace transform for Caputo-Fabrizio derivative defined in Equation (3) is given as [26]

$$\mathcal{L}\left[D_t^{(\alpha)} f(t)\right] = \frac{s\mathcal{L}\left[f(t) - f(0)\right]}{s + \alpha(1-s)} \tag{27}$$

So, the Laplace transform of the Equations (23) to (26) can be written respectively as

$$\tau^0(s) = \frac{-m\, g\, \Omega}{\rho_{eq} M_n \left(\Omega^2 + s^2\right)\left(\omega^2 + s^2\right)}, \text{ for } i = 0 \tag{28}$$

$$Q^0(s) = \frac{-g}{s\left(\kappa^2 + s^2\right)}, \text{ for } i = 0 \tag{29}$$

$$\tau^i(s) = \frac{-m}{\rho_{eq} M_n \left(\omega^2 + s^2\right)} \mathcal{L}\left[\sin(\Omega t) \frac{d^2 q^{i-1}(t)}{dt^2}\right] \\ -\frac{2\omega\zeta}{\left(\omega^2 + s^2\right)} \mathcal{L}\left[\frac{d^\alpha T_n^{i-1}(t)}{dt^\alpha}\right], \text{ for } i \geq 1 \tag{30}$$

$$Q^i(s) = \frac{\kappa^2}{\left(\kappa^2 + s^2\right)} \mathcal{L}\left[\sin(\Omega t)\; T^{i-1}(t)\right], \text{ for } i \geq 1 \tag{31}$$

where $\mathcal{L}[\bullet]$ stands for the Laplace transform for the term inside the square brackets. $\tau(s)$ and $Q(s)$ are the Laplace transform of $T(t)$ and $q(t)$, respectively. While, Ω is defined by $\Omega = \beta v$. The final solution can be obtained by inverse Laplace transform for Equations (28) to (31). In this solution the components of $T(t)$ and $q(t)$ given in Equations (19) and (20) are found in the following order: $T_n^0(t)$, $q^0(t)$ from Equations (28) and (29), and the other components ($T_n^1(t)$, $q^1(t)$, $T_n^2(t)$, $q^2(t)$,…) for $i \geq 1$ from Equations (30) and (31). The first terms of $T(t)$ and $q(t)$ can be obtained respectively as

$$T_n^0(t) = \frac{mg(\omega \sin(\Omega t) - \Omega \sin(\omega t))}{\rho_{eq} Mn(\Omega^2 - \omega^2)\omega} \tag{32}$$

$$q^0(t) = \frac{g(\cos(\kappa t) - 1)}{\kappa^2} \tag{33}$$

The Laplace inversions for the other components with $i \geq 1$ are obtained after inserting the numerical values of the problem. At this end, the generalized deflection functions $T_n(t)$ and the vertical displacement of the oscillator $q(t)$ can be found to obtain the solutions of the problem under consideration.

3. RESULTS AND DISCUSSION

The results when the beam is assumed to be isotropic and homogeneous are presented using the following dimensionless parameters; the length is normalized by the beam length L, and the time is normalized by (L/v), which is the maximum time needed for the oscillator to pass over the beam, and the deflection is normalized by the maximum static deflection $(mgL^3/48EI)$ at the mid-span of the beam. However, when the beam is considered to be FG, the deflection is normalized by $(mgL^3/4E_U bh^3)$, and the axial stress is normalized by $(1.5mgL/bh^2)$ which is the maximum flexural stress at the mid-span of the beam [22]. The upper surface of the FG beam is considered to be pure Aluminum and the lower is pure Alumina, the properties of the beam are clarified in Table 1 [20]. For the data presented in Table 1, Figure 2 depicts the variation of Young's modulus and the mass density along the thickness direction. The other geometric parameters of the beam are taken as: L= 30 m, b= 0.5 m and h= 0.5 m, while the oscillator has spring stiffness of 1000 N/m and mass of 50 kg. Before studying the general cases for obtaining the dynamic response of the beam, three verification studies were made to check the validity of the derived equations. The first one is for homogeneous undamped beam, the second is for an FG undamped beam while the third is for homogeneous fractionally damped beam.

Table 1. Material properties of the FG beam

Material	Young's Modulus (GPa)	Density (kg/m^3)
Aluminum	70	2700
Alumina	380	3800

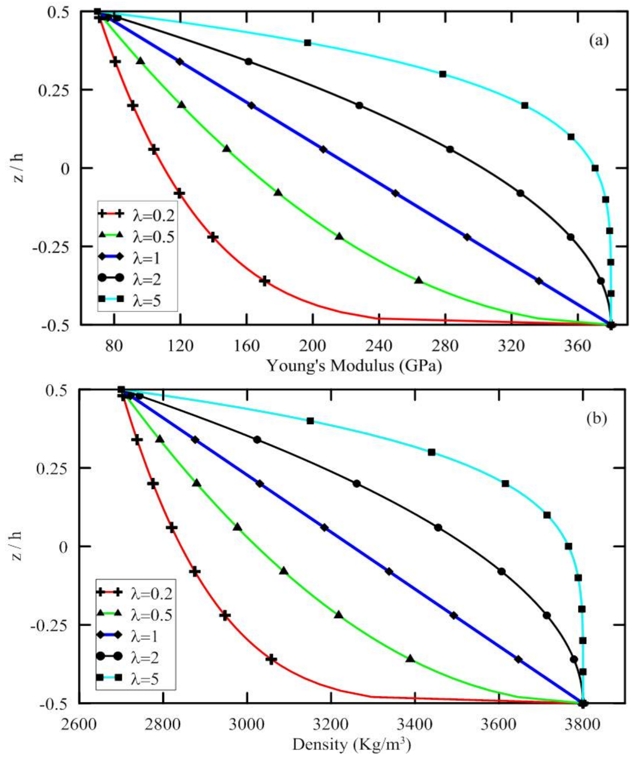

Figure 2. Variation of (a) Young's modulus and (b) mass density of the FG beam along the thickness direction according to the power law for different grading orders

3.1. Verification study I

In this verification problem, a homogeneous beam subjected to moving oscillator is considered. The properties and dimensions of the beam are $E = 2.8$ GPa, $I = 2.9$ m^4, $L = 25$ m, and mass per unit length of 2303 kg/m, whereas the oscillator mass is 5750 kg, and it is assumed to be moving with constant velocity of 27.78 m/s. The mid span deflection of the beam is compared with the results obtained by Majka and Hartnett [5], and as shown in Figure 3 there is a very good agreement between the two results. In addition, the results presented in Figure 3 have a good agreement with the experimental results provided by Biggs [34] for the same type of problems.

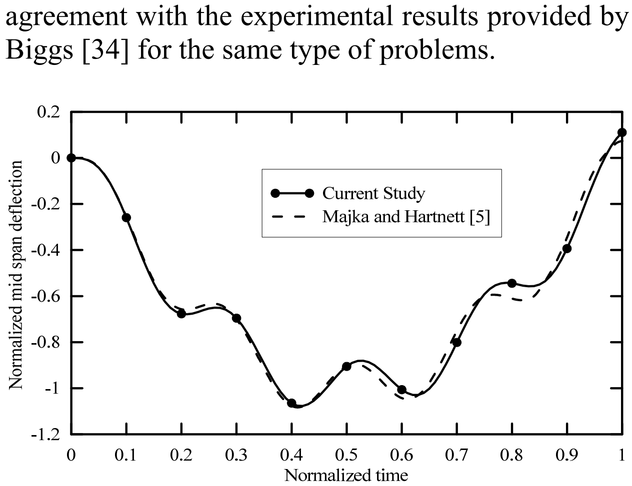

Figure 3. Normalized mid-span deflection of a homogenous beam subjected to moving oscillator

3.2. Verification study II

Here, the beam is considered to be functionally graded subjected to 100 kN point load moving with constant velocity of 80 m/s. The bottom surface is considered to be pure steel with $E_L = 210$ GPa and $\rho_L = 7800$ kg/m^3, while the top surface is made of Alumina with $E_U = 390$ GPa and $\rho_U = 3960$ kg/m^3. The beam is 20 m long, 0.4 m wide and thickness of 0.9 m. The mid-span deflection of the beam is compared with the results obtained by Simsek and Kocaturk [18]. As shown in Figure 4, the maximum deflection values in the two results agree well. However, there are slight differences as the load approaching the end of the beam due to the solution methods used, since Simsek and Kocaturk [18] solved using Newmark integration method.

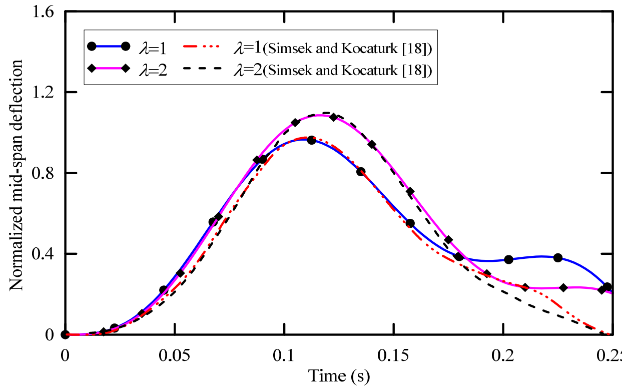

Figure 4. Normalized mid-span deflection of an FG beam subjected to moving point load with different grading orders

3.3. Verification study III

In this case, a comparison was made between Caputo's and Caputo-Fabrizio definitions of the fractional derivative. The homogeneous beam dimensions is assumed with $L = 20$ m, $A = 2.1\times10^{-3}$ m^2, $I = 3.953\times10^{-6}$ m^4, $E = 2.1$ GPa, $\rho = 7600$ kg/m^3 and $C_\alpha / E = 0.03$. It is supposed that a 100 N point load is crossing the beam with constant velocity equal to the half of the beam's critical velocity. As shown in Figure 5, there is excellent agreement with the solution obtained by Freundlich [14] based on Caputo's fractional derivative, which is considered as an evidence of how the same results may be obtained by simpler way using Caputo-Fabrizio fractional derivative without using Green's function in fractional form. Furthermore, this problem shows that the Caputo-Fabrizio fractional derivative model has the ability to describe the behavior internal beam damping with suitable scale of material heterogeneity, which can described by more complicated classical fractional derivate models.

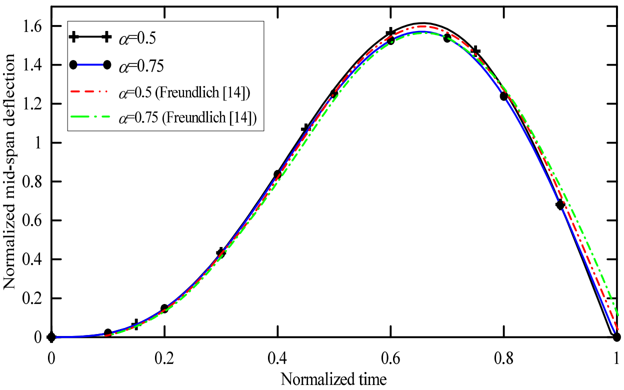

Figure 5. Normalized mid-span deflection of homogenous beam subjected to moving point load with different fractional derivative orders

3.4. Parametric study

A parametric study is performed to investigate the effects of the grading order, the damping ratio, and the Caputo-Fabrizio fractional derivative order on the dynamic response of the proposed beam. In Figure 6 the relation between the normalized deflection and the normalized time is demonstrated for $\zeta = 0.15$ and $\alpha = 0.25$ for $v = 15$ m/s and 40 m/s, respectively. As shown in the curves of Figure 6, the deflection is decreased by increasing the grading order, because the beam composition is varying from ductile material (Aluminum) at the upper surface to more rigid material (Alumina) at the lower surface. Also it is observed that by increasing the velocity, the oscillations and the deflection are decreased because the beam does not have much time to respond. The relation between the axial stress and the beam normalized thickness is depicted in Figure 7 for $v = 15$ m/s and 40 m/s at different grading orders. It is shown that the stress varies linearly when the beam is pure Aluminum or pure Alumina, also the tensile and the compressive stresses are equal, which means that the neutral axis is coincident with physical mid plane. However, when the beam is FG, the stress becomes nonlinear, and the shifting in the neutral axis position below the physical mid plane is obvious, because the lower material has modulus of elasticity greater than the upper. In Figure 8 the effect of the damping ratio on the beam deflection is depicted for $\alpha = 0.25$ and $\lambda = 1$ at two different velocities. It is observed that by increasing the damping ratio the deflection will decrease. Figure 9 shows the effect of the fractional derivative order on the dynamic response of the beam at $v = 15$ m/s and $v = 40$ m/s respectively, with $\zeta = 0.15$ and $\lambda = 1$. As noted from the curves of Figure 9, by increasing the fractional derivative order from 0.25 to 0.75 the deflection will decrease at both velocities. However, when the fractional derivative order becomes 1, the deflection will increase when $v = 15$ m/s and it will decrease when $v = 40$ m/s. The relation between the mid-span velocity and the normalized mid-span deflection is shown in Figure 10. In this figure, the loops represent the oscillations of the beam, which appear more obvious when $v = 15$ m/s than when $v = 40$ m/s. Also, it is noted that by increasing the grading order, the mid-span velocity is decreased. Moreover, by increasing the velocity of the oscillator, the mid-span velocity is increased. Figure 11 shows how the maximum normalized deflection is changing with the moving oscillator velocity at different grading orders. From the curves of Figure 11, it is observed that by increasing the grading order and the velocity of the moving oscillator the deflection is decreased. In this figure, a sudden change in the deflection is noted at $v = 77$ m/s. The sudden changes occurred because the oscillator velocity is very close to the beam critical velocity which is equal to 76.96 m/s when $\lambda = 0$. In Figure 12 it is observed that by increasing the damping ratio the normalized mid-span deflection is reduced. In Figure 13 it is noted that the deflection is decreased by increasing the fractional derivative order. However, when $\alpha = 1$ and the velocity is less than 26 m/s, the deflection will increase, which means that the effect of the fractional derivative order depends on the velocity of the oscillator.

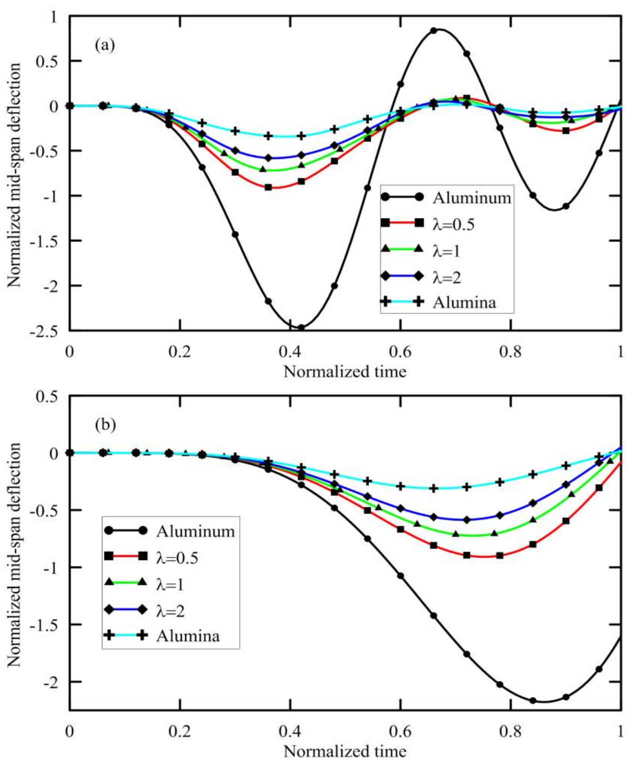

Figure 6. Normalized mid-span deflection of the FG beam with respect to normalized time at $\zeta = 0.15$, $\alpha = 0.25$ and various grading orders, (a) $v = 15$ m/s, (b) $v = 40$ m/s

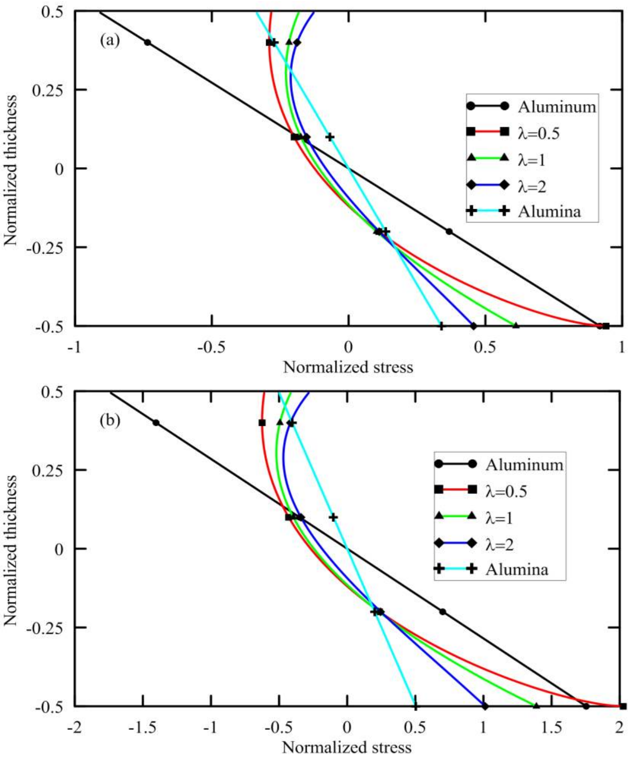

Figure 7. Variation of normalized stress with respect to the normalized thickness at the FG beam mid-span for $\zeta = 0.15$, $\alpha = 0.25$, $t = 0.9$ and various grading orders, (a) $v = 15$ m/s, (b) $v = 40$ m/s

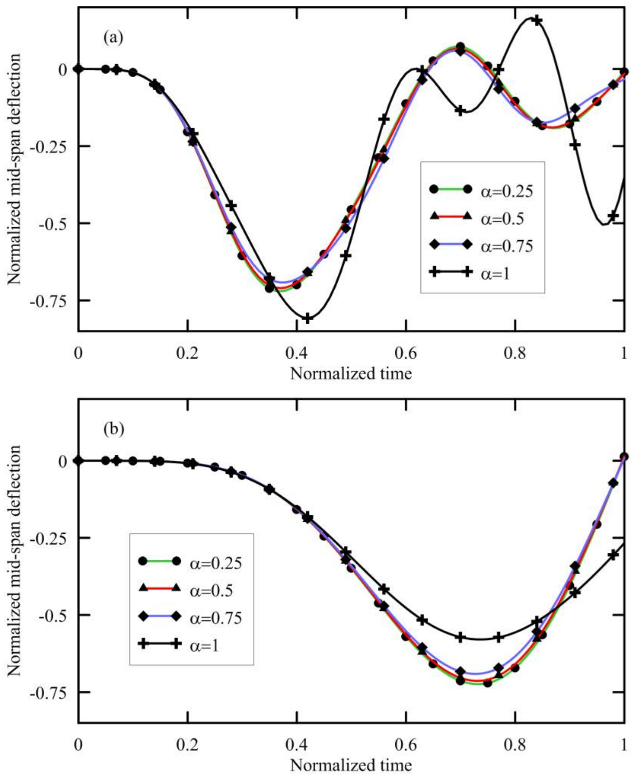

Figure 9. Normalized mid-span deflection of the FG beam with respect to normalized time at $\lambda = 1$, $\zeta = 0.15$ and various fractional derivative orders, (a) v=15 m/s, (b) v=40 m/s

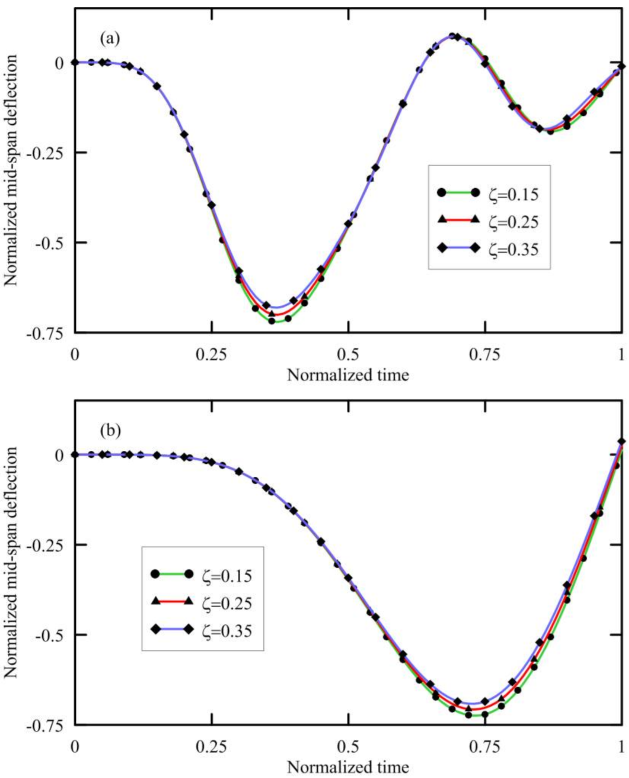

Figure 8. Normalized mid-span deflection of the FG beam with respect to normalized time at $\lambda = 1$, $\alpha = 0.25$ and various damping ratios, (a) $v = 15$ m/s, (b) $v = 40$ m/s

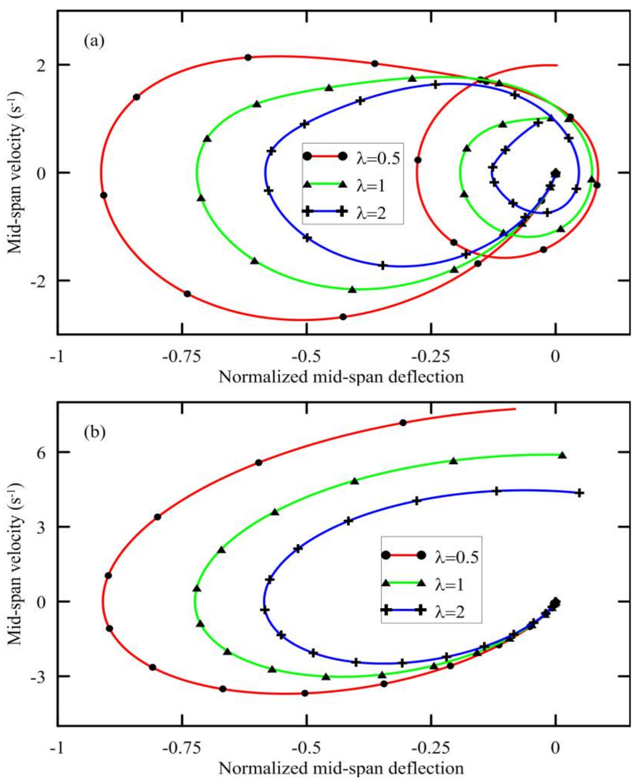

Figure 10. Mid-span velocity of the FG beam with respect to normalized mid-span deflection at $\alpha = 0.25$, $\zeta = 0.15$ and various grading orders, (a) v=15 m/s, (b) v=40 m/s

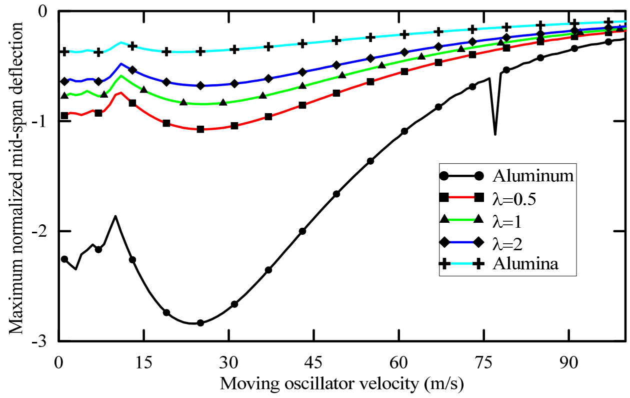

Figure 11. Maximum normalized mi-span deflection with respect to the velocity of the moving oscillator at $\alpha = 0.25$, $\zeta = 0.15$ and various grading orders

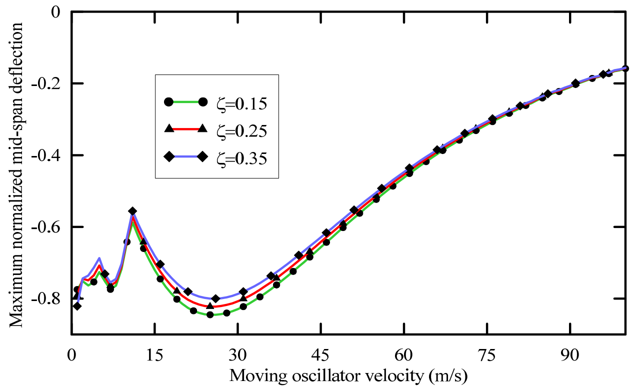

Figure 12. Maximum normalized mid-span deflection with respect to the velocity of the moving oscillator at $\lambda = 1$, $\alpha = 0.25$ and various damping ratios

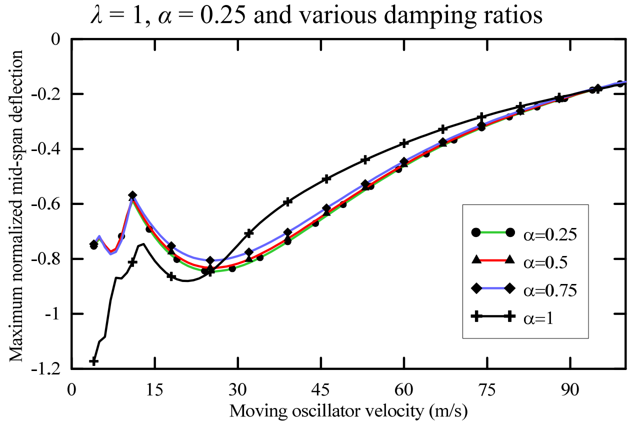

Figure 13. Maximum normalized mid-span deflection with respect to the velocity of the moving oscillator at $\lambda = 1$, $\zeta = 0.15$ and various fractional derivative orders

4. CONCLUSIONS

The dynamic response of functionally graded simply supported beam is examined. E-B theory is used with fractional Kelvin-Voigt model to describe the behavior of the beam. The material properties are varying along the thickness direction according to the power law. Caputo-Fabrizio definition for the fractional derivative was employed. It can be extracted from the verification studies that the applied method is valid as well as the Caputo-Fabrizio definition is suitable to describe the internal beam damping. The effects of the oscillator velocity, grading order, damping ratio and the fractional derivative order were examined. It is noted from the results that if the material composition varies from ductile to rigid material, the deflection will decrease by increasing the grading order. Also, it can be observed that the deflection is decreased when the oscillator velocity and the damping ratio are increased. Furthermore, if the fractional derivative order is increased the deflection will decrease except when the derivative order becomes unity, for this case the behavior of the beam will depend on the oscillator velocity. Furthermore, it is observed that the axial stress behave linearly when the beam is homogenous. However, when the beam becomes FG, the variation of the axial stress along the thickness becomes nonlinear and the neutral axis position is shifted from its physical mid-plane.

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