Evaluation of the Dissipated Energy in Vicinity of the Resonance, depending on the Nature of Dynamic Excitation

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Abstract: - The paper analyses the issue of functioning in the vicinity of the resonance of the mechanic equipment and/or devices with excitation sources the maximum perturbation force of which may be constant or variable related to the squared excitation pulsation.

In both cases, the dynamic system does not remain constantly, stably and stationary on resonance point, the technological functionality being characteristic only for the ante/post-resonance regimens. For this reason, the proper notion of functioning in the vicinity of the resonance is used.

There are dwelt with two distinct cases in this context, for a real dynamic model, from where it results the manner, we have to evaluate the maximal dissipated energy and maximal dissipated power related to the resonance regime.

In both cases, the monotone increasing variation of the excitation pulsation requires contribution of energy from the source at the passage from ante-resonance to resonance or from resonance to post-resonance.

Keywords: vicinity of the resonance, dissipated energy, dynamic excitation

1. INTRODUCTION

The evaluation of the maximum dissipated energy for a dynamic linear system (m, c, k) is performed for two distinct situations for the perturbing force, namely: for the dynamic excitation of the type $F = F_0 \sin \omega t$ where $F_0 =$ const. and for the dynamic excitation of the type $F = mr_0\, \omega^2 \sin \omega t$.

The conditions for the amplitude evolution in significant dynamic regimens have been established and, as a particular manner, the curves of the maximal amplitude peaks have been established. These curves represent the geometrical place of the amplitudes peaks in ante- or post- resonance, depending on the type of excitation.

2. RESPONSE CURVES AT THE DYNAMIC EXCITATION $F(t) = F_0 \sin \omega t$

The force $F(t)$ has constant amplitude, i.e. $F_0 =$ const. and dynamically excites the linear viscoelastic system (m, c, k).

In this case, the amplitude $A(\omega)$ of the instantaneous movement $x=x(t)$ of the forced vibration $x=x(t)=A \sin(\omega t- \varphi)$ is of the type:

$$A(\omega) = \frac{F_0}{\sqrt{(k-m\omega^2)^2+c^2\omega^2}} \tag{1}$$

In relation with the relative sizes $\Omega = \frac{\omega}{\omega_n}$, $\zeta = \frac{c}{2\sqrt{km}}$, the amplitude $A(\Omega)$ is given by the relation:

$$A(\Omega) = \frac{F_0}{k}\frac{1}{\sqrt{(1-\Omega^2)^2+(2\zeta\Omega)^2}} \tag{2}$$

2.1. Amplitude at resonance

The condition of resonance is $k - m\omega^2 = 0$ where $\omega_n{}^2 = \omega^2 = k/m$ or $\Omega = 1$, which leads to values of the amplitude at the resonance of the type

$$A^{rez}(\omega) = \frac{F_0}{c\omega_n} \tag{3}$$

$$A^{rez}(\Omega) = \frac{F_0}{k}\frac{1}{2\zeta} \tag{4}$$

Relations (3) are equivalent, therefore relation (3) may also be written as

$$A^{rez}(\omega) = \frac{F_0}{2\zeta mc\omega_n\omega_n} = \frac{F_0}{k}\frac{1}{2\zeta}$$

as $m\omega_n{}^2 = k$

2.2. Maximal value of the amplitude

The maximum of the $A(\omega)$ function in relation with ω is achieved by the condition $\frac{dA(\omega)}{d\omega} = 0$.

Thus, we have

$$\frac{dA(\omega)}{d\omega} = -F_0 \frac{1}{2} [(k - m\omega^2)^2 + c^2\omega^2]^{-3/2} [-4m\omega(k - m\omega^2) + 2c^2\omega]$$

or

$$\frac{dA(\omega)}{d\omega} = -F_0 \frac{2\omega[-2m(k-m\omega^2)+c^2]}{\sqrt{[(k-m\omega^2)^2+c^2\omega^2]^3}} = 0$$

thus

$$\omega^2 = \frac{2km - c^2}{2m^2}$$

or

$$\omega^2 = \omega_n^2(1 - 2\zeta^2)$$

We note $\omega = \omega_\alpha < \omega_n$ thus obtaining

$$\omega_\alpha = \omega_n\sqrt{1 - 2\zeta^2} \qquad (5)$$

On the other hand, it may also be written $\Omega_\alpha = \frac{\omega_\alpha}{\omega_n} < 1$ and, then, we have

$$\Omega_\alpha = \sqrt{1 - 2\zeta^2} \qquad (6)$$

with representation in figure 1.

Note For $\Omega = \Omega_\alpha$ the maximal value of the $A(\Omega)$ function is reached and it is also called "resonance to the left".

The maximum value of the amplitude A_α^{max} is $A_\alpha^{max} = A(\Omega_\alpha)$, i.e.

$$A_\alpha^{max} = \frac{F_0}{k} \frac{1}{2\zeta\sqrt{1-\zeta^2}} \qquad (7)$$

For values of $\zeta \in \left(0, \frac{\sqrt{2}}{2}\right)$ we have $0 < \sqrt{1 - \zeta^2} < 1$, from where resulting that $A_\alpha^{max} > A_{\Omega=1}^{rez}$

2.3. Geometrical location of the maximum peaks for the response curves at Ω = Ωα

Coordinates of the maximum peaks for $\Omega = \Omega_\alpha$ are

$$\begin{cases} \Omega_\alpha = \sqrt{1 - 2\zeta^2} \\ A_\alpha^{max} = \frac{F_0}{k} \frac{1}{2\zeta\sqrt{1-\zeta^2}} \end{cases} \qquad (8, 9)$$

Eliminating the parameter ζ between the two coordinates, the connection function between A_α^{max} and Ω_α is found, namely the curve describing the geometric place (the locus) of the maximum peaks.

Thus, we have from (8)

$$\zeta^2 = \frac{1 - \Omega_\alpha^2}{2}$$

which replaced in the relation (9), results into

$$A_\alpha^{max} = \frac{F_0}{k} \frac{1}{2\zeta\sqrt{1-\Omega_\alpha^4}} \qquad (10)$$

with the definition mathematic field $-1 < \Omega_\alpha < +1$ and the physics definition field $0 < \Omega_\alpha < +1$.

The graphic representation of $A_\alpha^{max}(\Omega_\alpha)$ function is presented in figure 2.

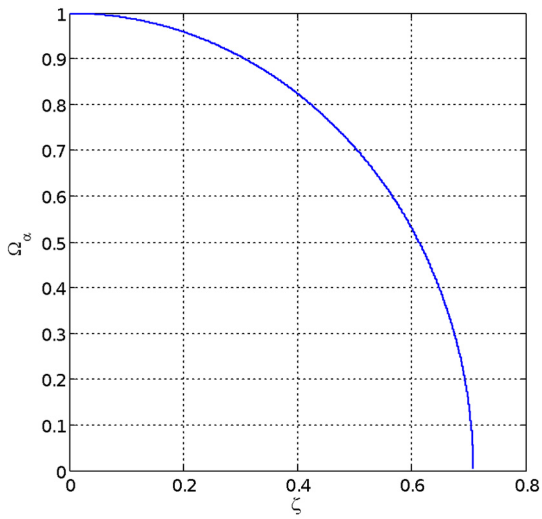

Figure 1. Variation of the relative pulsation Ω_α in relation with ζ for F_0 = const.

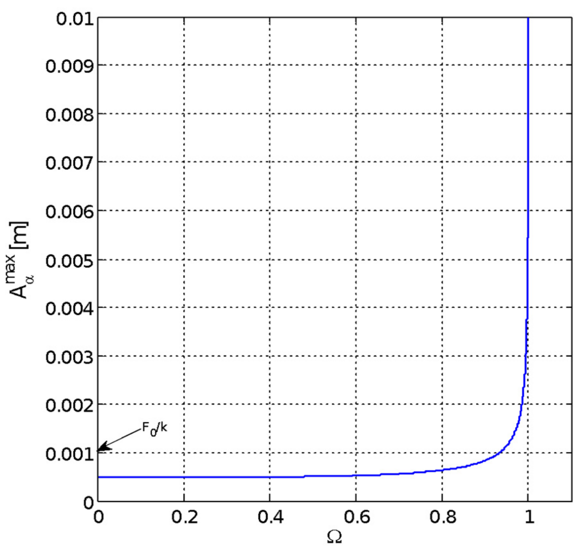

Figure 2. Curves of $A_\alpha^{max}(\Omega)$ amplitude peaksin relation with the relative pulsation Ω for F_0 = const.

2.4. Family of response curves $A(\Omega,\zeta)$

The function $A(\Omega)$ is represented by the family of parameterized response curves through the fraction from the critical amortization ζ, for the dynamic system with constant sizes F_0, k, m.

There are given the sizes $F_0=10^4$ N; $k=2\cdot10^7$ N/m; ζ =0,2; 0,4; 0,5; 0,6; 1,0; 2,0; 3,0.

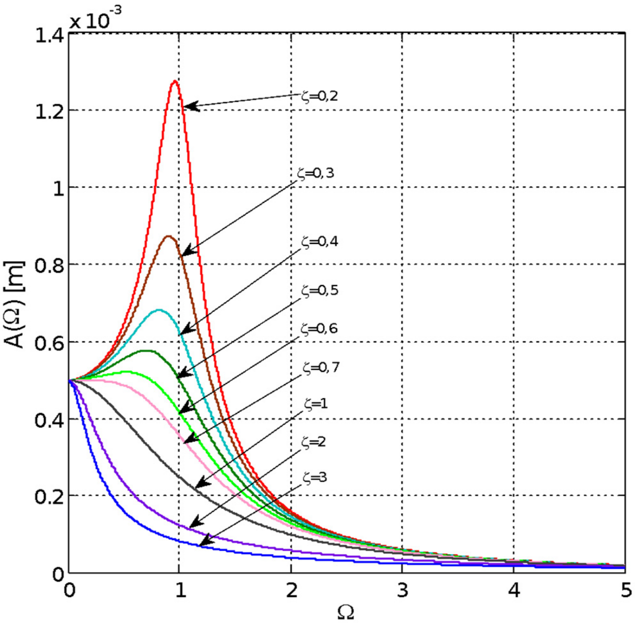

Figure 3. Family of curves of the amplitude $A(\Omega,\zeta)$ depending on ζ and Ω, for F_0= const.

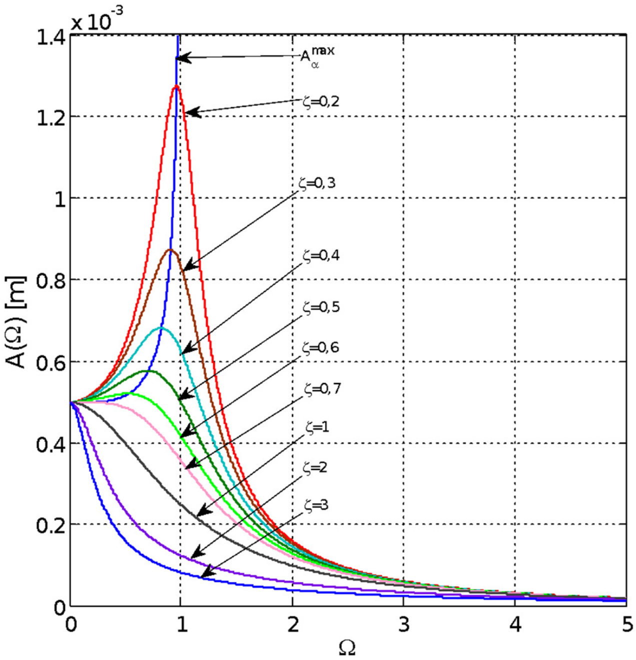

Figure 4. Family of curves of the amplitude $A(\Omega,\zeta)$ and the curve of the amplitude $A_\alpha^{max}(\Omega)$ peaks for F_0= const.

2.5. Dissipated energy

The dissipated energy at the **resonance** per cycle is of the type

$$W_{rez} = \pi c \omega_n A_{rez}^2$$

or

$$W_{rez} = \pi \frac{F_0}{k}\frac{1}{2\zeta} \tag{11}$$

Maximum dissipated energy at $\Omega_\alpha = \frac{\omega_\alpha}{\omega_n}$ may be written as

$$W_\alpha^{max} = \pi c \omega_\alpha (A_\alpha^{max})^2$$

or

$$W_\alpha^{max} = 2\pi\zeta\omega_n m \frac{F_0^2}{k^2}\frac{1}{4\zeta^2(1-\zeta^2)}$$

Therefore

$$W_\alpha^{max} = \pi \frac{F_0^2}{k}\frac{1}{2\zeta}\frac{\sqrt{1-2\zeta^2}}{1-\zeta^2} \tag{12}$$

We have of (11) and (12)

$$\frac{W_\alpha^{max}}{W_{rez}} = \frac{\sqrt{1-2\zeta^2}}{1-\tau^2} < 1$$

namely

$$W_\alpha^{max} < W_{rez} \tag{13}$$

2.6. Dissipated power

Dissipated power at **resonance** may be written as

$$P_d^{rez} = \frac{W_{rez}}{T} = \frac{W_{rez}}{2\pi}\omega_n$$

or

$$P_d^{rez} = \pi \frac{F_0^2}{k}\frac{1}{2\zeta}\frac{1}{2\pi}\omega_n$$

therefore

$$P_d^{rez} = \frac{1}{2}\frac{F_0^2}{c} \tag{14}$$

The maximum dissipated power in ante-resonance, considering (12), may be expressed as

$$P_{d,\alpha}^{max} = \frac{W_\alpha^{max}}{T} = \frac{W_\alpha^{max}}{2\pi}\omega_\alpha$$

or

$$P_{d,\alpha}^{max} = \frac{1}{2}\frac{\pi F_0^2}{k^2\zeta}\frac{\sqrt{1-2\zeta^2}}{1-\zeta^2}\omega_n\sqrt{1-2\zeta^2}$$

therefore

$$P_{d,\alpha}^{max} = \frac{1}{2}\frac{F_0^2}{c}\frac{1-2\zeta^2}{1-\zeta^2} \tag{15}$$

It is found that the maximum dissipated power is lower than the power dissipated in resonance, namely

$$\frac{P_{d,\alpha}^{max}}{P_{rez}} < 1 \text{ or } P_{d,\alpha}^{max} < P_{rez} \qquad (16)$$

This means that the passage from ante-resonance to resonance requires contribution of power from the excitation source with F_0 = const.

3. RESPONSE CURVES AT THE DYNAMIC EXCITATION *F(t)=m₀ rω² sinωt*

In this case, for a linear viscoelastic system (*m, c, k*), the amplitude of the forced vibration may be written as:

$$A(\omega) = \frac{m_0 r \omega^2}{\sqrt{(k - m\omega^2)^2 + c^2\omega^2}} \qquad (17)$$

or

$$A(\Omega) = A_0 \frac{\Omega^2}{\sqrt{(1-\Omega^2)^2 + (2\zeta\Omega)^2}} \qquad (18)$$

therefore $A_0 = \frac{m_0 r}{m}$ is the constant and stable amplitude for $\Omega \to \infty$.

3.1. The amplitude at resonance

From the resonance condition Ω =1, we have

$$A_{rez} = A_0 \frac{1}{2\zeta} \qquad (19)$$

3.2. Maximum value of the amplitude

We obtain from the condition $\frac{dA(\Omega)}{d\Omega} = 0$

$$1 - \Omega^2 + 2\zeta^2\Omega^2 = 0$$

where we have

$$\Omega = \Omega_\beta = \frac{1}{\sqrt{1-2\zeta^2}} \qquad (20)$$

which replaced in (18) leads to the maximum value $A = A^{max}{}_\beta$, thus

$$A_\beta^{max} = A_0 \frac{1}{2\zeta\sqrt{1-\zeta^2}} \qquad (21)$$

It is found that for $\Omega_\beta > 1$, and $A_\beta{}^{max} > A^{rez}$, and the achieved peak is moved to the right, reason why it is defined as "resonance to the right". It results from (21)

$$\frac{A_\beta^{max}}{A_{rez}} = \frac{1}{\sqrt{1-\zeta^2}} > 1$$

therefore

$$A_\beta^{max} > A_{rez}$$

The variation of Ω_β related to ζ is presented in figure 5.

3.3. Geometrical location (locus) of the maximum peaks for the response curves at $\Omega = \Omega_\beta$

Considering the relations (22) and (23), the coordinates of the maximum peaks result from the system

$$\begin{cases} \Omega_\beta = \frac{1}{\sqrt{1-2\zeta^2}} \\ A_\beta^{max} = A_0 \frac{1}{2\zeta\sqrt{1-2\zeta^2}} \end{cases} \qquad (22, 23)$$

Eliminating ζ of the equations (22) and (23), we obtain the function

$$A_\beta^{max} = A_0 \frac{\Omega_\beta^2}{\sqrt{\Omega_\beta^4 - 1}} \qquad (24)$$

It is specified that Ω□ has meaning only for $0 < \sqrt{1-2\zeta^2} < 1$ which is for $\in \left(0, \frac{\sqrt{2}}{2}\right)$, and the function (24) only for $\Omega_\beta \in [0,1)$.

Graphic representation of A_β^{max} is given in figure 6.

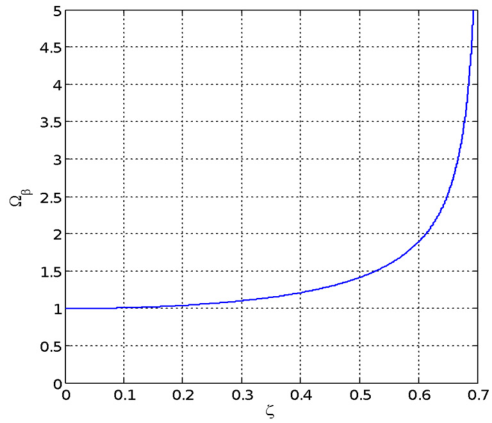

Figure 5. Variation of the relative pulsation Ω_β in relation with ζ

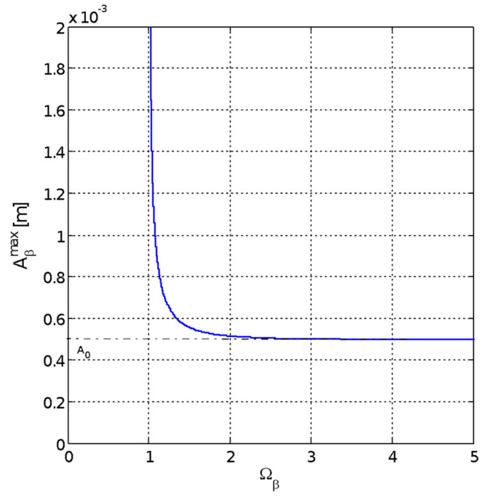

Figure 6. Curve of the amplitude $A_\beta^{max}(\Omega)$ peaks in relation with the relative pulsation Ω

3.4. Family of curves $A(\Omega, \zeta)$

The function $A(\Omega,\zeta)$ is represented in relation with Ω as current variable and discrete variable according to the relation (18) where $A_0 = 0{,}5\cdot 10^{-3}$ m, $\zeta = 0{,}2;\ 0{,}4;\ 0{,}5;\ 0{,}6;\ 1{,}0;\ 2{,}0;\ 3{,}0$, and $\Omega = 0...5$ with step at *0.1*. In figures 7 and 8, there are represented families of amplitude variation curves for $F = mr_0\,\omega^2$.

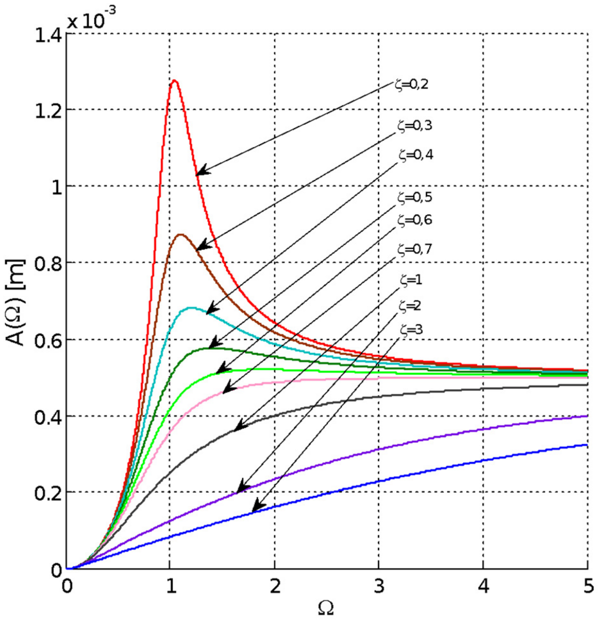

Figure 7. Family of curves of the amplitude $A(\Omega,\zeta)$ in relation of ζ and Ω, for $F_0 = m_0 r\omega^2$

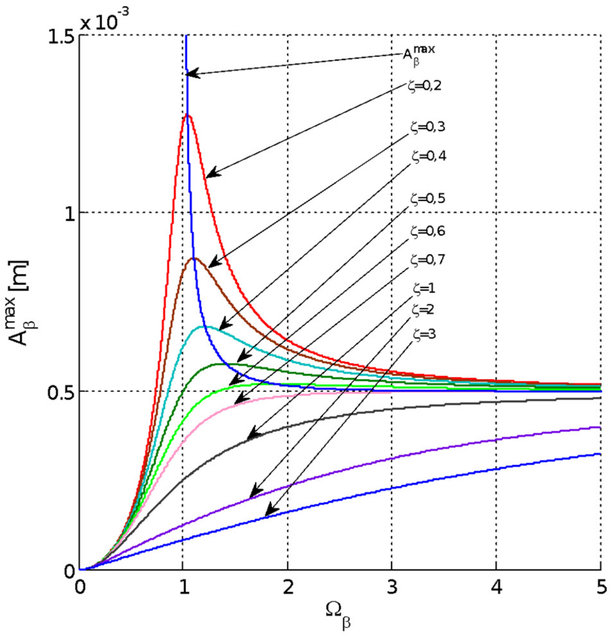

Figure 8. Family of curves of the amplitude $A(\Omega,\zeta)$ and the curve of the amplitude $A_\beta^{max}(\Omega)$, peaks for $F_0 = m_0 r\omega^2$

3.5. Dissipated energy

The dissipated energy in **resonance** is

$$W_{rez} = \pi c\omega_n A_{rez}^2$$

where we replace $c = 2\zeta\omega_n m$ și A_{rez} thus resulting

$$W_{rez} = \pi k A_0^2 \frac{1}{2\zeta} \tag{25}$$

The dissipated energy at Ω_β and A_β^{max} is given by the relation

$$W_\beta^{max} = \pi c\omega_\beta \left(A_\beta^{max}\right)^2$$

and, after replacing ω_β, c and A_β^{max} , we have

$$W_\beta^{max} = \frac{\pi k A_0^2}{2\zeta}\frac{1}{(1-\zeta^2)\sqrt{1-2\zeta^2}} \tag{26}$$

The ratio W_β^{max}/W_{rez} is of the type

$$\frac{W_\beta^{max}}{W_{rez}} = \frac{1}{(1-\zeta^2)\sqrt{1-2\zeta^2}} > 1$$

from where it results it is possible to write it as

$$W_\beta^{max} > W_{rez}$$

3.6. Dissipated power

The dissipated power in **resonance** is given by the relation

$$P_{rez} = \frac{W_{rez}}{2\pi}\omega_n$$

or

$$P_{rez} = \frac{1}{2}\frac{k^2 A_0^2}{c} \tag{27}$$

The maximum dissipated power in **postresonance** at Ω_β is of the type

$$P_\beta^{max} = \frac{W_\beta^{max}}{2\pi}\omega_\beta$$

or

$$P_\beta^{max} = \frac{1}{2}\frac{k^2 A_0^2}{c}\frac{1}{(1-\zeta^2)\sqrt{1-2\zeta^2}} \tag{28}$$

Thus, it results the following ratio of the powers

$$\frac{P_\beta^{max}}{P_{rez}} = \frac{1}{(1-\zeta^2)\sqrt{1-2\zeta^2}} > 1$$

thus

$$P_\beta^{max} > P_{rez}$$

This means that the passage through resonance for the monotonous increasing variation of the excitation pulsation requires power contribution for the source of the dynamic excitation system with the force $F = mr_0\,\omega^2$.

4. CONCLUSIONS

The analysis of the functioning in resonance for various categories of dynamic systems underlines that the resonance point for $\omega = \omega_n$ or $\Omega = 1$ cannot be maintained in a satisfying matte as a state of dynamic instability occurs. This is concretized by the "sliding~" of the point to the left or to the right, i.e in ante-resonance or post-resonance.

Depending on the type of dynamic excitation with the amplitude of the force being constant or variable related to the square of the excitation pulsation, the amplitude response curves and the curves of the maximal amplitudes peaks.

Thus, from the analysis of the dynamic behaviour, there have been assessed both dissipated energy and power in the vicinity of the resonance and at resonance. In this context, the following conclusions can be issued:

a. The dynamic excitation $F = F_0 \sin \omega t$ with F_0= const. is characterized by the fact that the maximum peaks for the response amplitude are on a curve places in ante-resonance;
b. The maximum dissipates energy and the maximum dissipated power in ante-resonance, respectively, have lower values than in resonance. For this, the passage from one regime to another at monotonous increase of the excitation pulsation is made with energy contribution
c. The dynamic excitation $F = mr_0\,\omega^2 \sin \omega t$ is characterized by the fact that the maximum peaks for the response amplitude are on a curve placed in post-resonance;
d. The maximum dissipated energy and the maximum dissipated power, respectively, in ante-resonance have higher values than in resonance. this leads to the need for energy contribution from the source, at the passage from resonance to post-resonance.

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