Assessing the Accuracy of a New Model for T-Shaped Cracks

Gilbert-Rainer GILLICH
"Eftimie Murgu" University fo Resita, P-ta Traian Vuia 1-4, 320085, Resita Romania, gr.gillich@uem.ro

Cristian TUFISI
"Eftimie Murgu" University fo Resita, P-ta Traian Vuia 1-4, 320085, Resita Romania, cristiantufisi@yahoo.com

David LUPU
"Eftimie Murgu" University of Resita, P-ta Traian Vuia 1-4, 320085 Resita, Romania, david.lupu@messer.ro

Oana Codruta HAMAT
"Eftimie Murgu" University of Resita, P-ta Traian Vuia 1-4, 320085, Resita Romania, c.hamat@uem.ro

Abstract: - The paper presents the investigations made to evaluate a model that predicts the frequency shifts due to T-shaped cracks. We developed the model basing on the energy loss due to the decrease of the beam's rigidity in the region of the crack which has a transversal and a longitudinal component, respectively. The algorithm includes a coefficient representing the energy fraction for the healthy and the damaged beam segment. These fractions are described involving the normalized square of the modal curvatures and get the form of an integral. To fast solve the integrals, we implemented the model in an application written in the Python programming language. This application solves numerically the integrals and a coefficient expressing the remained energy after damage results in form of a coefficient. We successfully tested the theoretical model against finite element simulation

Keywords: Damage assessment, frequency shift, cantilever beam, T-shaped crack, mathematical model.

1. INTRODUCTION

Vibration signals are intensively used to assess the health of engineering structures, because these signals are directly linked to the geometry and physics of the structure [1]. In the most of the cases, the transverse vibrations are taken into consideration for structural health monitoring [2]. However, it is important to know how effective these monitoring methods are for particular structures [3].

Frequency is the modal parameter that can be easily measured if proper estimation techniques are involved [4]. Therefore, methods using this feature are the most one presented in the literature; see for instance papers [5-8]. Transverse open or breathing cracks are analyzed with predilection, because of the lower complexity brought by this type of crack. The complexity increases if cracks with different shapes [9-14] or in changing environment [15,16] are considered.

In our prior research we studied the case of corrosion of metallic structures [17] as well as the branched cracks [18,19]. Because we observed a similar behavior for the beams with stiffness reduction irrespective to the way how this is achieved, we focused on developing a mathematical model that cover both corrosion and cracks with extent in the longitudinal direction.

This paper presents a model that can be used for cracks having a T-shape, i.e. having a transverse respectively longitudinal component. It is organized as follows: in the next section we present the mathematical model, in section three is introduced the application thatch solves numerically the model and in section four the simulation scheme. In the final part, in section five we present a comparison of the results obtained with the theoretical model and the finite element model (FEM) and the resulted conclusion.

2. PROBLEM FORMULATION

The model presented in this paper intends to predict the impact of the stiffness alteration on the natural frequencies of the cantilever beam with a T-shaped crack.

The first attempt is using a model that considers the stiffness decrease by maintaining the initial beam's mass. We achieved this by increasing the density for the segment with reduced stiffness (i.e. cross-section) as shown in Fig. 1.

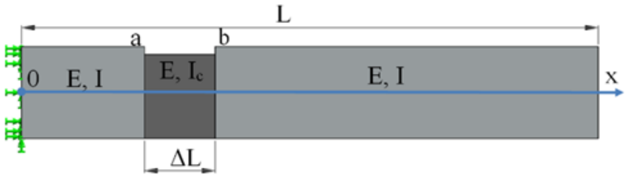

Figure 1. The reduced section model

For the intact cantilever beam the boundary conditions have been set and the dimensionless wave numbers for the first six modes of transversal vibration are given by solving the relation:

$$\cos(\alpha L)\cosh(\alpha L)+1=0 \tag{1}$$

where α is the dimensionless wave number.

After determining $\alpha L=\lambda$, we can calculate the natural frequencies of the undamaged beam by applying relation:

$$f_i=\frac{\lambda_i^2}{2\pi}\sqrt{\frac{EI_y}{mL^3}} \tag{2}$$

The relation for calculating the normalized mode shapes is:

$$\begin{aligned}\bar{\phi}_i(x)=0.5\{&\frac{\cos\alpha_i L+\cosh\alpha_i L}{\sin\alpha_i L+\sinh\alpha_i L}\cdot\\ &\cdot\left[\sin(\alpha_i x)-\sinh(\alpha_i x)\right]-\cos(\alpha_i x)+\cosh(\alpha_i x)\}\end{aligned} \tag{3}$$

The normalized modal curvature or bending moment is determined with the help of relation:

$$\begin{aligned}\bar{\phi}_i''(x)=0.5\{-&\frac{\cos\alpha_i L+\cosh\alpha_i L}{\sin\alpha_i L+\sinh\alpha_i L}\cdot\\ &\cdot\left[\sin(\alpha_i x)+\sinh(\alpha_i x)\right]+c\,os(\alpha_i x)+\cosh(\alpha_i x)\}\end{aligned} \tag{4}$$

The strain energy is expressed with the well-known relation:

$$\bar{U}_i=\frac{1}{2EI}\int_0^L\left[\bar{M}_i(x)\right]^2dx \tag{5}$$

From relation (5), we can derive for any vibration mode the strain energy coefficient

$$\kappa_i^{0-L}=\int_0^L\left[\bar{\phi}_i''(x)\right]^2dx=0.25 \tag{6}$$

For a segment a-b, the coefficient becomes:

$$\kappa_i^{a-b}=\int_a^b\left[\bar{\phi}_i''(x)\right]^2dx \tag{7}$$

Knowing the relation between the natural frequencies and strain energy is $f_i\approx\sqrt{U_i}$, the frequency for the beam with reduced cross-section becomes:

$$f_{Ci}=f_i\sqrt{1-4\kappa_i^{a-b}\frac{I-I_C}{I}} \tag{8}$$

Relation (8) can be expressed also as:

$$f_{Ci}=f_i\sqrt{\frac{\kappa_i^{0-a}+\frac{I_C}{I}\kappa_i^{a-b}+\kappa_i^{b-L}}{\kappa_i^{0-L}}}=c_i^{a-b}\cdot f_i \tag{9}$$

The relation is implemented in an application using the Python language, which solves the coefficients by numerical integration. To test the reliability of the stiffness reduction method, we compare the results obtained in the Python software with the natural frequencies obtained through the ANSYS simulation software for the cantilever beam affected by the T-shaped damage.

3. MATERIALS AND METHODS

The modal FEM study was carried out with the help of the ANSYS simulation software. The structure considered for this study is a cantilever beam with the physical properties of structural steel in the software library. These properties are given in Table 1.

Table 1. Cantilevers main dimensions and properties

Mass density [kg/m3]	Young modulus [N/m2]	Moment of inertia I [mm^4]
7850	$2\cdot10^{11}$	520.8333

The main dimensions of the analyzed beam are presented in Table 2. These confer the possibility to use the Euler-Bernoulli model for this beam.

Table 2. Cantilevers beam dimensions

Length L [mm]	Width B [mm]	Thick. H [mm]
1000	50	5

The damage considered is a T-shaped crack, placed at different locations and composed of a transversal crack and a longitudinal delamination.

The depth of the transverse component is *d* and is taken from the top surface. The delamination has the ends at distances *a* and *b* taken from the fixed end of the beam. The transversal component is always in the middle of the delamination. The crack along with its main dimensions is presented in Figure 2.

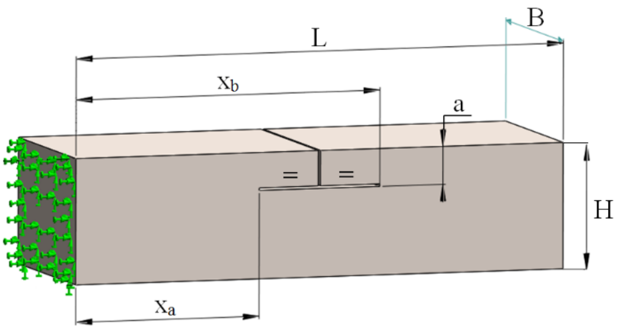

Figure 2. A zoom on the cantilever beam altered by a T-shaped crack

The beam and crack geometry used in the simulation was modeled with the help of the CAD software Solidworks. After importing the geometry in ANSYS, modal simulations were performed for all scenarios by applying the required boundary conditions. For accuracy, a fine mesh of hexahedral elements of maximum 2 mm size was used. The results obtained are the natural frequencies obtained for the cantilever beam in undamaged as well as in damaged state.

The locations of the crack were chosen arbitrary and they are presented in Table 3.

Table 3. The cracks locations and main dimensions

Damage interval *a-b* [mm]	Delamination length *T* [mm]	Damage depth *d* [mm]
300-350	50	1 & 1,5
350-400		
400-450		
280-230		
330-380		

As it can be observed from the previous table, the crack is positioned in five locations, firstly having a depth *d*=1 mm and in the second scenario having *d*=1.5 mm. For all damage locations and depth the analytical and FEM natural frequencies have been identified.

We developed software to automatize the calculus of the stiffness coefficients, described method is Python with the algorithm program and its interface presented in Figure 2.

The main menu makes it possible to select the type of boundary conditions for the beam, meaning: cantilever, double-clamped, free-free and simple-supported.

The transversal vibrating mode number is easily selected from the drop-down menu; the maximum mode that the software can calculate is ten.

The stiffness decrease portion of the structure is introduced as an interval value with the start of the damage location set up as the first value and the end is being the second value. The third value in the input box is the beam length.

As a last step before executing the program by pushing the „Integrare" button is to insert the values of the thickness of the beam, as a three values input type. The stiffness reduction value is set up as the ratio between the reduced section value and the constant beam value.

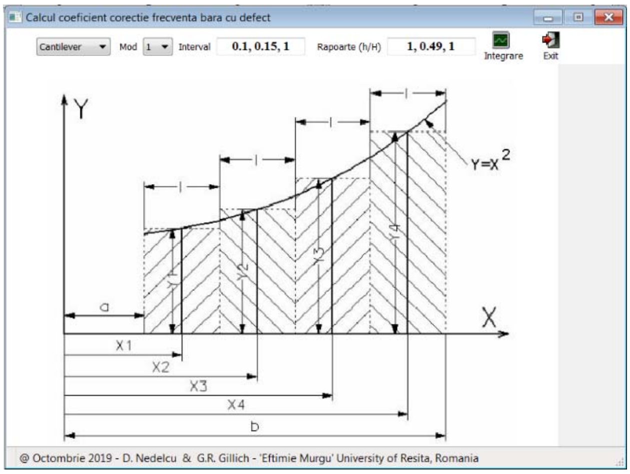

Figure 3. The developed Python software interface

As it can be observed in Figure 3, after executing the program for the desired reduced stiffness interval and depth, the curvature of the beam is presented is shown graphically and the stiffness reduction coefficient κ_i^{a-b} is depicted.

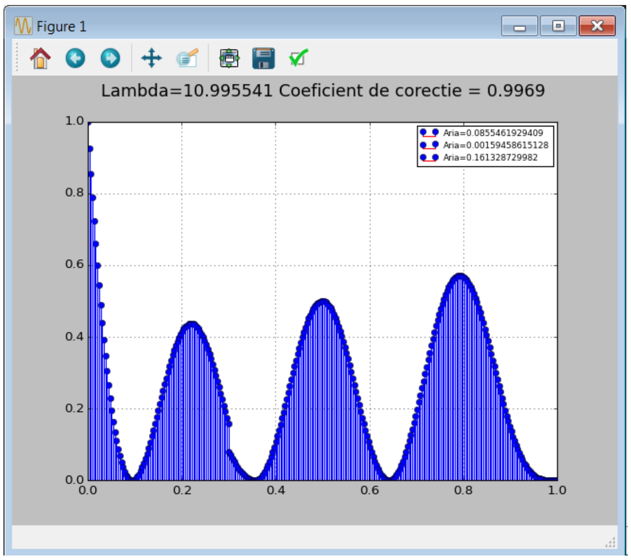

Figure 4. Energy distribution for mode four

After obtaining the stiffness reduction coefficient it is possible to calculate using relation (11) the natural frequencies of the beam with reduced section and constant mass

4. COMPARISON OF THE RESULTS

To test the reliability of the stiffness reduction method, we compare the results obtained in the developed software with the natural frequencies obtained through the ANSYS simulation software for the cantilever beam affected by the damage described above.

The first step was to set the proposed parameters for the reduced section, meaning the damage intervals and depth. The stiffness reduction coefficients have been recorded, for the first six modes of transversal vibration from the Python embedded algorithm and the natural frequencies of the cantilever have been calculated. In order to demonstrate the accuracy of the developed method, modal simulations were made using FEM analysis for the same cantilever beam, altered by the described T-shaped crack.

The percent differences between the developed analytical model and FEM simulations are shown in Table 4 for the crack having a depth of a=1 mm and in Table 5 for the crack with a=1.5 mm depth.

Table 4. Differences between the analytical model and the FEM simulations for the T-shaped crack with *a=1*

Mode no.	*a=300 b=350*	*a=350 b=400*	*a=400 b=450*	*a=280 b=330*	*a=330 b=380*
1	-1.35%	-1.06%	-0.81%	-1.48%	-1.18%
2	-0.61%	-1.11%	-1.56%	-0.42%	-0.90%
3	-1.48%	-1.11%	-0.54%	-1.52%	-1.30%
4	-0.25%	-0.13%	-0.83%	-0.49%	-0.07%
5	-0.72%	-1.53%	-1.19%	-0.34%	-1.32%
6	-1.61%	-0.78%	-0.47%	-1.68%	-1.15%

Table 5. Differences between the analytical model and the FEM simulations for the T-shaped crack with *a=1.5*

Mode no.	*a=300 b=350*	*a=350 b=400*	*a=400 b=450*	*a=280 b=330*	*a=330 b=380*
1	-3.64%	-2.90%	-2.24%	-3.97%	-3.18%
2	-1.61%	-2.91%	-4.10%	-1.12%	-2.39%
3	-3.71%	-2.75%	-1.32%	-3.83%	-3.22%
4	-0.60%	-0.34%	-2.07%	-1.15%	-0.18%
5	-1.80%	-3.62%	-2.72%	-0.85%	-3.20%
6	-3.28%	-1.47%	-0.87%	-3.54%	-2.25%

As we can observe from tables 4 and 5, the maximum error obtained is 1.56% for the crack having a depth of d=1 mm with the location in the interval 400-450 mm. As the crack has a larger depth we can observe a maximum error of 4.1% in the interval 300-350.

5. IMPROVEMENT OF THE MODEL

To obtain more accurate results, we studied the behavior of the damaged beam and observed a supplementary slope at the cross-section changes (at the T-crack ends). An image of the deformed state is shown in Fig. 5.

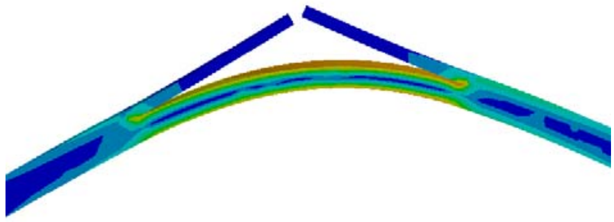

Figure 5. Supplementary bending moments at the damage location

To obtain the slope, we include in the model a supplementary bending moment at each damage end, as shown in Fig. 6. These bending moments are proportional with the squared normalized modal curvature at those locations.

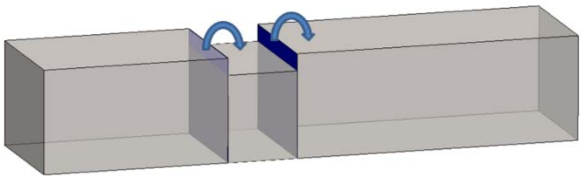

Figure 6. Supplementary bending moments at the damage extremities

We empirically deduced the supplementary severity $s(d)$ which are integrated in the correction coefficients c_i^a and c_i^b. These coefficients are used to multiply the obtained frequency correction coefficient c_i^{a-b} to give more accurate frequency estimation. These are found to be:

$$c_i^a = 1 - s(d)\left[\bar{\phi}_i''(a)\right]^2 = 1 - 7\gamma(d)\left[\bar{\phi}_i''(a)\right]^2 \quad (10)$$

$$c_i^b = 1 - s(d)\left[\bar{\phi}_i''(b)\right]^2 = 1 - 7\gamma(d)\left[\bar{\phi}_i''(b)\right]^2 \quad (11)$$

where $\gamma(d)$ is the severity of the transverse crack with a given depth d.

The proposed Python-severity coefficient is:

$$c_s = c_i^a \cdot c_i^{a-b} \cdot c_i^b \quad (12)$$

which considers the stiffness loss due to cross-section reduction between points a and b respectively the supplementary bending moments at the damage extremities.

The deflections of the beam under dead mass were depicted with the help of the ANSYS static simulation software. We have considered only the transversal element of the crack, meaning for the first case the crack with a=1 mm and second for d=1.5 mm. The severity $\gamma(d)$ results from the undamaged and damaged beam deflections, according to the following relation:

$$\gamma = \frac{\sqrt{\delta_D} - \sqrt{\delta_U}}{\sqrt{\delta D}} \tag{13}$$

As demonstrated in paper [19], if too close to the fixed end, the crack determines deflections and frequencies that do not follow the analytical solutions. For this reason, for calculating the beam deflection in damaged state we consider the location of the crack at x=3 mm from the fixed end and remove it by a step of 3 mm until it reaches x=18 mm from the fixed end. The deflection is estimated from the regression curve plotted on the achieved points [21], which are given in Fig. 7 for the 1 mm crack and in Fig 8 for the 1.5 mm crack. In these figures, the mathematical relation for the regression curve is also shown.

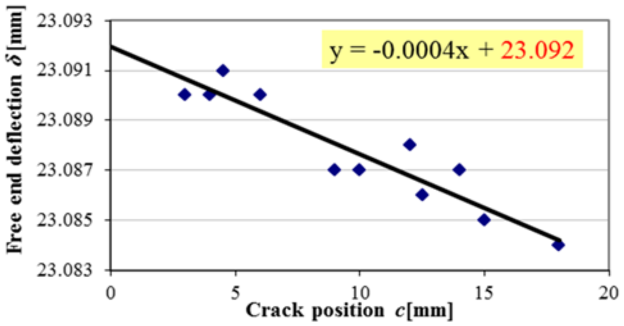

Figure 7. The trend-line plotted for the deflections of the damaged beam with different crack locations at depth d=1 mm

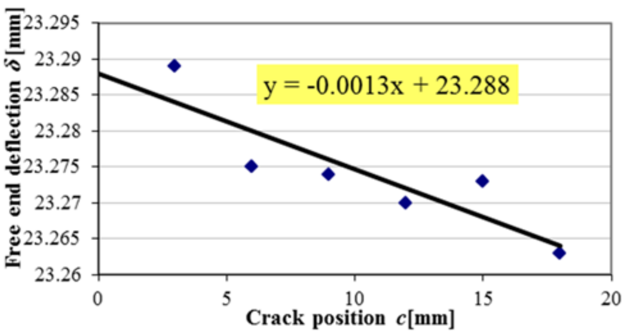

Figure 8. The trend-line plotted for the deflections of the damaged beam with different crack locations at depth d=1.5 mm

After obtaining the static deflection values from the mathematical relation using the regression curve, the severity is calculated using relation (13).

For the crack of depth d=1 mm we found the severity $s(1)$=0.02186 and for depth d=1.5 mm we found $s(1.5)$=0.006.

The results obtained by employing relation (12) are presented in Table 6 for the T-shaped crack with 1mm depth and in Table 7 for the crack with 1.5 mm depth.

Table 6. Differences between the proposed model and the FEM simulations for the T-shaped crack with d=1 mm

Mode no.	a=300 b=350	a=350 b=400	a=400 b=450	a=280 b=330	a=330 b=380
1	0.02%	0.01%	0.00%	0.03%	0.01%
2	0.11%	0.15%	0.21%	0.09%	0.14%
3	0.32%	0.22%	0.09%	0.31%	0.27%
4	0.10%	0.12%	0.21%	0.15%	0.09%
5	0.21%	0.37%	0.35%	0.18%	0.30%
6	0.20%	0.03%	0.02%	0.09%	0.15%

Table 7. Differences between the proposed model and the FEM simulations for the T-shaped crack with d=1.5 mm

Mode no.	a=300 b=350	a=350 b=400	a=400 b=450	a=280 b=330	a=330 b=380
1	0.36%	0.35%	0.30%	0.35%	0.35%
2	0.08%	0.08%	0.13%	0.09%	0.08%
3	0.57%	0.41%	0.15%	0.54%	0.51%
4	0.21%	0.23%	0.41%	0.35%	0.19%
5	0.42%	0.91%	0.94%	0.38%	0.65%
6	1.01%	0.45%	0.27%	0.67%	0.83%

As indicated in tables 6 and 7, the maximum error obtained is 0.37% for the crack having a depth of d=1 mm with the location in the interval 350-400 mm. As the crack has a larger depth we can observe a maximum error of 1.01% in the interval 300-350.

4. CONCLUSIONS

The paper presents the investigations made to evaluate a model that predicts the frequency shifts due to T-shaped cracks.

We considered the actual bending moment which is the moment that acts on the constant section beam but having the same effect for the small section bar as a consequence of the apparent modification of the bending moment in the reduced area.

Expressing the distribution of energy loss along the beam and taking into account the energy ratio we have obtained the stiffness reduction coefficients which allows us to calculate the frequencies for the narrow section beam, a relation valid for any type of support. Beside the stiffness reduction coefficients, by applying the supplementary bending moments for the reduced section we have obtained an accurate prediction of the natural frequencies for the beam with a T-shaped crack.

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