Analysis of Dynamic Earth Stiffness depending on Structural Parameters in the Process of Vibration Compaction

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Abstract: - The paper addresses the problem of the rigidity of the foundation earth under the action of dynamic compaction vibrations. For this, based on the Voigt-Kelvin linear viscoelastic model and the vibrator roll -terrain interaction, the calculation relation and the curvature families were established for the variation of rigidity in relation to the elastic and viscous structural parameters. Thus, the influence of viscosity and mass on continuous frequency variation was highlighted.

Keywords: - Dynamic Earth Stiffness, Vibration Compaction.

1. INTRODUCTION

The behavior of the structural systems based on the Voigt-Kelvin linear model is characterized by the fact that, at excitation frequencies in post-resonance, the stiffness $k=k_{dyn}$ is significantly higher than the static stiffness k_0.

This is shown both numerically and experimentally by lower amplitudes of the vibrations at high frequencies, i.e. for $\omega > 3\omega_n$, where $\omega = 2\pi f$ is the excitation frequency and $\omega_n = \sqrt{\frac{k}{m}}$ is the structural system's internal (natural) frequency, with a single degree of freedom. Furthermore, the increase of the damping ratio ζ or the increase of mass does not make for an interesting case in the assessment of dynamic stiffness.

Essentially, the analysis of the dynamic stiffness k_{dyn} is interesting when the excitation frequency (pulse) is continuously variable, and the damping or inertial parameters (masses) are discretely variable. Thus, we can calculate the influence from the modification of the pairs of parameters (ω, ζ) with a constant mass or of the parameters (ω, m) for constant damping.

2. MODIFICATION OF THE DYNAMIC STIFFNESS WITH THE CONTINUOUS VARIATION OF THE EXCITATION FREQUENCY AND THE DISCRETE VARIATION OF THE DAMPING OR OF THE MASS

The dynamic stiffness of the Voigt-Kelvin linear structural system, of the type (m, c, k_0) is given by the relation

$$k(\omega) = k_{dyn} = \sqrt{(k_0 - m\omega^2)^2 + c^2\omega^2} \quad (1)$$

where the fractional entry of the critical damping or the damping ratio ζ so that we have

$$k_{dyn}(\omega,\zeta) = \sqrt{(k_0 - m\omega^2)^2 + 4k_0 m\zeta^2\omega^2} \quad (2)$$

where the fact that $c = 2\zeta m\omega_n$ and $k_0 = m\omega_n^2$ was considered.

For the continuous variation of the excitation frequency ω and the discrete variation of the damping ratio ζ, the analytical plotting of the function $k_{dyn}(\omega,\zeta)$ is shown in figure 1.

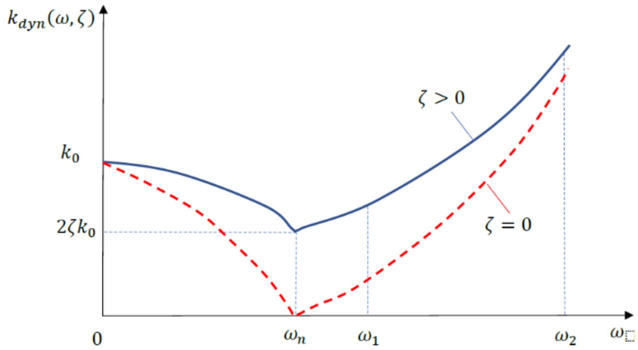

Figure 1. Variation of the dynamic stiffness depending on the excitation frequency ω and the damping ratio ζ with the constant mass m

For the continuous variation of the excitation frequency ω and the discrete variation of the mass m with the constant preservation of the damping ratio ζ, the function $k_{dyn}(\omega, m)$ is shown in figure 2.

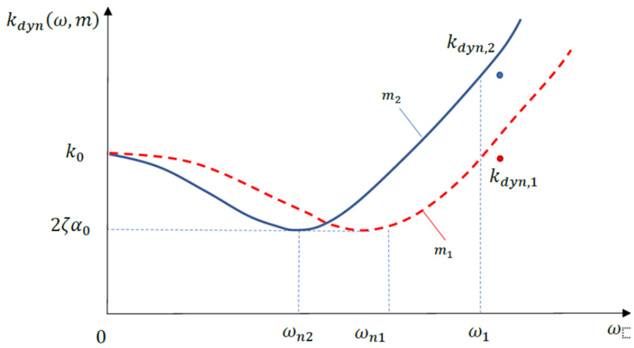

Figure 2. Variation of the dynamic stiffness depending on the excitation frequency ω and the mass m at constant damping

We can see that, for the discrete variation of the damping (fig.1), the dynamic stiffness for the frequency $\omega_1$in the proximity of ω_n increases with the increase of ζ. In the post-resonance area, *i.e.* for $\omega_2 \gg \omega_n$ the influence of damping is negligible, but the dynamic stiffness increases significantly at the increase of the resonance frequency.

At the discrete modification of the mass by ascending values, we can see that, at the excitation frequency in post-resonance $\omega_1 \gg \omega_n$, the dynamic stiffness increases markedly. This is possible only when, at vibration, based on technological causes, the mass m is modified by upward values. In this case, the vibration amplitude $A_2(m_2)$ is considerably lower than the vibration amplitude $A_1(m_1)$, which means that we have $k_{dyn,2} \gg k_{dyn,1}$, following the ***significant input of additional mass during the vibration***. This situation is specific to the variable mass systems, such as: soils massifs, vibrating fresh concrete, vibrating asphalt mixture, transport of granulated materials by vibration, the classification of granulated materials on vibrating screens, etc.

3. FAMILIES OF CURVES OF THE DYNAMIC STIFFNESS

In the case of the dynamic processes with discretely variable mass m or with damping that changes during vibration, by the discrete variation of the fraction of the equivalent critical damping also called equivalent damping ratio ζ, the Voigt-Kelvin model can be applied sequentially for stages of discretely variable values, both for m, and for ζ. At the continuous variation of the excitation frequency ω across the whole range of technological existence of the dynamic process, for which a discrete variable parameter is considered, while the other is kept constant, allows the drawing of the family of curves $k_{dyn} = k_{dyn}(\omega, \zeta)$ with constant m or $k_{dyn} = k_{dyn}(\omega, m)$ with constant ζ.

3.1. Variation of the dynamic stiffness depending on the frequency ω and on the damping ζ.

When, sequentially, ζ is modified upwardly by discrete values, and the excitation frequency varies continuously across the whole functional range, in figure 3 we can analyze the family of curves $k_{dyn}(\omega, \zeta)$ with constant m.

We can see that, in the proximity of the natural resonance frequency $\omega = 60$ rad/s, the dynamic stiffness decreases with the increase of the fraction of the critical damping, and the minima are arranged on a curve that draws away to the left from the ordinate line of the frequency $\omega = \omega_n = 60$ rad/s.

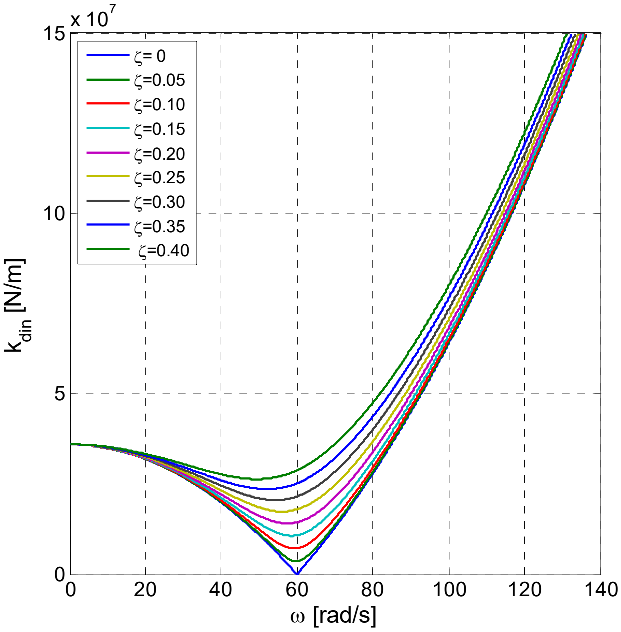

Figure 3. Family of curves $k_{dyn} - \omega$ for the discrete variation of ζ. Constant values for $m = 10^4$ kg, $k_0 = 36 \cdot 10^6$ N/m

At high frequencies, above 60 rad/s, the effect of the damping on the dynamic stiffness is significantly diminished.

Subsequently, at excitation frequencies in the proximity of the resonance $\omega = \omega_n$ the effect of the damping on the dynamic stiffness is consistent, while at frequencies in post-resonance, the dynamic stiffness depends on the inertial factor m, with very low influences of the damping, ensuring a low level of the vibration amplitude. This is the physical effect of the increase of the dynamic stiffness at high frequencies (with $\omega \gg \omega_n$).

3.2. Variation of the dynamic stiffness with the variation of the frequency ω and of the mass m.

For the continuous variation of the frequency in the range (0….200) rad/s and the discrete variation of the mass m from $5 \cdot 10^3$kg to $25 \cdot 10^3$ kg, the variation curves of the dynamic stiffness in figure 4 were plotted.

We can see that the family of curves has the shape of the diagrams in figure 4, where the minimum dynamic stiffness is kept constant, at the value $2\zeta k_0 = 1.44 \cdot 10^7$ N/m regardless of the values of ω and m. In this case, the minimum moves (slides) on the straight line $k_{dyn} = 2\zeta k_0$, depending on the current values of ω and on the discrete values of m. Thus, we can see that, for the same value of ω, the dynamic stiffness increase significantly with the discrete increase of the mass m, which is in a vibrating motion.

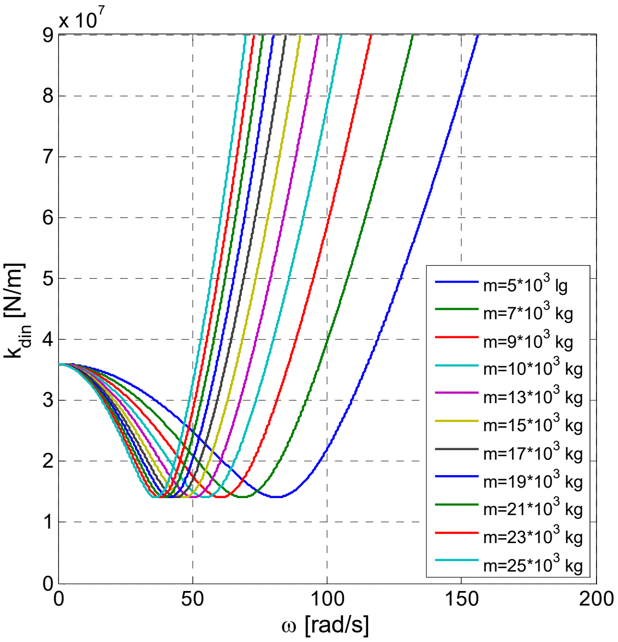

Figure 4. Family of curves $\mathrm{k_{dyn}} - \omega$ for the discrete variation of m. Constant values for $\mathrm{k_0} = 36 \cdot 10^6$ N/m; $\zeta = 0.20$

4. CONCLUSIONS

The analysis of the behavior of dynamic compaction equipment, of linear viscoelastic systems with one degree of freedom, of the vibrating systems for the processing of variable mass materials, in various technical application across industries, constructions, chemistry, biomechanical systems, etc., shows the modification of the dynamic stiffness depending on the variation of the excitation frequency, the viscous damping ratio and the system mass engaged in a vibrating motion. Following the experimental investigations, the laboratory and *in situ* tests, and the dynamic response analysis, on numerical and instrumental grounds, the following conclusions can be expressed:

a) For constant values of the mass m and of the stiffness k_0, at the continuous variation of the frequency ω and the discrete variation of the damping ratio ζ, families of curves of the dynamic stiffness with minimum values in the proximity of the natural frequency ω_n may be shown, as illustrated in figure 3. Here, this means that, in post-resonance $\omega > \omega_n$, the system's dynamic stiffness increases significantly depending on the excitation frequency (pulse). At excitation frequencies $\omega > 120$ rad/s, the influence of the damping is negligible.

b) For constant values of the stiffness k_0 and damping ζ, at the continuous variation of the excitation frequency ω and the discrete variation of the mass m, the dynamic stiffness has a minimum that is maintained on the straight line $k_{dyn} = 2\zeta k_0$ parallel with the horizontal axis for various values of ω and m (fig.4). We can see that, for the same value of ω (constant excitation) the dynamic stiffness increases, in an accelerated manner, with the discrete increase of the mass in vibrating motion.

Thus, for $\omega = 100$ rad/s, the variation of the mass m with discrete quantities (5, 7, 9, 10)·10^3 kg leads to the increase of the dynamic stiffness at the discrete values (2.1; 4.0; 6.0; 8.0)·10^7 N/m. This means that a double mass leads to a 4-time multiplication of the dynamic stiffness.

Given the above-mentioned aspects, the analysis of the dynamic stiffness, depending on the structural parameters of damping and mass with the continuous variation of the excitation frequency, becomes important in the assessment of the behavior expressed by the dynamic system with variable parameters.

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