Dynamic Rigidity of The Linear Voigt - Kelvin Viscoelastic Systems

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Abstract: - The paper addresses the problem of dynamic rigidity for structural systems under forced vibration. Thus, depending on the intrinsic elastic characteristics, masses and viscous damping coefficients, the variation of the dynamic rigidity can be highlighted depending on the frequency of the vibrations, as well as the variation of the mass or the damping regime. Essentially, the main families of curves are presented, which highlight the evolution modality of the dynamic rigidity.

Keywords: - Forced vibration, viscous damping, dynamic rigidity, discrete mass variation, natural pulses, excitation pulses.

1. INTRODUCTION

The analysis of the linear structural systems with a degree of freedom, Voigt - Kelvin modelled, with harmonic excitation, schematized (m, c, k), implies, under certain excitation conditions, the evaluation of dynamic rigidity. In this case, this involves checking the modality of the way of modifying the dynamic rigidity according to the variation of the excitation frequency for discrete values of mass and of the amortization of the structural system. Consequently, the study should highlight the variation laws and the curves related to the modification of the dynamic rigidity for the continuous increasing variation of the excitation frequency with the discrete variation of the amortization or of the mass, so that the ante-resonance, resonance and post-resonance regimes may be highlighted.

2. THE VOIGT - KELVIN LINEAR DYNAMIC STRUCTURAL RIGIDITY

For the linear system with a degree of freedom (m, c, k_0) dynamically excited with the harmonic force $F(t) = F_0 \sin \omega t$ where F_0 is the amplitude of the force, and ω the excitation pulse and the frequency $f = \frac{\omega}{2\pi}$. In this case, with the given static rigidity k_0, mass m and amortization c, in turn discrete variables, with continuous variation in the increasing direction of pulse ω , the dynamic rigidity is given by the following relations:

$$k_{dyn}^{v} = k_{dyn}^{v}(\omega, c) = \sqrt{(k_0 - m\boldsymbol{\omega}^2)^2 + \boldsymbol{c}^2\boldsymbol{\omega}^2} \quad (1)$$

with the discrete variation of the linear viscous amortization coefficient c (amortization steps c_1, c_2,...,c_n) with m and k_0 are constant.

$$k_{dyn}^{v} = k_{dyn}^{v}(\omega, m) = \sqrt{(k_0 - \boldsymbol{m}\omega^2)^2 + \boldsymbol{c}^2\omega^2} \quad (2)$$

with the discrete variation of mass m, either continuous, (either in inertial loading steps m_1, m_2, ...m_n) when k_0, c and ω are constant.

Minimum dynamic rigidity $k_m^v = k_{dyn,min}^v$ is obtained from the condition $\frac{dk_{dyn}^v}{dt} = 0$ for $\omega = \omega_m^v$ with the expression:

$$\omega_{m,v} = \sqrt{\frac{k_0}{m} - \frac{c^2}{2m^2}} \quad (3)$$

with the condition

$$\frac{k_0}{m} - \frac{c^2}{2m^2} \geq 0 \quad (4)$$

from where it emerges the domain of possible variation for c, as:

$$c \in \left[0, \sqrt{2k_0 m}\right] \quad (5)$$

In this case, the minimum of function (1) for $\omega = \omega_m^v$ este k_m^v, given by the relation:

$$k_m^v = c\sqrt{\frac{k_0}{m} - \frac{c^2}{4m^2}} \quad (6)$$

with the condition

$$\frac{k_0}{m} - \frac{c^2}{4m^2} \geq 0 \quad (7)$$

from where it emerges the domain of possible variation for c, as follows:

$$c \in \left[0, 2\sqrt{k_0 m}\right] \quad (8)$$

For the simultaneous existence of functions (3) and (6) it is necessary that from the intersection of the sets from relations (5) and (8), the possible values of c satisfy the following requirement:

$$c \in \left[0, \sqrt{2k_0 m}\right] \quad (9)$$

The minimum of function $k_{dyn}^{v}(\omega)$ is defined by point $V(\omega_{m,v};\ k_m^v)$ with the coordinates $\omega_{m,v}$ and k_m^v given by relations (3), respectively (6).

The equation of the curve of the points of minimum V_i, i = 1, 2, 3,..., s is the geometric place of all points V_i and it emerges by eliminating amortization c between equations (3) and (6). Thus, we have:

$$k_m^v = \sqrt{2k_0 m\left(\omega_n^2 - \omega_{m,v}^2\right) - m^2\left(\omega_n^2 - \omega_{m,v}^2\right)^2} \quad (10)$$

2.1. Effect of viscous amortization

The curves of parameters variation according to the increasing and continuous variation of ω, for discrete values of amortization c, are represented by analytical functions defined by the previous calculation relations. In this case the mass m and rigidity k_0 are constant.

a) **The curve of variation of pulse $\omega_{m,v}$** with respect to the variation of the amortization is represented on the basis of relation (3) in figure 1.

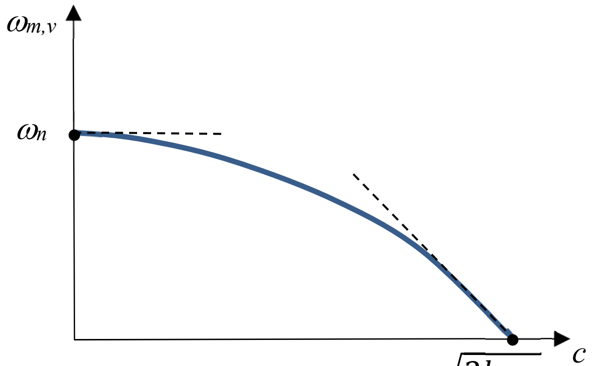

Figure 1. Variation of the pulse of the minimum point V in relation with amortization c

b) **The curves of variation of the dynamic resistance k_{dyn}^{v}**

They are presented in the form of the parametrized family rpin by amortization c with discrete variation and continuous variation of the pulse, in accordance with relation (1). The family of curves is represented in figure 2.

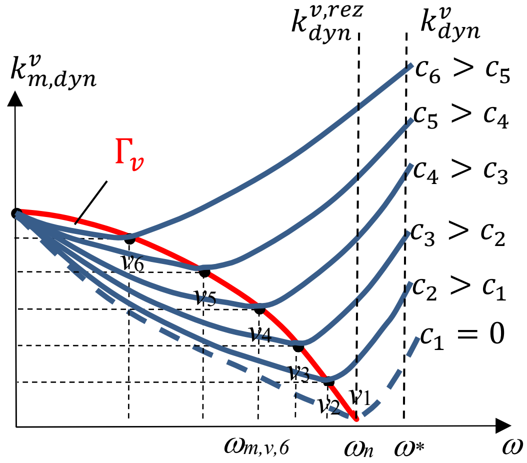

Figure 2. The family of curves for dynamic rigidity to the continuous variation of ω and the discrete variation of c

• V_i, i = 1, 2, 3,..., 6 minimum points for the curves parameterized by amortization $c_1, c_2, ..., c_6$

•• $k_{dyn}^{v,rez}$ is the dynamic rigidity at resonance for $\omega = \omega_n$

••• Γ_v is the geometric place of the minimum points for $\omega_{m,v,i}$ with i = 1, 2,..., 6.

The ante-resonance regime $\omega < \omega_n$ is materialized by placing the minimum points V_i, i = 1, 2, 3,..., s on curve Γ_v whose equation is given by relation (10), with the following particularities:

- for $\omega_{m,v} = \omega \equiv 0$ emerges $k_m^v = k_0$
- for $\omega_{m,v} = \omega_n \equiv 0$ emerges $k_m^v = 0$ at resonance the minimum point is $V_1(\omega_n, 0)$
- for $\omega_{m,v} \neq 0$ and $\omega_{m,v} \neq \omega_n$ curve Γ_v has equation (10) and presents itself as in figure 3.

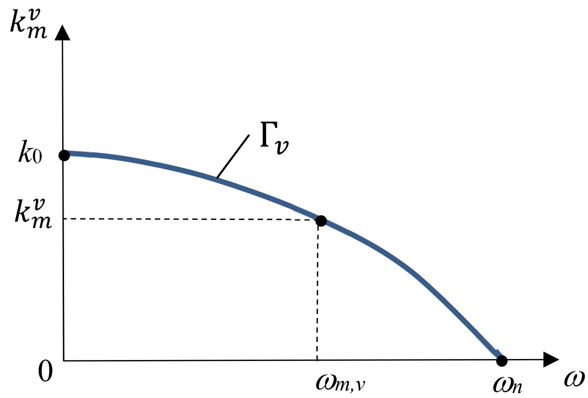

Figure 3. Variation of the minimum dynamic rigidity k_m^v according to the variation of $\omega_{m,v} = \omega$ emerged from curve Γ_v

The resonance regime $\omega = \omega_n$ corresponds to the case where $k_0 - m\omega^2 = 0$, for $\omega = \omega_n = \sqrt{\frac{k_0}{m}}$. It is represented by the vertical right $k_{dyn}^{v,rez} = c\omega_n$ for $\omega = \omega_n$ wit discrete values according to the discrete variation of amortization c ($c_1, c_2, ..., c_6$), as in figure 2.

It is found that at resonance the dynamic rigidity is given only by the viscous rigidity $c\omega_n$.

The post-resonance regime corresponds to the case for which e $\omega > \omega_n$ and for a given value of $\omega = \omega^*$ the dynamic rigidity k_{dyn}^v can be determined on the vertical right $\omega = \omega^*$ with values $k_{dyn,i}^v$ where i = 1, 2,..., s, according to the discrete variation of c_i , i = 1, 2,..., s as in figure 2. It is found that as the excitatory pulsation becomes larger, obviously the dynamic rigidity is significantly increasing.

2.2. The effect of mass

For certain categories of structural systems mass m may modify continuously or discreetly depending on the specific technology. The continuous variation of the mass takes place in the materials processing technologies with the help of the vibrating equipment, such as: sorting, grinding, compacting, mixing, dosing.

The discontinuous or discrete variation of the mass is significant in the technologies where at the initial mass, after certain periods of time, the technological masses in certain quantities are supplemented or decreased.

The variation of mass m occurs only for technical conditions where rigidity k_0, amortization c and the excitation pulse $\omega = \omega^*$ are constant, at significant working regimes.

Based on relation (2), the minimum condition of dynamic rigidity k_{dyn}^v is given by $\frac{dk_{dyn}^v}{dm} = 0$, which leads to $m_{min} = \frac{k}{\omega^2}$, and $k_{dyn,min}^v = k_{dyn}^m$, as follows:

$$k_{dyn}^m = c\omega \qquad (11)$$

That it is obtained precisely **the viscous rigidity** dependent only on c for constant $\omega = \omega^*$.

Figure 4 shows the variation of the dynamic rigidity k_{dyn}^v only according to the variation of mass m, when k_0, c and $\omega = \omega^*$ are constant.

It is found that for $m = 0$, in the absence of mass, the inertial effect is non-existent, the rigidity is $k_1 = \sqrt{k_0^2 + c_1^2\omega^{*2}}$, respectively $k_2 = \sqrt{k_0^2 + c_2^2\omega^{*2}}$. Also, in the presence of mass $m > 0$, with significant inertial effect, the minimum rigidity is $k_{dyn}^m = c\omega^*$ for $m_{min} = \frac{k}{\omega^{*2}}$, after which the increase of mass $m > m_{min}$ highlights a marked increase in the dynamic rigidity.

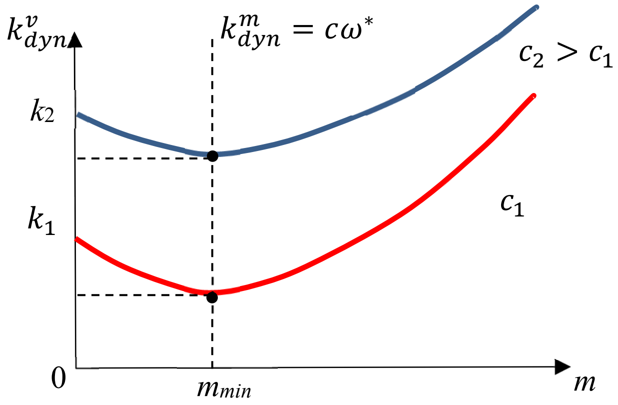

Figure 4. Family of curves of variation k_{dyn}^v according to mass m and parameter c

3. ANALYSIS OF THE VISCOUS AMORTIZATION EFFECT

The quantification of the effect of the discrete variation of the viscous amortization c_i, i =1,..., 4, of the continuous variation of the excitation pulse ω, of maintaining constant mass m and rigidity k_0, is numerically conducted based on relations (1) and (10). Thus, the initial data are: m = 10^4 kg; k_0 =36 $\cdot 10^6$N/m; c = (3; 4; 5; 6) $\cdot$ 10^5Ns/m and ω = (0...500) rad/s. Thus, figure 5 presents the family of curves $k^v = k_{dyn}^v(\omega, c)$, and figure 6 shows curve Γ_v of minimum rigidity k_m^v according to ω, with the specification that this curve is the geometric place of the minimum points in figure 5.

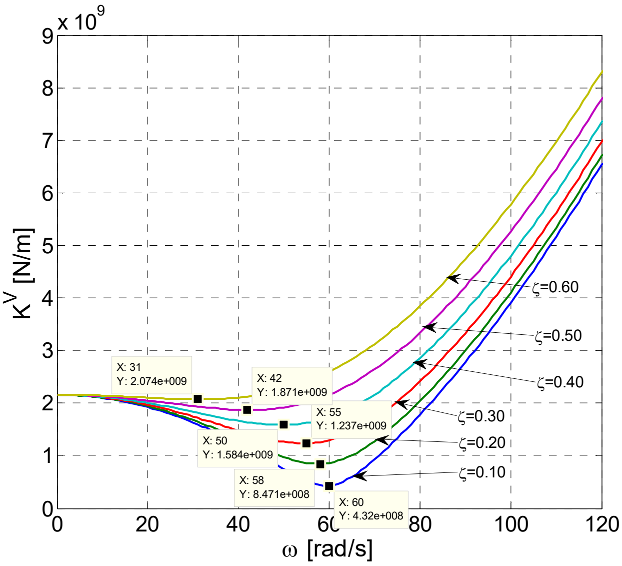

Figure 5. Dynamic rigidity variation $k^v = k_{dyn}^v$. Continuous variation of excitation pulse ω and at discrete variation of viscous amortization c

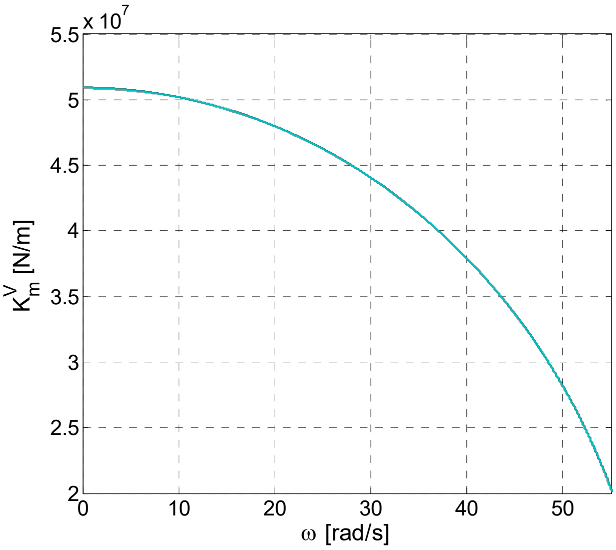

Figure 6. Curve Γ_v of the minimum points (k_m^v, ω_m)

It is found that in the vicinity of the resonance for $\omega = \omega_n = 60$ rad/s the effect of amortization is significant with variations of the dynamic rigidity from $1.8 \cdot 10^7$ N/m to $2.8 \cdot 10^7$ N/m. Also, in the post-resonance for $\omega > 80$ rad/s, the viscous amortization becomes insignificant by the fact that the curve bundle gets very close until overlapping.

4. ANALYSIS OF MASS EFFECT

The analysed case is characterized by the following parametric measures: $\omega = \omega^* = 300$ rad/s, c = (3; 4; 5; 6) · 10^5 Ns/m, k_0 =36 ·10^6 N/m, and mass is continuously variable, that is m = (0. 5000) kg. Based on relations (2), (3), and (6) there were numerically determined the values of $k_{dyn}^v = k^v(m), \omega_m(m) = \omega_{min}$ and $k_{min}^v(m)$. Thus figures 7, 8 and 9 show the corresponding families of curves.

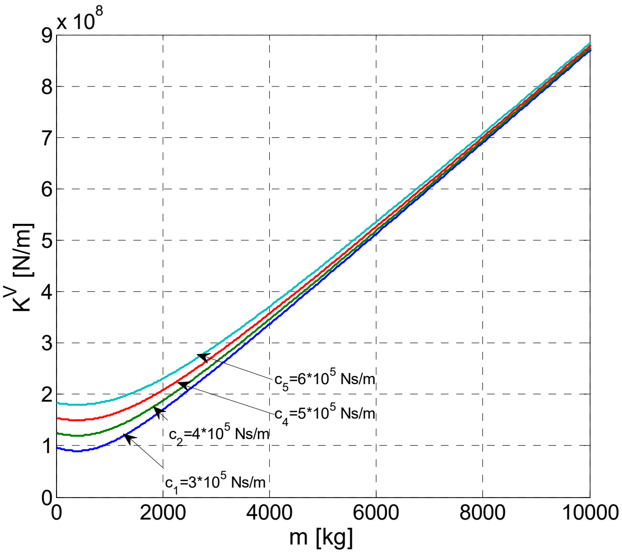

Figure 7. Variation of dynamic rigidity according to variation of mass and amortization

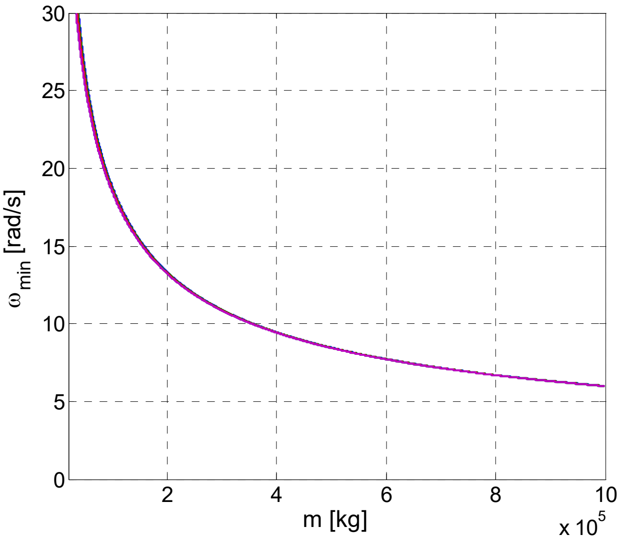

Figure 8. Variation of pulse of minimum point of dynamic rigidity according to mass

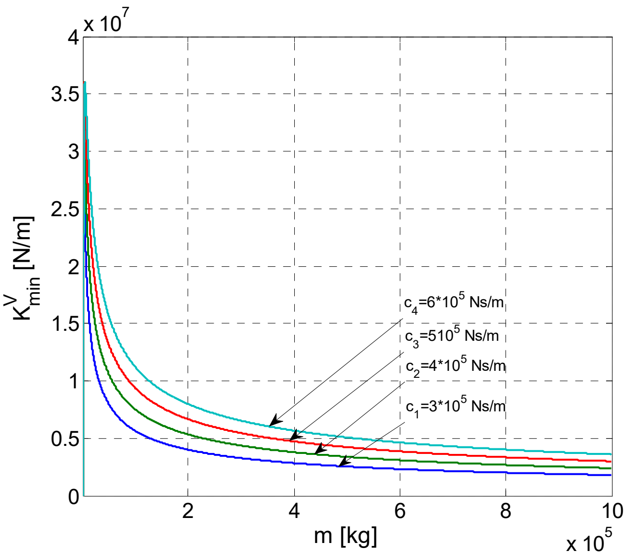

Figure 9. Variation of minimum rigidity according to mass and amortization

5. CONCLUSIONS

The analysis of the dynamic rigidity for a Voigt - Kelvin modelled structural system emphasizes the presence of the mass m, with inertial effect, as well as the dynamic excitation regime by pulse ω. As a result of the analytical study, numbered, as well as the use of experimental results, the initial data for the structural parameters were established, based on which the representative diagrams of figures 5, 6, 7, 8 and 9 were drawn.

In this context, the following conclusions can be systematized:

a) the dynamic rigidity, for a given mass and static rigidity system, modifies according to the viscous amortization and the excitation pulse. Thus, for excitation pulses in ante-resonance, it is found that the dynamic rigidity is decreasing to a minimum point, after which it increases in post-resonance at significantly high values. In this case, it emerges that at high excitation pulses (frequencies), in post-resonance regime, the dynamic rigidity has high values, which causes the displacements or the deformations to decrease significantly (fig. 5).

b) For rigidity k_0 given also pulse $\omega = \omega^*$ defined by the dynamic process, the variation of mass m leads to the rapid increase of the dynamic rigidity for values $m > m_{min}$, to the significant decrease of the pulse of the minimum points $\omega_m = \omega_{min}$ and to the slow decrease of the minimum dynamic rigidity k_{min}^v as it emerges from figures 7, 8 and 9.

Given the above-mentioned, it emerges for each Voigt – Kelvin modelled structural system that it is necessary to analyse the dynamic rigidity in order to optimize the domain in harmonic excitation and viscous amortization regime.

REFERENCES

[1] Bratu, P., Stuparu A., Leopa A., Popa S., *The dynamic analyses of a construction with the base insulation consisting in anti-seismic devices modeled as a Hooke-Voigt-Kelvin linear rheological system*, Acta Tehnica Napocensis, series: Applied Mathematics, and Engineering Vol 60, Issue IV, pp. 465-472, November (2018)

[2] Bratu, P., Stuparu A., Popa S., Iacob N., Voicu O., Iacob N., Spanu G., *The dynamic isolation performances analysis of the vibrating equipment with elastic links to a fixed base*, Acta Tehnica Napocensis, series: Applied Mathematics, and Engineering Vol 61, Issue I, March, pp. 23-28, 2018;

[3] Dobrescu, C. F., *Analysis of the dynamic regime of forced vibrations in the dynamic compacting process with vibrating roller*, Acta Technica Napocensis - Applied Mathematics, Mechanics and Engineering, Vol 62, No 1 (2019)

[4] Bratu, P., Dobrescu, C. F,, *Evaluation of the Dissipated Energy in Vicinity of the Resonance, depending on the Nature of Dynamic Excitation*, Romanian Journal of Acoustics and Vibration pp. 66-71, Vol 16, No 1 (2019):

[5] Dobrescu C. F. *Highlighting the Change of the Dynamic Response to Discrete Variation of Soil Stiffness in the Process of Dynamic Compaction with Roller Compactors Based on Linear Rheological Modeling*, in: Herisanu N., Marinca V. (Editors), Applied Mechanics and Materials, vol. 801: Acoustics & Vibration of Mechanical Structures II, 350 page., (ISBN 978-3-03835-628-8), page. 242-248, doi: 10.4028/www.scientific.net/AMM.801.242, (2015)

[6] Dobrescu C.F., Brăguţă E., *Optimization of Vibro-Compaction Technological Process Considering Rheological Properties*, Proceedings of the 14th AVMS Conference, Timisoara, Romania, May 25–26, 2017, Springer Proceedings in Physics 198, Nicolae Herisanu Vasile Marinca Editors, pp 287-293, ISSN 0930-8989 ISSN 1867-4941 (electronic) Springer Proceedings in Physics ISBN 978-3-319-69822-9 ISBN 978-3-319-69823-6 (eBook) https://doi.org/10.1007/978-3-319-69823-6.

[7] Mitu, A.M., Sireteanu, T., Ghita, G., *Passive and Semi-Active Bracing Systems for Seismic Protection: A Comparative Study*, *Romanian Journal of Acoustics and Vibration,* Volume: 12 Issue: 1 Pages: 49-56, 2015

[8] Potirniche, A., *Assessments Regarding Effective Configurations of Vibration Isolators Based on Displacements Analysis*, Romanian Journal of Acoustics and Vibration, Volume: 12 Issue: 2 Pages: 126-131, 2015

[9] Sireteanu, T., *Smart Suspension Systems*, Romanian Journal of Acoustics and Vibration, Volume: 13 Issue: 1.Pages: 2-2, 2016

[10] Stanescu, N.D., *Vibrations of a Shell with Clearances, neo-Hookean Stiffness, and Harmonic Excitations*, *Romanian Journal of Acoustics and Vibration,* Volume: 13 Issue: 2, Pages: 104 -111, 2016

[11] Vasile, O., *Active Vibration Control for Viscoelastic Damping Systems under the Action of Inertial Forces*, *Romanian Journal of Acoustics and Vibration,* Volume: 14 Issue 1, Pages 54-58, 2017

[12] Xi-Yuan Zhou, Miao Han, Lin Yang, *Study on protection measures for seismic isolation rubber bearings*, ISET Jurnal of Earthquake Technology, Paper no.436, vol.40, no.2-4, June-December 2003, pp 137-160