Stability of Triangular Equilibrium Points in the Photogravitational R3BP When both Primaries are Oblate Spheroid and Effect of Radiation

Avdhesh KUMAR
Department of Mathematics, Jaglal Chaudhary College, Chapra, (A Constituent unit of J. P. University, Chapra) - INDIA, avdheshsahani@yahoo.com

Ashish Kumar SHARMA
Department of Mathematics, IEC, University Baddi, H.P-INDIA, ashishk482@gmail.com

Abstract: - In this study, we have examined the stability of triangular equilibrium points in the photogravitational restricted three- body problems (R3BP) as well as the effect of radiation, where both primaries are oblate spheroid. The results confirm the position of triangular equilibrium points of our problem and it also depicts that the equations of motion are affected by radiation pressure force, oblateness and sources of radiation. Stability conditions were discussed using the characteristic equation. All classical results involving photogravitational and oblateness in R3BP may be verified in the near future employing this result.

Keywords: - Stability, Equilibrium Points, Photogravitational R3BP, Oblate Spheroid

1. INTRODUCTION

The simplest form of the three-body problem is called the restricted three-body problem (R3BP), in which a particle of infinitesimal mass moves in the gravitational field of two massive bodies (known as the primaries, or primary and secondary).These massive bodies has been revolved in accordance with the exact solution of the two-body problem. The particle with infinitesimal mass does not perturb the motions of the two massive bodies. There are enormous reports available citing to this problem including both analytic and numerical developments. The analytic work was dedicated typically to the circular and planar R3BP, where all particles are confined to a plane as well as the two finite masses. The finite masses are in circular orbits around their centre of mass. On the other hand, numerical developments allowed consideration of the more general problem. The R3BP were comprised of simulations and used in order to study some physical systems alongwith their approximation. We have examined the motion of a body having low mass under the influence of two different bodies having extremely high masses. In this type of scaffold, the mass of one of the bodies is negligible compared to the masses of the other two i.e., motion of the two massive bodies is Keplerian

When the orbits of the primaries are circular R3BP the first integral (Jacobi constant) happens and when a positive eccentricity has been added, the orbits turn to elliptic. In other words, we do not have circular R3BP anymore. However, the simulator used in this project computes the orbit of the third body where primaries move in an elliptical orbit with an eccentricity. It can also be circulated R3BP.

The R3BP possesses five equilibrium points, three collinear and two triangular. In the linear sense, the collinear points L_1, L_2, L_3 are unstable for any value of the mass ratio. Triangular points L_4, L_5 are stable if the mass ratio μ of the finite bodies is less than μ_0=0.03852… (Szebehely, 1967). In the case of R3BP, both the primaries are oblate spheroids whose equatorial plane coincide with the plane of motion. The location of libration points and their stability in the Liapunov sense has been studied by Vidyakin (1974). In addition, Subba Rao and Sharma (1975) have deliberated the stability of the libration points, assuming a condition where the bigger primary is an oblate spheroid. Furthermore, Khanna, Bhatnagar and Hallan (1978) calculated the effect of perturbation in the centrifugal and Coriolis forces. Bhatnagar and Hallan (1979) examined the effect of perturbed potentials on the linear stability of libration points in the restricted three-body problem. Bhatnagar and Gupta (1986) also studied the existence and stability of the equilibrium points of a triaxial rigid body moving around another triaxial rigid body. In recent years, Khanna and Bhatnagar (1998) studied the linear stability of L4 in the restricted three-body problem when the smaller primary is a triaxial rigid body.

In this study, we have examined the stability of triangular equilibrium points in the photogravitational

R3BP when both primaries are oblate spheroid, considering one of its axes as the axis of symmetry and its equatorial plane as the plane of motion. In addition, we assumed that the primaries are moving without rotation in circular orbits around their centre of mass.

2. LOCATION OF TRIANGULAR EQUILIBRIUM

The equations of motion in the vector form become

$$\frac{\partial^2 \vec{r}}{\partial t^2} + 2\vec{w} \times \left(\frac{\partial \vec{r}}{\partial t}\right) + \vec{w} \times (\vec{w} \times \vec{r}) = \left(\frac{-\partial V_1}{\partial r_1}\right)\hat{r}_1 + \left(\frac{-\partial V_2}{\partial r_2}\right)\hat{r}_2 \qquad \text{(McCusky, page165)}$$

Where, $\vec{w} = nk$

Then the equation of motion becomes

$$\left(\ddot{\gamma}\hat{i} + \ddot{\kappa}\hat{j}\right) - 2n(\dot{\kappa}\hat{i} - \dot{\gamma}\hat{j}) - n^2(\gamma\hat{i} + \kappa\hat{j}) = -(V_{1\gamma} + V_{2\gamma})\hat{i} - (V_{1\kappa} + V_{2\kappa})\hat{j}$$

Comparing the coefficient of $\hat{i}$ and $\hat{j}$ on both sides, we get

$$\ddot{\gamma} - 2n\dot{\kappa} - n^2\gamma = -(V_{1\gamma} + V_{2\gamma})$$

$$\ddot{\kappa} + 2n\dot{\gamma} - n^2\kappa = -(V_{1\kappa} + V_{2\kappa})$$

Now the of motion in Cartesian form can be written as

$$\ddot{\gamma} - 2n\dot{\kappa} = \Phi_\gamma \qquad (1)$$

$$\ddot{\kappa} + 2n\dot{\gamma} = \Phi_\kappa \qquad (2)$$

Where,

$$\Phi_\gamma = n^2\gamma - \sum_{i=1}^{2}\left[\frac{1}{r_i^3}\{H_i\,\mu_i(\gamma - \gamma_i)\} - \frac{1}{2r_i^5}\{3H_i\,\mu_i (2\rho_{1i} - \rho_{2i})(\gamma - \gamma_i)\} - \frac{1}{2r_i^5}\{3H_i\,(\gamma - \gamma_i)\}A_i + \frac{1}{2r_i^7}\{15H_i\,(\rho_{1i} - \rho_{2i})(\gamma - \gamma_i)\kappa^2\}\right]$$

$$\Phi_\kappa = n^2\kappa + \sum_{i=1}^{2}\left[\frac{1}{r_i^3}\{H_i\,\kappa\} - \frac{1}{2r_i^5}\{3H_i\,(4\rho_{1i} - 3\rho_{2i})\kappa\} + \frac{1}{2r_i^7}\{15\,H_i\,(\rho_{1i} - \rho_{2i})\kappa^3\} - \frac{1}{2r_i^5}\{3\,H_i\,\kappa\}A_i\right]$$

Where,

$H_1 = (1-p)(1-\mu)$, $H_2 = \mu$, 'p' is source of radiation, $A_1 = \frac{r_e^2 - r_p^2}{5r^2}, A_2 = \frac{r_e'^2 - r_p'^2}{5r^2}$ is the oblateness coefficient r_e, r_p and r_e', r_p' is the equatorial and polar radii respectively r is the distance between primaries.

$\rho_{1i} = \zeta_{1i} - \zeta_{3i}, \rho_{2i} = \zeta_{2i} - \zeta_{3i}, \zeta_{1i} = \frac{x_i^2}{5R^2}, \zeta_{2i} = \frac{y_i^2}{5R^2}$,

$\zeta_{3i} = \frac{z_i^2}{5R^2}, x_i, y_i, z_i\ (i = 1,2)$ as the length of its semi-axis, R is the distance between the primaries and the mean motion

$$n^2 = 1 + \sum_{i=1}^{2}[\frac{3}{2}(2\rho_{1i} - \rho_{2i}) + \frac{3}{2}A_i] \qquad (3)$$

$$r_1^2 = (\gamma - \mu)^2 + \kappa^2, \qquad r_2^2 = (\gamma - \mu + 1)^2 + \kappa^2$$

$$\gamma_1 = \mu, \quad \gamma_2 = \mu - 1,\ \mu_1 = 1 - \mu,\ \mu_2 = \mu$$

$\mu = \frac{m_2}{m_1 + m_2} \le \frac{1}{2}$ with $m_1 \ge m_2$ being the masses of the primaries

The equation permits the Jacobi Integral.

Multiplying equations (1) & (2) by $\dot{\gamma}$ and $\dot{\kappa}$ respectively, then after adding two yields

$\dot{\gamma}\ddot{\gamma} + \dot{\kappa}\ddot{\kappa} = \dot{\gamma}\Phi_\gamma + \dot{\kappa}\Phi_\kappa = d\Phi$, by integrating yields

$\dot{\gamma}^2 + \dot{\kappa}^2 - 2\Phi + C = 0$, C is the constant of integration

The equilibrium points are the singularities of the manifold

$f(\gamma, \kappa, \dot{\gamma}, \dot{\kappa}) = \dot{\gamma}^2 + \dot{\kappa}^2 - 2\Phi + C = 0$ given by

$f_\gamma = f_\kappa = f_{\dot{\gamma}} = f_{\dot{\kappa}} = 0$, and $\Phi_\gamma = 0 = \Phi_\kappa$

Two cases arise:

(i) Triangular equilibrium points

(ii) Collinear equilibrium points

Case (i)

Triangular equilibrium points are given by $\Phi_\gamma = 0 = \Phi_\kappa$, $\kappa \ne 0$, then we have

$$\gamma = \mu - \frac{1}{2} + \frac{3p}{8} + \frac{1}{8}\mu^{-1}(4 - \mu)\rho_{11} - \frac{1}{8}\mu^{-1}(4 + 3\mu)\rho_{21} - \frac{1}{8}(1-\mu)^{-1}(3 + \mu)\rho_{12} + \frac{1}{8}(1-\mu)^{-1}(7 - 3\mu)\rho_{22} - \frac{1}{2}(A_1 - A_2) \qquad (4)$$

$$\kappa = \pm\frac{\sqrt{3}}{2}\left[1 + \frac{2}{3}\left\{-\frac{p}{3} + \frac{1}{8}\mu^{-1}(4 - 23\mu)\rho_{11} + \frac{1}{8}\mu^{-1}(-4 + 19\mu)\rho_{21} + \frac{1}{8}(1-\mu)^{-1}(-19 + 23\mu)\rho_{12} + \frac{1}{8}(1-\mu)^{-1}(15 - 19\mu)\rho_{22} - \frac{1}{2}A_1\right\}\right] \qquad (5)$$

Case (ii)

Collinear equilibrium points are the solution of the equations ($\kappa = 0$)

$$\vartheta(\gamma)=n^2\gamma-\sum_{i=1}^{2}\left[\frac{1}{r_i^{\,3}}\{H_i\mu_i(\gamma-\gamma_i)\}-\frac{1}{2r_i^{\,5}}\{3H_i\mu_i\right.$$

$$(2\rho_{1i}-\rho_{2i})(\gamma-\gamma_i)\}+\frac{1}{2r_i^{\,7}}\{15H_i(2\rho_{1i}-\rho_{2i})\kappa^2\}$$

$$\left.-\frac{3H_i(\gamma-\mu)}{2r_i^{\,5}}A_i\right] \qquad (6)$$

Where, $r_i=\left|\gamma-\gamma_i\right|$, (i= 1, 2)

Obviously, these equilibrium points lie on the x-axis and their abscissa are given by the roots of equation (6), since $\vartheta(\gamma)>0$ in each of the open interval (-∞, μ-1), (μ-1,μ) and (μ,∞), the function ϑ is strictly increasing in each of them.

Also $\vartheta(\gamma)\to$ -∞ as x → - ∞, (μ-1) +0 or μ+0, $\vartheta(\gamma)\to$ +∞ As γ → +∞, (μ-1)-0 or μ-0. There exist, one and only one value of γ in each of the above interval such that $\vartheta(\gamma)=0$ Further $\vartheta(\mu-2)<0$, $\vartheta(0)\geq 0$ and $\vartheta(\mu+1)>0$ .Therefore, there are only three real roots of the equation (6) one lying in each of the interval (μ-2,μ-1), (μ-1, 0) and (μ, μ+1).

Thus there are three triangular equilibrium points.

3. STABILITY OF TRIANGULAR EQUILIBRIUM POINTS

Suppose $\psi=X\,e^{\lambda t}$, $\phi=Y\,e^{\lambda t}$ is the small displacement of Lagrangian points $\left(\gamma_0,\kappa_0\right)$, X , Y, λ are parameters, $\gamma=\gamma_0+\psi$, $\kappa=\kappa_0+\phi$ and therefore the equations of perturbed motion corresponding to the system of equation (1), (2) may be written as

$$\ddot{\psi}-2n\dot{\phi}=\Phi_{\gamma}^{0}+\psi\,\Phi_{\gamma\gamma}^{0}+\phi\,\Phi_{\gamma\kappa}^{0} \qquad (7)$$

$$\ddot{\phi}+2n\dot{\psi}=\Phi_{\kappa}^{0}+\psi\,\Phi_{\kappa\gamma}^{0}+\phi\,\Phi_{\kappa\kappa}^{0} \qquad (8)$$

Above system of equation can be written as,

$$X\left(\lambda^2-\Phi_{\gamma\gamma}^{0}\right)+\left(-2n\lambda-\Phi_{\gamma\kappa}^{0}\right)Y=0,$$

$$X\left(2n\lambda-\Phi_{\kappa\gamma}^{0}\right)+\left(\lambda^2-\Phi_{\kappa\kappa}^{0}\right)Y=0$$

These will have a non-trivial solution if

$$\det\begin{pmatrix}\lambda^2-\Phi_{\gamma\gamma}^{0} & -2n\lambda-\Phi_{\gamma\kappa}^{0}\\ 2n\lambda-\Phi_{\kappa\lambda}^{0} & \lambda^2-\Phi_{\kappa\kappa}^{0}\end{pmatrix}=0$$

$$\Rightarrow \lambda^4-\left(\Phi_{\gamma\gamma}^{0}+\Phi_{\kappa\kappa}^{0}-4n^2\right)\lambda^2+\Phi_{\gamma\gamma}^{0}\Phi_{\kappa\kappa}^{0}$$

$$-\left(\Phi_{\kappa\kappa}^{0}\right)^2=0 \qquad (9)$$

(a) $0\leq\mu\angle\mu_{crit}$

Replacing λ^2 by Λ in the equation (9)

$$\Lambda^2+X\Lambda+Y=0 \qquad (10)$$

Where

$$X=1+3\rho_{11}+\frac{3}{2}\left(-3+2\mu\right)\rho_{21}+3\rho_{12}-\frac{3}{2}\left(1+2\mu\right)$$

$$\rho_{22}-\frac{3}{4}(3-4\mu)A_1-\frac{9}{8}(11-\mu)A_2>0$$

$$Y=\frac{27}{4}\mu\left(1-\mu\right)+\frac{3}{2}\mu\left(1-\mu\right)p+\frac{9}{16}(-10+99\mu$$

$$-89\mu^2)\rho_{11}+\frac{9}{16}\left(10-47\mu+37\mu^2\right)\rho_{21}+\frac{9}{16}(-284$$

$$+1056\mu-864\mu^2)\rho_{12}+\frac{9}{16}\mu\left(-27+37\mu\right)\rho_{22}$$

$$+\frac{1}{32}(-153+10638\mu-14580\mu^2)A_1+\frac{1}{32}(-1057$$

$$+2421\mu-324\mu^2)A_2 \qquad (11)$$

$$\wedge_{1,2}=\frac{1}{2}(-X\pm\sqrt{X^2-4Y})$$

Consequently, the roots $\lambda_1=+\sqrt{\wedge_1}$, $\lambda_2=-\sqrt{\wedge_1}$ $\lambda_3=+\sqrt{\wedge_2}$, $\lambda_4=-\sqrt{\wedge_2}$ depend on a simple manner, on the value of the mass parameter $\mu,\rho_{1i}\,,\rho_{2i}\;A_i\;\left(i=1,2\right)$. Now the discriminant of the equation (10) is zero if $X^2-4Y=0$.

$$1-27\mu\left(1-\mu\right)-6\mu\left(1-\mu\right)p-\frac{3}{4}(-38+297\mu$$

$$-267\mu^2)\rho_{11}-\frac{3}{4}(42-149\mu+111\mu^2)\rho_{21}$$

$$+\frac{3}{4}(8-237\mu+267\mu^2)\rho_{12}+\frac{3}{4}(-4+73\mu$$

$$-111\mu^2)\rho_{22}+\frac{1}{8}(121-10590\mu+14580\mu^2)\,A_1$$

$$+\frac{1}{8}(-931-21771\mu-288\mu^2)A_2=0 \qquad (12)$$

If $A_i,p,\rho_{1i},\rho_{2i}\;\left(i=1,2\right)$ are equal to zero, then $\mu=\mu_0$ is a roots of the equation (12) where $\mu_0=0.0385208965...$(Szebehely, 1967). When $A_i,p,\rho_{1i},\rho_{2i}\;\;\left(i=1,2\right)$ are not equal to zero, we suppose, $\mu=\mu_{crit}=\mu_0+\gamma_1\rho_{11}+\gamma_2\rho_{21}+\gamma_3\rho_{12}+\gamma_5A_1+\gamma_6A_2+\gamma_7\,p+\gamma_4\rho_{22}$ as the roots of the equation (12), where $\gamma_1,\gamma_2,\gamma_3,\gamma_4,\gamma_5,\gamma_6,\gamma_7$ are to be determined in such a manner that $X^2-4Y=0$

$$1-27(\mu_0+\gamma_1\sigma_{11}+\gamma_2\sigma_{21}+\gamma_3\sigma_{12}+\gamma_4\sigma_{22}+\gamma_5A_1$$

$$+\gamma_6A_2+\gamma_7\,p\;)\;(1-\mu_0-\gamma_1\sigma_{11}-\gamma_2\sigma_{21}-\gamma_2\sigma_{21}-$$

$$\gamma_4\sigma_{22}-\gamma_5\,A_1-\gamma_6A_2-\gamma_7\,p)+P_1\sigma_{11}+P_2\sigma_{21}+$$

$$P'_1\sigma_{12}+P'_2\sigma_{22}+P_3A_1+P_4A_2+P_5p=0 \qquad (13)$$

Hence, $\mu=\mu_{crit}=\mu_0+\gamma_1\rho_{11}+\gamma_2\rho_{21}+\gamma_3\rho_{12}+\gamma_4\rho_{22}+\gamma_5A_1+\gamma_6A_2+\gamma_7\,p$

$$\mu = 0.038520895 .... + 0.81126474\rho_{11} - 1.09626653\rho_{21}$$
$$- 0.02206859\rho_{12} - 0.04071097\rho_{22} - 0.1330773375 A_1$$
$$- 0.0465436517 A_2 - 0.0089174706\, p \qquad (14)$$

But $X > 0$, therefore $\wedge_1$ and $\wedge_2$ are negative.

Therefore, in this case, the four roots of the characteristic equation are written as

$$\lambda_{1,2} = \pm i\sqrt{(-\wedge_1)} = \pm i s_1$$

and

$$\lambda_{3,4} = \pm i\sqrt{(-\wedge_2)} = \pm i s_2 \qquad (15)$$

This shows that the equilibrium point is stable.

The solution of the equation (7) & (8) is given by

$$\psi = \sum_{i=1}^{2} \chi_i \cos s_i t + S_i \sin s_i t$$

$$\phi = \sum_{i=1}^{2} \overline{\chi}_i \cos s_i t + \overline{S}_i \sin s_i t$$

$$\ddot{\psi} = -\sum_{i=1}^{2} (\chi_i s_i^2 \cos s_i t + S_i s_i^2 \sin s_i t)$$

$$\ddot{\phi} = -\sum_{i=1}^{2} (\overline{\chi}_i s_i^2 \cos s_i t + \overline{S}_i s_i^2 \sin s_i t)$$

Where,

$$\overline{\chi}_i = \tau_i (2nS_i s_i - \Phi^0{}_{\gamma\kappa} \chi_i)$$
$$\overline{S}_i = -\tau_i (2n\chi_i s_i + \Phi^0{}_{\gamma\kappa} S_i)$$

(Szebehely, 1967, pp.250)

$$\tau_i = \frac{1}{\Phi^0{}_{\kappa\kappa} + S_i^2} \rangle\, 0, \qquad (i = 1,2)$$

Now, we introduce the variable $\overline{\psi}$, $\overline{\phi}$ by the transformation

$$\psi = \overline{\psi} \cos\alpha - \overline{\phi} \sin\alpha$$
$$\phi = \overline{\psi} \sin\alpha + \overline{\phi} \cos\alpha$$

This is equivalent to the rotation of the co-ordinate system by α.

We choose α in such a way that the term containing $\overline{\psi}$, $\overline{\phi}$ in $\Phi = 0$

The new quadratic form becomes

$$\Phi = \overline{U}\psi^2 + \overline{V}\phi^2 + \overline{W} \qquad (16)$$

$$\overline{U} = \frac{3}{8} + \frac{1}{4}\{(-1+3\mu)\cos 2\alpha + \frac{\sqrt{3}}{3}(1+\mu)\sin 2\alpha\}p$$
$$-\frac{3\sqrt{3}}{8}(1-2\mu)\sin 2\alpha + \frac{3}{32}\mu^{-1}[19\mu + 10\mu\sin^2\alpha$$
$$+(-8+15\mu)\cos 2\alpha + \frac{\sqrt{3}}{3}(8-47\mu+89\mu^2)\sin 2\alpha$$
$$]\rho_{11} + \frac{3}{32}\mu^{-1}[-\mu - 16\mu^2\cos^2\alpha - 6\mu\sin^2\alpha - (8$$
$$-15\mu^2)\cos 2\alpha + \frac{\sqrt{3}}{3}(-8+9\mu-37\mu^2)\sin 2\alpha]\rho_{21}$$
$$+\frac{3}{32}(1-\mu)^{-1}[22 - 15\mu^2 + 4(1-2\mu)\cos^2\alpha -$$
$$\mu\cos 2\alpha + \frac{\sqrt{3}}{3}(-50+131\mu-89\mu^2)\sin 2\alpha]\rho_{12}$$
$$+\frac{3}{32}(1-\mu)^{-1}[8(1-\mu)(-3+2\mu)\cos^2\alpha - (23\mu$$
$$-15\mu^2)\mu\cos 2\alpha + \frac{\sqrt{3}}{3}(36-65\mu+37\mu^2)\sin 2\alpha]\rho_{22}$$
$$+\frac{3}{16}(9-8\mu)\cos^2\alpha\, A_1 + \frac{3}{16}(14-3\mu)\cos^2\alpha\, A_2$$
$$-\frac{3\sqrt{3}}{16}(6-8\mu)\sin 2\alpha\, A_1 - \frac{3\sqrt{3}}{16}(21-33\mu)\sin 2\alpha\, A_2$$
$$+\frac{39}{16}\sin^2\alpha\, A_1 + \frac{105}{16}\sin^2\alpha\, A_2$$

$$\overline{V} = \frac{3}{8} + \frac{1}{4}\{(-1+3\mu)\cos 2\alpha + \frac{\sqrt{3}}{3}(1+\mu)$$
$$\sin 2\alpha\}p - \frac{3\sqrt{3}}{8}(1-2\mu)\sin 2\alpha + \frac{3}{32}\mu^{-1}$$
$$[19\mu + 10\mu\cos^2\alpha + (8-15\mu^2)\cos 2\alpha - \frac{\sqrt{3}}{3}$$
$$(8-47\mu+89\mu^2)\sin 2\alpha]\rho_{11} + \frac{3}{32}\mu^{-1}$$
$$[-\mu - 16\mu^2\cos^2\alpha - 6\mu\sin^2\alpha + (-8+15\mu^2)$$
$$\cos 2\alpha - \frac{\sqrt{3}}{3}(-8-9\mu-37\mu^2)\sin 2\alpha]\rho_{21}$$
$$+\frac{3}{32}(1-\mu)^{-1}[22 - 15\mu^2 + 4(1-2\mu)\sin^2\alpha$$
$$+\mu\cos 2\alpha - \frac{\sqrt{3}}{3}(-50+131\mu-89\mu^2)$$
$$\sin 2\alpha]\rho_{12} + \frac{3}{32}(1-\mu)^{-1}[8(1-\mu)(-3+$$
$$2\mu)\sin^2\alpha + (-23+15\mu^2)\cos 2\alpha - \frac{\sqrt{3}}{3}$$
$$(36+65\mu+37\mu^2)\sin 2\alpha]\rho_{22} + \frac{3}{16}(9$$
$$-8\mu)\sin^2\alpha\, A_1 + \frac{3}{16}(14-3\mu)\sin^2\alpha\, A_2 +$$
$$\frac{3\sqrt{3}}{16}(6-8\mu)\sin 2\alpha\, A_1 + \frac{3\sqrt{3}}{16}(21-33\mu)$$
$$\sin 2\alpha\, A_2 + \frac{39}{16}\cos^2\alpha\, A_1 + \frac{105}{16}\cos^2\alpha\, A_2$$

$$\overline{W} = \frac{3}{2} + (-1+\mu)p + \frac{1}{8}(11+\mu)\rho_{11} - \frac{1}{8}$$

$$(1+5\mu)\rho_{21}+\frac{1}{8}(12-\mu)\rho_{12}-\frac{1}{8}(6+\mu)$$

$$\rho_{22}+\frac{1}{8}(5-2\mu)A_1+\frac{1}{8}(3-2\mu)A_2 \ \tan 2\alpha=\frac{Q}{R}$$

$$Q=-\frac{3\sqrt{3}}{2}\{\mu-\frac{1}{2}+\frac{1}{9}(1+\mu)p+\frac{1}{24}\mu^{-1}$$

$$(8-47\mu+89\mu^2)\rho_{11}+\frac{1}{24}\mu^{-1}(-8+9\mu$$

$$-37\mu^2)\rho_{21}+\frac{1}{24}(1-\mu)^{-1}(-50+131\mu$$

$$-89\mu^2)\rho_{12}+\frac{1}{24}(1-\mu)^{-1}(36-65\mu+37$$

$$\mu^2)\rho_{22}-\frac{1}{4}(7-10\mu)A_1-\frac{1}{4}(21-33\mu)A_2\}$$

$$R=\{\frac{3}{4}++\frac{1}{2}(1-3\mu)p+\frac{3}{16}\mu^{-1}(8+5\mu-$$

$$15\mu^2)\rho_{11}+\frac{3}{16}\mu^{-1}(-8-3\mu+23\mu^2)\rho_{21}+$$

$$\frac{3}{16}(1-\mu)^{-1}(-2+25\mu)\rho_{12}+\frac{3}{16}(1-\mu)^{-1}(12$$

$$-43\mu+23\mu^2)\rho_{22}-\frac{3}{16}(15_1-8\mu)A_1+\frac{39}{16}A_1$$

$$-\frac{3}{16}(14-3\mu)A_2+\frac{105}{16}A_2\} \tag{17}$$

Using the Jacobi constant, we have

$$J=2\Phi=2\,\overline{U}\,\psi^2+2\overline{V}\phi^2+2\overline{W} \tag{18}$$

Hence, it follows that the above curve is an ellipse and the direction α of the major axis is given by the equation (3.12).The length of semi-major and semi-minor axis are given by

$$A_{sm}=\sqrt{\left(\frac{J-2\overline{W}}{2\overline{U}}\right)} \ \ and \ \ A_{sm}=\sqrt{\left(\frac{J-2\overline{W}}{2\,\overline{V}}\right)} \tag{19}$$

Where $\overline{U}, \overline{V}, \overline{W}$ are given by the equation (16) and C depends upon the initial conditions.

(b) $\mu_{crit}\angle\mu\angle\frac{1}{2}$

This discriminant of the characteristic equation is negative.

Also $\Lambda_{1,2}=\frac{1}{2}(-X\pm\sqrt{D})$

Where X is given by the equation (11) and $D=X^2-4Y$, $\Lambda_{1,2}=\frac{1}{2}(-X\pm i\delta)$

Where $0\angle\delta=\sqrt{D}$ and is given by

$$\delta=[27\mu(1-\mu)-1+6\mu(1-\mu)p-\frac{3}{4}(38-297\mu$$

$$+267\mu^2)\rho_{11}-\frac{3}{4}(-42+149\mu-111\mu^2)\rho_{21}-\frac{3}{4}$$

$$(8-237\mu-267\mu^2)\rho_{12}-\frac{3}{4}(-4+73\mu-111\mu^2)$$

$$\rho_{22}-\frac{1}{8}(121-10590\mu+14580\mu^2)A_1-\frac{1}{8}(931+$$

$$21771\mu-288\mu^2)A_2]^{\frac{1}{2}} \tag{20}$$

So, the roots of the characteristic equation are $\Lambda_{1,2}=\pm\sqrt{\Lambda_1}$, $\Lambda_{3,4}=\pm\sqrt{\Lambda_2}$

These roots are equal and are given by

$$|\lambda|=|\lambda_{1,2,3,4}|=\sqrt[4]{\frac{(X^2+\delta^2)}{4}} \tag{21}$$

Where X and δ are given by the equation (11) and (17)

Let

$$\alpha_1+i\beta_1=\frac{1}{\sqrt{2}}\sqrt{-X+i\delta}=rcis\frac{\theta}{2}$$

$$=r(\cos\frac{\theta}{2}+i\sin\frac{\theta}{2})$$

Squaring both sides, we get

$$\frac{1}{2}(-X+i\delta)=r^2(\cos\theta+i\sin\theta)$$

Equating real and imaginary parts on both sides

$$\frac{-X}{2}=r^2\cos\theta, \quad \frac{\delta}{2}=r^2\sin\theta$$

$$\frac{-\delta}{X}=\tan\theta=\frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}$$

$$\delta\tan^2\frac{\theta}{2}-2X\tan\frac{\theta}{2}-\delta=0$$

$$\tan\frac{\theta}{2}=\frac{X\pm\sqrt{X^2+\delta^2}}{\delta}$$

$$\frac{\theta}{2}=Arc\tan\left(\frac{X\pm\sqrt{X^2+\delta^2}}{\delta}\right)$$

Therefore, the principal argument of the first root is

$$\frac{\theta}{2}=\frac{\theta_1}{2}=Arc\tan\left(\frac{X\pm\sqrt{X^2+\delta^2}}{\delta}\right)$$

So, we see that the argument is related by

$$\theta=\theta_1=\theta_2-\pi=2\pi-\theta_3=\pi-\theta_4$$

The real and imaginary parts of the roots α_i and β_i (i=1, 2, 3, 4) are related by

$$\alpha=\alpha_1=-\alpha_2=\alpha_3=-\alpha_4$$

$$\beta=\beta_1=-\beta_2=-\beta_3=\beta_4$$

$$\alpha=\frac{1}{2}\sqrt{\frac{\delta^2}{2|\lambda|^2+X}}\rangle 0, \quad \beta=\sqrt{\frac{X+2|\lambda|^2}{4}}\rangle 0$$

Therefore, it follows that the real parts of two of the characteristic roots are positive and equal and so the equilibrium point in this case is unstable.

(c) $\mu = \mu_{crit}$, Consequently, $\Lambda_{1,2} = \frac{-X}{2}$,

$$\lambda_1 = \lambda_3 = i\sqrt{\frac{X}{2}},\ \ \lambda_2 = \lambda_4 = -i\sqrt{\frac{X}{2}}$$

The double roots give secular term in the solution of the equations of motion and so the equilibrium point is unstable.

4. CONCLUSIONS

In this research article, we have demonstrated the stability of triangular equilibrium points in the photogravitational restricted three-body problem (R3BP) and the effect of radiation, where both primaries are oblate spheroid.

(i) The co-ordinates of the triangular equilibrium points are given in equation (4) & (5). We found that the displacement of the new triangular equilibrium points from the classical triangular equilibrium points is small and it depends upon the quantities.

$$\rho_{1i} = \zeta_{1i} - \zeta_{3i},\ \ \rho_{2i} = \zeta_{2i} - \zeta_{3i}$$

$$H_1 = (1-p)(1-\mu),\quad H_2 = \mu,$$

$$\zeta_{1i} = \frac{x_i^2}{5R^2},\ \ \zeta_{2i} = \frac{y_i^2}{5R^2},\ \ \zeta_{3i} = \frac{z_i^2}{5R^2}$$

Where $x_i, y_i, z_i\ (i=1,2)$ as the length of its semi-axis, R is the distance between the primaries.

(ii) The mean motion 'n' of the primaries is given the equation, $n^2 = 1 + \sum_{i=1}^{2}[\frac{3}{2}(2\rho_{1i} - \rho_{2i}) + \frac{3}{2}A_i]$

(iii) When both bodies are spherical in shape $p = \rho_{1i} = \rho_{2i} = A_i = 0$, $(i=1,2)$, $\gamma = \mu - \frac{1}{2}$, $\kappa = \pm\frac{\sqrt{3}}{2}$, the results obtained are in agreement with those of the classical problem.

(iv) The stability of $L_{4,5}$ depends upon value of equation (14) such that **(a)** $0 \le \mu \langle \mu_{crit}$ $L_{4,5}$ is stable. It may be noted that the range of stability decreases when compared to the classical case **(b)** $\mu_{crit} \langle \mu \langle \frac{1}{2}$ $L_{4,5}$ is unstable and **(c)** $\mu = \mu_{crit}$ $L_{4,5}$ is unstable.

(v) We also found that near the triangular equilibrium points there are long or short periodic elliptical orbits for the mass parameter $0 \le \mu \langle \mu_{crit}$, the direction α of the major axis of the ellipse is given by $\tan 2\alpha = \frac{Q}{R}$, where Q and R are presented in equation (17). In addition, we have calculated the lengths of the semi-major and semi-minor axes of the ellipse given by the equation (19).

REFERENCES

[1] Zahra K, Z. Awad, H.R. Dwidar and M. Radwan, *On stability of triangular equilibrium points of the restricted relativistic elliptic three body problem with triaxial and oblate primaries.*Serb.Astron.J.N.195, 2017, 47-52

[2] Jain Preeti, Rajiv Aggarwal, Amit Mittal, Abdullah *Periodic Orbits in the Photogravitational restricted three body problem when primaries are triaxial rigid bodies,* International Journal of Astronomy and Astrophysics.6,2016,111-121

[3] Jain, M and Aggarwal, R, *A study of Non-Collinear Libration points in Restricted three-body problem with stokes Drag effect when smaller primary is an oblate spheroid*, Astronomy and Astrophysics.358,2015, 1-8.

[4] Perdios, E.A, Kalantonis, V. S, Perdiou, A. E and Nikaki, A .A *Equilibrium points and Related Periodic motions in restricted three body problem with Angular Velocity Radiation effects,* Advances in Astronomy,2015, 1-21.

[5] Kumar Avdhesh, and Ishwar, B *Linear stability of triangular equilibrium points in the Photogravitational restricted three body problem with triaxial rigid bodies with bigger one an oblate spheroid and source of radiation,* KAS.Vol.30, 2015, 297-299.

[6] Mittal, A., Aggarwal, R. and Bhatnagar, K.B. *Periodic orbits around L_4 in the photogravitational restricted problem with oblate primaries' WSE4S 6th* International conference Proceedings on optics Astrophysics and Astrology, Article ID: 650927,201.1

[7] Mittal, Amit; Ahmad, Iqbal and Bhatnagar, K. B,*'Periodic orbits in the photogravitational restricted problem with the smaller primary an oblate body,* Astrophysics and Space Science, Volume 323,2009, pp.65-73.

[8] Mittal, A, Iqbal and Bhatnagar, K. B., *'Periodic orbits Generated by Lagrangian Solution of the restricted problem when one of the primaries is an oblate body'.* Astr. & Space Science, Vol. 319, 2008, pp.63-73.

[9] Kumar Avdhesh, J.P. Sharma and Ishwar, B, *'Linear stability of triangular equilibrium points in the photogravitational restricted three body problem with poynting-Robertson drag when both primaries are oblate spheroid,* Journal of dynamical system & Geometric Theories, Vol.5,2007, 193-2002.

[10] Ishwar, B. Tripathi and Deepak Kumar *'Linear stability of triangular equilibrium points in the photogravitational restricted three body problem with poynting-Robertson drag',* News Bull. Cal. Math. Soc., 28, 2005.

[11] Sharma, R.K. Taqvi, Z.A. and Bhatnagar, K. B. *'Effect of perturbed potential on the stability of libration points in the restricted problem',* Celest. Mech. & Dyn. Astr. 79,2001, pp. 119-133.

[12] Ishwer, B., Elipe, A.. *Secular solutions at triangular equilibrium point in the generalized photogravitational Restricted Three Body Problem'.* Astrophysics and Space Science, Volume 277, 2001, pp.437-446.

[13] Khanna, M. and Bhatnagar, K. B., *'Existence stability of libration points in the restricted three-body problem when the smaller primary is a triaxial rigid body'* Indian J. Pure. Appl. Math29 (10), 1998, 1011.