Modal Amortization Rate Equivalent to a Structural System with Elastomer Insulators

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Abstract: - This article addresses the amortization rate equivalent or composed on own ways of the structural system. This situation is specific to structural systems consisting of several mechanically coupled components. In this context, there is analyzed the case of dynamic insulation of a road bridge structure with different groups of elastomeric anti-seismic devices. On this basis, there are treated the real cases with parametric values experimentally obtained and with a pre-compressed 40-m reinforced concrete beam configuration conducted for the first time in Romania.

Keywords: dynamic insulation, anti-seismic devices, composed amortization, equivalent amortization rate.

1. INTRODUCTION

For a structural system with elastic and viscous linear amortization connections of different parametric values it is necessary to assess the equivalent modal amortization also named equivalent modal amortization for a viaduct leaned at ends on two parametrically distinct linear viscoelastic systems. The dominant movement at seismic actions is horizontal so that only the horizontal translation work according to the length of the viaduct shall be taken into account.

It is emphasized the combined effect of the amortization rate ζ_i and of the stiffness k_i, at the horizontal translational displacement, for each bearing i =1, 2, on the equivalent modal amortization rate ζ_{eq}^r according to r vibration mode. For the given situation there is only one mode r = 1 of vibration corresponding to the horizontal translation of the viaduct.

In the case of elastomeric bearing systems, as anti-seismic devices, the hysteretic amortization coefficient η is used and the complex stiffness $k(1+j\eta)$ where $j=\sqrt{-1}$ is the imaginary unit.

2. EQUIVALENT MODAL AMORTIZATION RATE

We consider a system consisting of i discrete elastic and amortization components, for the r vibration mode, so that the dissipated energy E_i^r can be written as:

$$E_i^r = \pi c_i \omega X_r^2 \qquad (1)$$

where ω is the excitation pulsation on r mode which can be put in the form $\omega=\omega_r$;

c_i - the linear viscous amortization coefficient for component i, which can be written as $c_i = 2\zeta_i^r m_i^r \omega_r$;

X_r - modal displacement amplitude r.

The maximum potential energy of the modal deformation V_i^r is given by the relation

$$V_i^r = \frac{1}{2} k_i^r X_r^2 \qquad (2)$$

Taking into account that modal stiffness $k_i^r = m_i^r \omega_r^2$, relation (1) can be written as

$$E_i^r = 2\pi\zeta_i^r k_i^r X_r^2$$

or

$$E_i^r = 4\pi\zeta_i^r V_i^r \qquad (3)$$

For the entire system consisting of i components, the total dissipated energy $E_i^r = 4\pi\zeta_{eq}^r V_t^r = \sum_i E_i^r$, which leads to the relation

$$4\pi\zeta_{eq}^r V_t^r = 4\pi \sum_i \zeta_i^r V_i^r$$

where $V_t^r = \sum_i V_i^r$ and eventually it emerges ζ_{eq}^r as

$$\zeta_{eq}^r = \frac{\sum_i \zeta_i^r V_i^r}{\sum_i V_i^r} \qquad (4)$$

In the case of the system with translational displacement only on the longitudinal axis, that is of a system with a single degree of freedom, therefore with a single mode r =1 of vibration, relation (4) can be written as

$$\zeta_{eq}^1 = \frac{\sum_i \zeta_i^1 \frac{1}{2} k_i X_1^2}{\sum_i \frac{1}{2} k_i X_1^2}$$

or

$$\zeta_{eq}^1 = \frac{\sum \zeta_i^1 k_i}{\sum k_i} \qquad (5)$$

Relation (5) is valid for the situation when the elastomeric insulator i has the parametric characteristics k_i and ζ_i experimentally raised on a dynamic test bench with the excitation pulsation ω and amplitude $x_0 = x_1$ corresponding to the r =1 mode, the single mode of excitation and forced vibration. Hysteretic behavior is characterized by the fact that the hysteretic loss factor $\eta_{0,i}$ for the elastomeric insulator i and the vibration isolator r can be written as

$$\eta_{0,i} = 2\zeta_{0,i}\Omega_0 \tag{6}$$

where $\zeta_{0,i}$ is the amortization rate of the insulator i for the vibration mode r =1 with pulsation excitation $\omega \neq \omega_n$

$\Omega_0 = \frac{\omega}{\omega_n}$ - relative pulsation of the dynamic load on the test bench;

$\omega_n = \sqrt{\frac{k_i}{m_i}}$ - the own (natural) pulsation of the elastomeric insulator i assembled in the test system of the bench with the corresponding mass m_i.

The test on the dynamic bench is performed for the resonance case that is when Ω_0 =1 which leads to the modification of relation (6) thus

$$\eta_{0,i} = 2\zeta_{0,i} \tag{7}$$

The $E_{0,i}$ energy dissipated by insulator i when tested on the bench is as

$$E_0^i = \pi\eta_{0,i}k_iX_0^2 \tag{8}$$

or taking into account relation (7) we have

$$E_0^i = 2\pi\zeta_{0,i}k_iX_0^2 \tag{9}$$

If in relation (9) we insert the potential maximum deformation energy $V_i^{max} = \frac{1}{2}k_iX_0^2$, then we obtain

$$\zeta_{0,i} = \frac{E_0^i}{4\pi V_i^{max}} \tag{10}$$

Relation (10) is used for the calculation of $\zeta_{0,i}$ based on raising the hysteretic loop. This is characterized by the area of the closed loop which is proportional to $E_{0,i}$, by the maximum lateral displacement X_0 and by $k_i = k_{eff} = \frac{F_{max}}{X_0}$, where F_{max} is the maximum value of the elastic deformation force with X_0 amplitude.

3. DETERMINATION OF THE EQUIVALENT MODAL AMORTIZATION RATE BASED ON EXPERIMENTAL RESULTS.

The test of the elastomeric insulators is conducted based on the provisions of the EN15129 European standard regarding the anti-seismic devices. In this case the test frequencies are set to values that ensure the results processing in accordance with relations (7), (9) and (10) that is for Ω_0 =1, at a known ω excitation pulsation.

In reality, the structural system with elastomeric insulators $i \gg 2$, may have its own pulsation $\omega_r \neq \omega$ for the r vibration mode. This situation requires adequate analysis regarding the determination of the amortization rate ζ_i^r for the r mode when the elastomeric insulators have parameters $\zeta_{0,i}$ and k_i as test bench measured values.

For an elastomeric insulator to be used, it is necessary to ensure the equivalent dissipated energy in both cases, that is $E_{0,i}$ on the test bench and E_i^r for the structural system with insulator i corresponding to the vibration mode r. Thus, the equivalence relation may be written down as

$$E_0^i = E_i^r \tag{11}$$

where E_i^r is the dissipated energy on r mode

But E_i^r can be written as

$$E_i^r = 2\pi\zeta_i^r k_i\Omega X_r^2 \tag{12}$$

where $\Omega = \frac{\omega}{\omega_r}$ is the relative pulsation at r mode;

X_r – lateral displacement amplitude (in horizontal translation) for r mode.

Imposing condition (11) based on relations (8), (12) we obtain

$$\zeta_{0,i}X_0^2 = \zeta_i^r\Omega X_r^2 \tag{13}$$

from where we have

$$\zeta_i^r = \frac{\omega_r}{\omega}\zeta_{0,i}\left(\frac{X_0}{X_r}\right)^2 \tag{14}$$

For the given structural system with i =1,2 and r =1 there emerge the partial amortizations ζ_1' and ζ_2' thus

$$\zeta_1' = \frac{\omega_1}{\omega}\zeta_{0,1}\left(\frac{X_0}{X_1}\right)^2 \tag{15}$$

$$\zeta_2' = \frac{\omega_2}{\omega}\zeta_{0,2}\left(\frac{X_0}{X_2}\right)^2 \tag{16}$$

In this case, the equivalent amortization rate ζ_{eq}^1 based on relations (5), (15) and (16), can be written down as

$$\zeta_{eq}^1 = \frac{\omega_1}{\omega_2}\left(\frac{X_0}{X_1}\right)^2 \frac{\zeta_{0,1}k_1+\zeta_{0,2}k_2}{k_1+k_2} \quad (17)$$

and for several components $i \gg 2$ we have

$$\zeta_{eq}^r = \frac{\omega_r}{\omega}\left(\frac{X_0}{X_r}\right)^2 \frac{\sum_i \zeta_{0,i}k_i}{\sum_i k_i},\ r=1 \quad (18)$$

Relation (18) highlights the following specific cases:

a) if the anti-seismic device is tested on a dynamic test bench with values of the modal parameters $\omega_r = \omega_1$ and $X_r = X_1$, then the equivalent amortization rate ζ_{eq}^1 can be calculated as follows

$$\zeta_{eq,a}^1 = \frac{\sum_i \zeta_{0,i}k_i}{\sum_i k_i} \quad (19)$$

b) when the anti-seismic device is tested on a dynamic test bench with values of the modal parameters $\omega_r = \omega_1 = \omega$ and $X_r = X_1 \neq X_0$ from relation (18) we have

$$\zeta_{eq,b}^1 = \left(\frac{X_0}{X_1}\right)^2 \frac{\sum_i \zeta_{0,i}k_i}{\sum_i k_i} \quad (20)$$

c) when the anti-seismic device is tested on a dynamic test bench with values of the modal parameters $\omega_r = \omega_1 \neq \omega$ and $X_r = X_1 = X_0$, from relation (18) we obtain

$$\zeta_{eq,c}^1 = \frac{\omega_1}{\omega}\frac{\sum_i \zeta_{0,i}k_i}{\sum_i k_i} \quad (21)$$

The specific shear deformation is given by the relation

$$\gamma = \frac{x}{T_q} \quad (22)$$

where x is the instantaneous horizontal displacement.

In this case the amortization reports on the bench $\zeta_{0,1}$, $\zeta_{0,2}$ at the maximum test displacement $x_{max} = X_0 = \gamma_0 T_q$ can be formulated as follows

$$\zeta_{0,1} = \frac{E_{d1}}{2\pi k_1 X_0^2} \quad (23)$$

$$\zeta_{0,2} = \frac{E_{d2}}{2\pi k_2 X_0^2} \quad (24)$$

Based on relations (23) and (24), taking into account that $X_r = X_1 = \gamma T_q$,, relation (18) for the compound amortization of insulator groups I and II can be written as

$$\zeta_{eq}^1 = \frac{\omega_1}{\omega}\frac{X_0^2}{X_r^2}\frac{\frac{E_{d1}}{2\pi k_1 X_0^2}2k_1+\frac{E_{d2}}{2\pi k_2 X_0^2}2k_2}{2k_1+2k_2}$$

from where

$$\zeta_{eq}^1 = \frac{\omega_1}{\omega}\frac{1}{X_r^2}\frac{E_{d1}+E_{d2}}{2\pi(k_1+k_2)}$$

where by replacing $X_r^2 = X_1^2 = \gamma^2 T_q^2$ we have

$$\zeta_{eq}^1 = \frac{\omega_1}{\omega}\frac{E_{d1}+E_{d2}}{2\pi(k_1+k_2)\gamma^2 T_q^2} \quad (25)$$

4. CASE STUDY

It is considered the reinforced and prestressed I-shaped concrete beam with, length L = 40 m, height h = 2 m and base width b = 0.96 m, leaned at its ends against two different groups of elastomeric insulators (HDRB) manufactured by SOMMA company from Italy.

Table 1.

No.	Mechanic characteristics		Elastomeric insulator (HDRB)	
	Parameter	**M. U.**	**ISI-S** 350/150	**ISI–N** 350/150
1	Maximum seismic displacement, d_{Ed}	mm	300	300
2	Elastomer diameter, D	mm	350	350
3	Shear mode, G	MP$_a$	0.4	0.8
4	Total elastomer thickness (multilayer), T_q	mm	150	150
5	Total insulator height, H	mm	263	263
6	Effective horizontal stiffness, k_h	kN/mm	0.26	0.51
7	Effective vertical stiffness, k_v	kN/mm	312	496
8	Maximum vertical force at earthquake (ULS), N_{Ed}	kN	320	320
9	Maximum static vertical force (ULS) F_{zd}	kN	905	1811
10	Dissipated energy per cycle, E_d	kJ	3.75	15.86
11	Test displacement amplitude,, X_0	mm	150	150
12	Dynamic test frequency, f	H$_z$	0.50	0.50
13	Shear maximum specific deformation, γ^{max}	%	200	200
14	Effective amortization rate, ζ_0 at γ=100%	%	10.2	22.0

Each bearing group consists of two identical elastomeric insulators. Thus, at one end there is the I group consisting of two elastomeric insulators type ISI-S350/150, and at the other end there is the II group consisting of two elastomeric insulators type ISI-N350/150, both of them having the characteristics in Table 1. The equipped beam has its mass m = 80t, and its own pulsation for the horizontal translation mode is ω_1 =4.38 rad/s. The test bench generates harmonic movements with a frequency of 0.5Hz.

For the case study, we have group I of elastomeric insulators with the characteristics $k_1 = 0.28\cdot10^6$ N/m, E_{d1}=3.75 kJ, T_q= 0.15 m, and group II have the following characteristics $k_2 = 0.51\cdot10^6$ N/m, E_{d2}=15.86 kJ, T_q= 0.15 m. At the specific shear deformation γ=100%, based on relation (25) we have

$$\zeta_{eq}^1(\gamma = 1) = \frac{4.38}{3.14}\frac{(3.75 + 15.86)10^3}{2\pi(0.26 + 0.51)10^6 \cdot 1^2 \cdot 0.15^2} = 0.25$$

If the dissipated energy E_{d1} and E_{d2} represents the possible maximum values of the elastomeric insulators and can be considered constant measures, then the amortization report at specific shear displacements for γ_1 = 1,5 and γ_2 = 2,0 it emerges as $\zeta_{eq}^1(\gamma = 0.15) = 0.11$ and respectively $\zeta_{eq}^1(\gamma = 2.0) = 0.0625$.

If the excitation pulsation ω on the dynamic test coincides with the vibration pulsation of the own mode $\omega_r = \omega_1$, that is $\omega = \omega_1$, then for $\gamma = 1$, it emerges

$$\zeta_{eq}^1 \Big|_{\omega_1 = \omega}^{\gamma = 1} = \frac{(3.75 + 15.86)10^3}{2\pi(0.26 + 0.51)10^6 \cdot 1^2 \cdot 0.15^2} = 0.179$$

When the modal resonant pulsation $\omega_1 = \frac{1}{2}\omega$ it emerges

$$\zeta_{eq}^1 \Big|_{\omega_1 = \frac{1}{2}\omega}^{\gamma = 1} = 0.0895$$

5. CONCLUSIONS

In the case of the anti-seismic devices with hysteretic characteristic, the amortization report is determined experimentally in accordance with the provisions of the harmonized European standard EN15129: 2009. The value of the amortization report ζ_0, for the sinusoidal regime translation displacements, corresponds to the maximum horizontal displacement (amplitude) X_0 and to the excitation pulsation ω so that the dynamic system may perform complete cycles raising the hysteretic loops.

a) For an insulation system composed of distinct elastomeric insulator groups with stiffness and amortization characteristics k_i, ζ_i, with different values, the compound amortization or the modal amortization report for the whole system can be determined with relation (5) when pulsation ω of the sinusoidal dynamic regime of the test bench is equal to the pulsation of the own vibration mode of the structural system.

b) If the excitation pulsation of the dynamic bench is different from the pulsation of its own vibration mode then relation (18) applies.

c) When the bearing system is made exclusively of elastomeric insulators deformed in the horizontal direction at shear the amplitude of the translational displacement is given by γT_q. In this case, the modal amortization rate can be calculated with the relation (25).

Given the above, it is found that the compound amortization or the equivalent modal amortization report is a system parameter that must be assessed according to the parametric values ζ_i, k_i, in the laboratory and the modal parametric conditions of the structural system, that is own pulsations and own vectors.

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