A New Model for Milling Circular Work-pieces

Elena-Larisa PREDEL
University of Piteşti, street Târgul din Vale, no 1, 110040, ROMANIA, e-mail: predel_elenalarisa@yahoo.com

Nicolae-Doru STĂNESCU
University of Piteşti, street Târgul din Vale, no 1, 110040, ROMANIA, e-mail: s_doru@yahoo.com

Nicolae PANDREA
University of Piteşti, street Târgul din Vale, no 1, 110040, ROMANIA, e-mail: nicolae_pandrea37@yahoo.com

Abstract: - In this paper we present a non-linear model for the investigation of the milling of circular work-pieces. Some hypotheses are used in this model including the non-variation of the direction of the cutting force in function of the engagement of the teeth. The model has four degrees of freedom. The non-linearity has as main causes the variation of the depth of cut and the expression of the components of the cutting forces in function of the depth of cut. Details of the integration of the non-linear system are also provided. A numerical example highlights the theory.

Keywords: - non-linear model, milling, Heaviside function, integration

1. INTRODUCTION

Different models and studies are published in the field of milling manufacturing [1 – 16]. The first model for the milling operations were developed in the fundamental books of Tobias [15], Tlusty [16] and Altintas [14].

The mechanical models are applied to up- or end-milling and they deal with rectilinear miling [1 – 16]. The chatter has as main cause the variation of the depth of cut due to the discontinuity of the milling process and the vibrations of the technological system [1 – 16]. The mechanics of milling is described with the aid of a planar mechanical system with four degrees of freedom [1, 3, 7, 10, 12, 13]. The cutting force is a sum of the cutting forces given by each tooth [1, 3, 7, 10, 12, 13] multiplied by a step function that takes into account whether or not the current tooth cuts. Based on these hypotheses and the non-linearity of the technological system the authors [1 – 16] focus their attention on the determination of the stability diagram in different situations (unequal tooth pitch [1], nonlinearity of the active force [2, 9] etc.). Isolating the cutting force that acts upon the tool, authors [2] determine the stability lobes and propose a method for active vibration control resulting in the increase of the stability zones. For the mechanical system with four degrees of freedom the theoretical results are compared to the experimental ones [1 – 8, 10 – 16]. The vibration signals in end-milling are analyzed [4] by using mathematical transformation and the numerical results are comparing to the experimental ones obtained for thin parallelepiped work-pieces. The avoidance of chatter for the spindle speed are presented in [5] using numerical results. Dynamics of milling using helical mills and the determination of forced vibration and the stability zones are presented in [6]. Non-linear regeneration in milling is discussed in [7] as well some aspects concerning bifurcation and hysteresis. Some particular effects like serration [8] are the main goal of some papers. Problems that appear in micro-milling are treated in [9] and the validation is offered by numerical simulation. The chaotic dynamics of milling is investigated in [10]. Reference [11] treats the spatial case of milling in a particular case discussing the influence of different parameters on the stability zones. The improvement of the stability zones by orientated transfer functions is presented in [12]. The influence of variable pitch and helix cutters is discussed in 13].

In this paper we present a four degrees of freedom non-linear model for the milling of the cylindrical work-pieces.

2. PROBLEM FORMULATION

The mechanical model is captured in Fig. 1. It consists in the work-piece of center O_w and the tool of center O_t. In the initial position we added the index i, while in an intermediate position we added the index f.

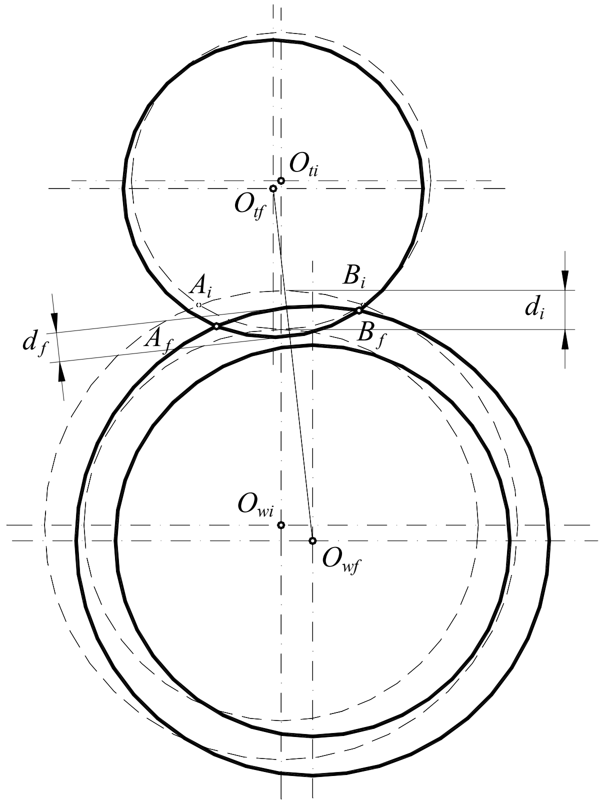

Figure 1. Mechanical model.

Because of the flexibility of the axes of tool and work-piece the centers move from the initial positions O_{ti} and O_{wi} to the new positions O_{tf} and O_{wf}, respectively.

In the initial position the cutting takes place in the zone delimitated by the points A_i and B_i, while in the intermediate position in the zone delimitated by the points A_f and B_f. Consequently, the maxim depth of cut varies from d_i to d_f.

The initial position is marked by dashed line, while the intermediate position is marked by continuous line.

We assume that the cutting force can be decomposed along the straight line $O_{tf}O_{wf}$ and perpendicular to this direction. Moreover, the cutting force is proportional with d_f^p, where p is a positive exponent.

The system has four degrees of freedom: the displacements x_t and y_t of the tool and the displacement x_w and y_w of the work-piece.

One asks for the determination of motion for the tool and work-piece.

3. DEPTH OF CUT

The reader is asked to refer to Fig. 2. We denote with E the point of intersection between the periphery of the work-piece and the straight line O_tO_w, and with D the point of intersection between the periphery of the tool and the straight line O_tO_w.

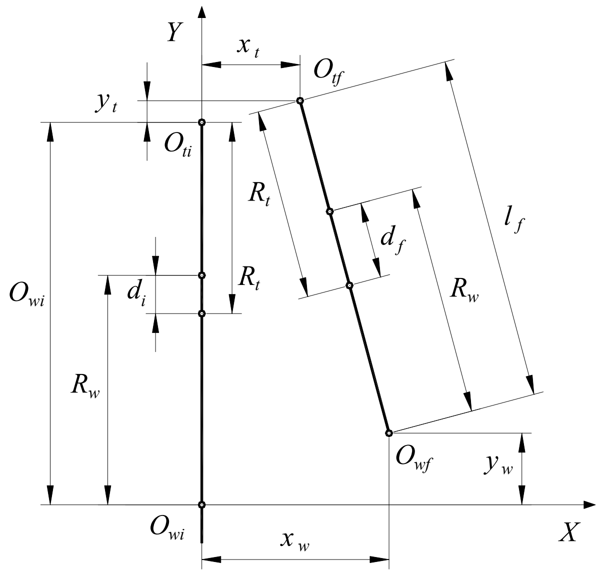

Figure 2. The depth of cut.

In the initial position (the position at rest) one way write

$$d_i = R_w + R_t - l_i, \tag{1}$$

where d_i is the initial depth of cut, and l_i is the initial distance between O_{ti} and O_{wi}.

Similarly, in the intermediate position, one has

$$d_f = R_w + R_t - l_f. \tag{2}$$

In the previous formulae R_w and R_t are the radii of the work-piece and tool, respectively.

From the figure one gets

$$l_f = \sqrt{(x_w - x_t)^2 + (l_i + y_t - y_w)^2}, \tag{3}$$

where x_t, y_t, x_w, y_w are the displacements of the tool and work-piece along the axes OX and OY, respectively.

One also may write

$$O_{ti}D_i = O_{tf}D_f, \; O_{wi}E_i = O_{wf}E_f. \tag{4}$$

4. CUTTING FORCE

According to our assumptions, we have

$$F_t = C d_f^p , \tag{5}$$

where F_t is the component of the cutting force perpendicular to the direction $O_{tf}O_{wf}$, C is a constant and p a positive exponent which depend on the cutting conditions.

Taking into account the previous expressions, one way write

$$F_t = C\left[R_w + R_t - \sqrt{(x_w - x_t)^2 + (l_i + y_t - y_w)^2} \right]^p \tag{6}$$

.

References assume that between the components F_t and F_r of the cutting force there exists the relation

$$F_r = K F_t , \tag{7}$$

where K depends on the cutting conditions.

The relation (6) and, consequently, (7), are true if and only if $d_f > 0$, that is, the expression (6) must be corrected as

$$F_t = C d_f^p \Theta(d_f), \tag{8}$$

where $\Theta(\cdot)$ is the Heaviside step function

$$\Theta(z) = \begin{cases} 1, \text{if } z > 0 , \\ 0, \text{if } z \le 0 . \end{cases} \tag{9}$$

In conclusion

$$F_t = C \cdot \cdot\left[R_w + R_t + \sqrt{(x_w - x_t)^2 + (l_i + y_t - y_w)^2} \right]^p \cdot \cdot\Theta\left(R_w + R_t + \sqrt{(x_w - x_t)^2 + (l_i + y_t - y_w)^2} \right), \tag{10}$$

$$F_r = CK \cdot \cdot\left[R_w + R_t + \sqrt{(x_w - x_t)^2 + (l_i + y_t - y_w)^2} \right]^p \cdot \cdot\Theta\left(R_w + R_t + \sqrt{(x_w - x_t)^2 + (l_i + y_t - y_w)^2} \right). \tag{11}$$

5. EQUATIONS OF MOTION

Let us denote by α the angle between the axis OY and the straight line O_tO_w (Fig. 3).

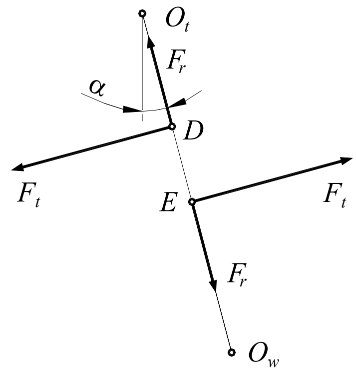

Figure 3. Equations of motion.

The theorem of momentum applied to the two rigid bodies (the tool and the work-piece) leads to the following differential equations

$$m_t \ddot{x}_t = -F_t \sin\alpha - F_r \cos\alpha , \quad m_t \ddot{y}_t = F_r \sin\alpha - F_t \cos\alpha , \tag{12}$$

$$m_w \ddot{x}_w = F_t \cos\alpha + F_r \sin\alpha , \quad m_w \ddot{y}_w = F_t \sin\alpha - F_r \cos\alpha , \tag{13}$$

where

$$\sin\alpha = \frac{x_w - x_t}{l_f}, \ \cos\alpha = \frac{l_i + y_t - y_w}{l_f}, \tag{14}$$

while m_t and m_w are the masses of the tool and work-piece, respectively.

Equations (12) and (13) form a system of four non-linear differential equations of second order.

Denoting

$$z_1 = x_t , \ z_2 = y_t , \ z_3 = x_w , \ z_4 = y_w , \ z_5 = \dot{x}_t , \ z_6 = \dot{y}_t , \ z_7 = \dot{x}_w , \ z_8 = \dot{y}_w \tag{15}$$

one obtains a system of eight first order non-linear differential equations

$$\frac{dz_1}{dt} = z_5 , \tag{16}$$

$$\frac{dz_2}{dt} = z_6 \tag{17}$$

$$\frac{dz_3}{dt} = z_7 \quad (18)$$

$$\frac{dz_4}{dt} = z_8 \quad (19)$$

$$\frac{dz_5}{dt} = \frac{1}{m_t}\left(-F_t \sin\alpha - F_r \cos\alpha - c_{tx} z_5 - k_{tx} z_1\right), \quad (20)$$

$$\frac{dz_6}{dt} = \frac{1}{m_t}\left(F_r \sin\alpha - F_t \cos\alpha - c_{ty} z_6 - k_{ty} z_2\right), \quad (21)$$

$$\frac{dz_7}{dt} = \frac{1}{m_w}\left(F_t \cos\alpha + F_r \sin\alpha - c_{wx} z_7 - k_{wx} z_3\right), \quad (22)$$

$$\frac{dz_8}{dt} = \frac{1}{m_w}\left(F_t \sin\alpha - F_r \cos\alpha - c_{wy} z_8 - k_{wy} z_4\right). \quad (23)$$

In the previous equations, c_{tx}, c_{ty}, c_{wx}, c_{wy} are the damping coefficients for the tool and work-piece along the axes OX and OY. Similarly, k_{tx}, k_{ty}, k_{wx}, k_{wy} are the stiffness of the tool and work-piece along the same axes.

The system of equations (16) … (23) can be numerically integrated by the fourth order Runge-Kutta method.

6. NUMERICAL SIMULATION

For the numerical simulation we consider the following two cases.

The first case is defined by $l_i = 54.8\ \text{mm}$, $R_w = 30.0\ \text{mm}$, $R_t = 25.0\ \text{mm}$, $p = 1.2$, $C = 1000$, $K = 0.7$, $k_{tx} = 1000\ \text{N/mm}$, $k_{ty} = 5000\ \text{N/mm}$, $k_{wx} = 500\ \text{N/mm}$, $k_{wy} = 500\ \text{N/mm}$, $c_{tx} = 0\ \text{Ns/mm}$, $c_{ty} = 0\ \text{Ns/mm}$, $c_{wx} = 0\ \text{Ns/mm}$, $c_{wy} = 0\ \text{Ns/mm}$, $m_t = 100\ \text{kg}$, $m_w = 5\ \text{kg}$.

The second case is characterized by the same parameters excepting $C = 1200$, $K = 0.8$.

In both cases the initial values are: $x_t = 0\ \text{mm}$, $y_t = 0\ \text{mm}$, $x_w = 0\ \text{mm}$, $y_w = 0\ \text{mm}$, $\dot{x}_t = 0\ \text{mm/s}$, $\dot{y}_t = 0\ \text{mm/s}$, $\dot{x}_w = 0\ \text{mm/s}$, $\dot{y}_w = 0\ \text{mm/s}$.

The time of simulation is $t_{\max} = 5\ \text{s}$, while the step of integration is $\Delta t = 10^{-3}\ \text{s}$.

The results of the simulation are captured in the next diagrams.

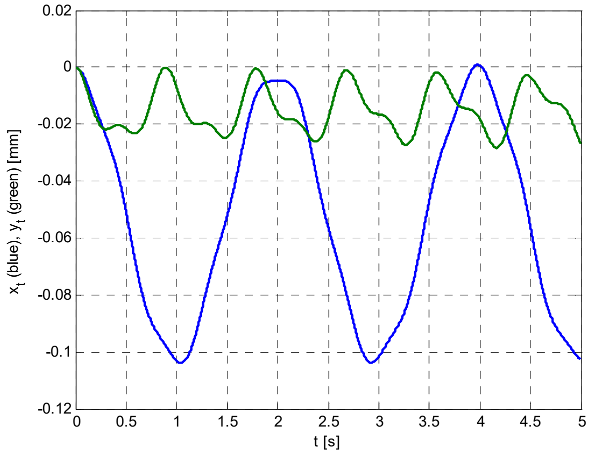

Figure 4. Time histories for the displacements of tool in the first case.

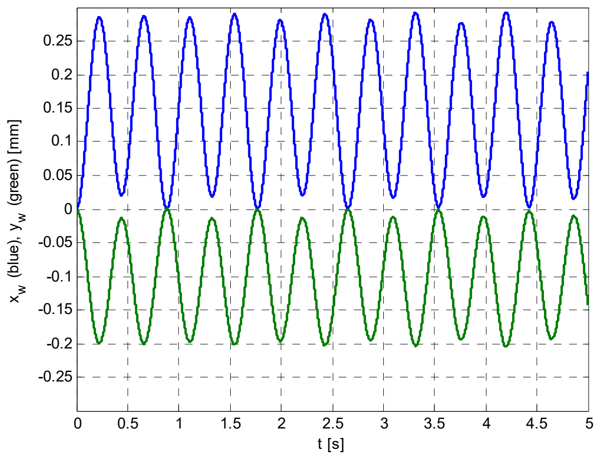

Figure 5. Time histories for the displacements of the work-piece in the first case.

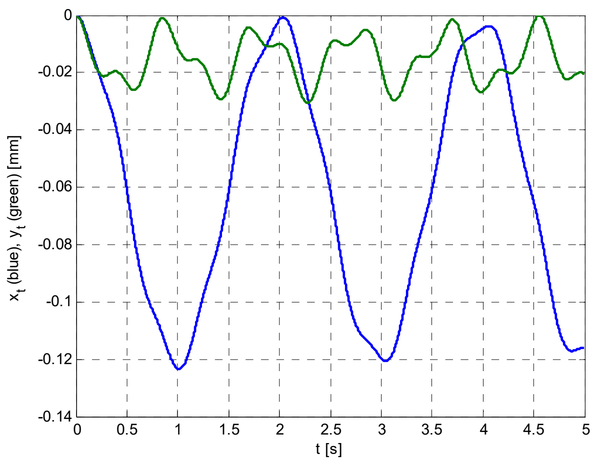

Figure 6. Time histories for the displacements of tool in the second case.

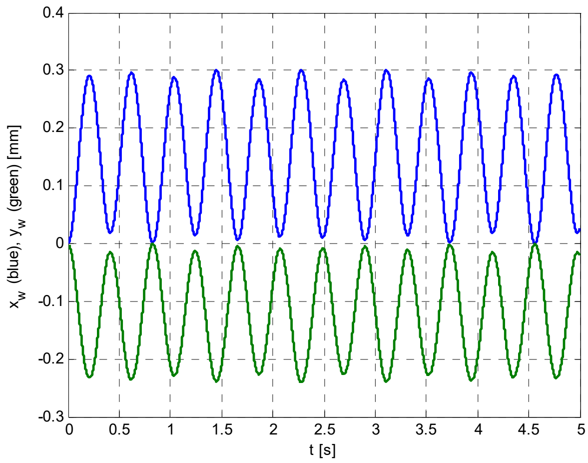

Figure 7. Time histories for the displacements of the work-piece in the first case.

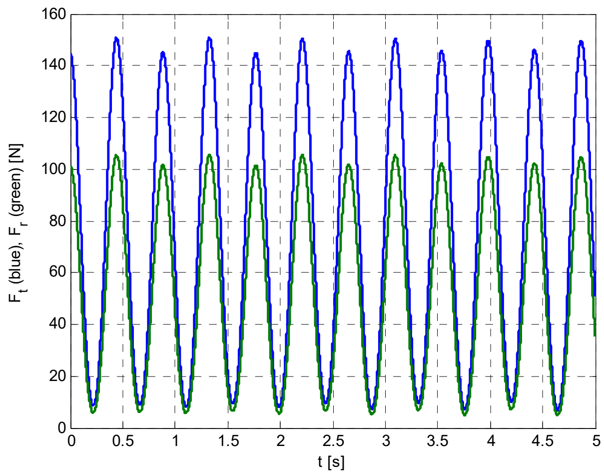

Figure 8. Time histories for the components of the cutting forces in the first case.

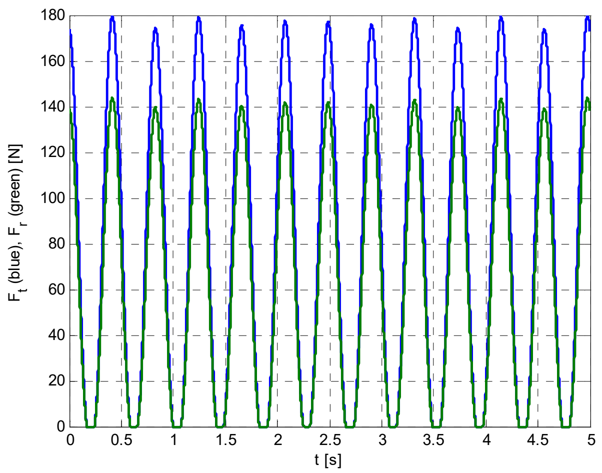

Figure 9. Time histories for the components of the cutting forces in the second case.

One may observe that the motion is periodic for the work-piece, but not for the tool. In the first case the contact between the tool and the work-piece is not lost (the depth of cut and the components of the cutting force do not vanish). This is not the situation in the second case when the depth of cut becomes for some periods of time equal to zero; consequently the components of the cutting force vanish too. It results that the tool and the work-piece have independent motion, that is, the system of equations may be divided in two uncoupled systems of equations of motion.

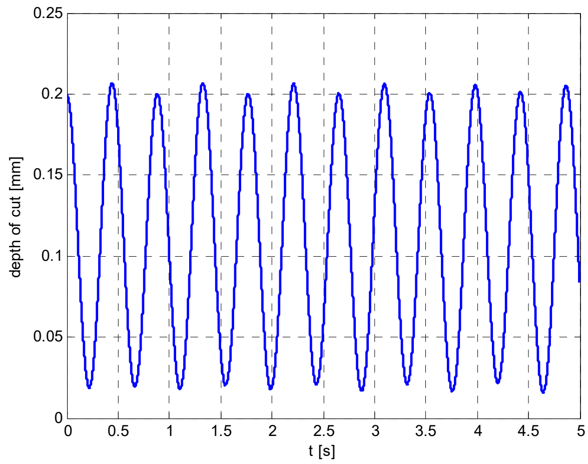

Figure 10. Time history for the depth of the cut in the first case.

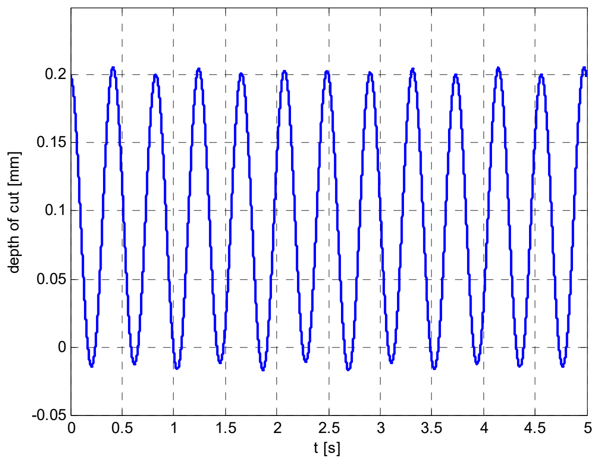

Figure 11. Time history for the depth of the cut in the second case.

7. CONCLUSIONS

This dynamic model for milling is a new one and may be used for small depth of cut. Moreover, one requires that the arc AB in Fig. 1 has to be small enough in order to neglect the angular variation of the cutting force. The loose of contact between the tool and the work-piece may appear for high cutting force and small stiffness for the technological system. Consequently, we may state that our model may simulate the behavior of a milling process in some important practical cases. One of such cases is the manufacturing by milling of the helix screws for the machine tools.

In our future works we will generalize the model for the three dimensional cases.

REFERENCES

[1] Sellmeier V., Denkena B., Stable islands in the stability chart of milling processes due to unequal tooth pitch, *International Journal of Machine Tools & Manufacture*, Vol. 51, 2011, pp. 152-164.

[2] Parus A., Powalka B., Marchelek K., Domek S., Hoffmann M., Active vibration control in milling flexible workpieces, *Journal of Vibration and Control*, Vol. 19, No. 7, 2012, pp. 1103-1120.

[3] Balachandran, B., Zhao M.X., A Mechanics Based Model for Study of Dynamics of Milling Opperations, *Meccanica*, Vol. 35, 2000, pp. 89-109.

[4] Susanto A., Liu C.-H., Yamada K., Hwang Y.-R., Tanaka R., Sekiya K., Application of Hilbert-Huang transform for vibration signal analysis in end-milling, *Precision Engineering*, Vol. 53, 2018, pp. 263-277.

[5] Bak P.A., Jemielniak K., Self-excited vibrations avoidance methodology in non-linear numerical simulation environment, *Procedia CIRP*, Vol. 62, 2017, pp. 245-249.

[6] Bachrathy D., Munoa J., Stepan G., Experimental validation of appropriate axial immersions for helical mills, *International journal of advanced manufacturing technology*, Vol. 84, 2016, pp. 1295-1302.

[7] Mann B.P., Garg N.K., Young K.A., Helvey A.M., Milling Bifurcations from Structural Asymetry and Nonlinear Regeneration, *Nonlinear Dynamics*, Vol. 42, 2005, pp. 319-337.

[8] Dombovari Z., Altintas Y., Stepan G., The effect of serration on mechanics and stability of milling cutters, *International Journal of Machine Tools & Manufacture*, Vol. 50, 2010, pp. 511-520.

[9] Liu M.-K., Halfmann E.B., Suh C.S., Multi-dimensional time-frequency control of micro-milling instability, *Journal of Vibration and Control*, Vol. 20, No. 5, 2014, pp. 643-660.

[10] Banihasan M., Bakhtari-Nejad F., Chaotic vibrations in high-speed milling, *Nonlinear Dynamics*, Vol. 66, 2011, pp. 557-574.

[11] Liu X., Li R., Yang L, Yue C., A prediction method of milling chater stability for complex surface mold, *International journal of advanced manufacturing technology*, Vol. 89, 2017, pp. 2637-2648.

[12] Geng Z., Ridgway K., Turner S., Linear improvement of the machining stability lobes and application in milling process prediction, *Proceedings of the Institution of Mechanical Engineers, Part B: Journal of Engineering Manufacture*, Vol. 221, 2007, pp. 369-378.

[13] Jin G., Zhang Q., Hao S., Xie Q., Stability prediction of milling process with variable pitch and variable helix cutters, *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, Vol. 228, No. 2, 2014, pp. 281-293.

[14] Altintas Y., *Manufacturing Automation: Metal Cutting Mechanics, Machine Tool Vibrations, and CNC Design*, 2nd ed., Cambridge University Press, 2012.

[15] Tobias S.A., *Machine Tool Vibrations*, Wiley, 1965.

[16] Tlusty G., *Manufacturing Process and Equipment*, Pearson, 1999.