# Free Vibration of Orthotropic Thin Plates with Cantilever and Clamped Boundary Conditions

**Muhammad IMRAN**
*Department of Mechanical Engineering, International Islamic University, Islamabad, muhammad.imran@iiu.edu.pk*

**Rafiullah KHAN**
*Department of Mechanical Engineering, International Islamic University, Islamabad, rafiullah.khan@iiu.edu.pk*

**Abdur RAFAI**
*Department of Mechanical Engineering, International Islamic University, Islamabad, abdur.rafai@iiu.edu.com*

*Abstract:* - In this research paper, analytical analysis, experimental work and finite element simulations are combined to analyze the vibration behavior at different delamination sizes and different stacking sequences. The boundary condition in this investigation were all sides clamped and cantilever supported. Rayleigh-Ritz method was used to derive the governing equations to find the natural frequencies and the results were computed using Matlab tool. Experiments have been conducted to study the vibration characteristics of carbon fiber reinforced polymer (CFRP) composite plate. The results from analytical, experimental and finite element analysis were then compared and studied. It is seen that the natural frequencies of carbon fiber reinforced polymer decreased with an increase in delamination size. Stacking sequence of (0/90/45/90) showed higher values of natural frequencies subjected to all-sided clamped boundary conditions. It was interesting to know that there were small differences in values of natural frequencies for lower modes but the difference gradually increased in case of higher modes.

*Keywords:* Finite element analysis, composites, delamination, experimental vibration

**Nomenclature**

A, B, C and D are coefficients of frequency
a, b and ρ are width, height and density of the plate respectively
$D_x$, $D_y$, $D_{xy}$ are the stiffness terms and depends on the orientation of the fibers
$E_x$ :Young'sModulus–Longitudinal
$E_y$ : Young's Modulus – Transverse
m, n: Eigen vectors for mode shapes and mode numbers
ω : Angular velocity, $f_1$ : first natural frequency
$G_{xy}$ : Shear modulus - in-plane
$v_{xy}$ : Poisson's ratio

## 1. INTRODUCTION

The application of composites in various fields of sporting equipment, aerospace, marine and agricultural products have increased tremendously [1] due to their multi-dimensional, attractive and novel properties [2-11] along with low maintenance of composite materials [7-9, 12, 13]. One of the most critical defect in composites is delamination that can badly affect the behavior of composite structures [13, 14] so it is equally important to study the effect of ply layups, boundary conditions and delaminated region on the vibration characteristics of composite structures. Delamination is comparatively most complex problem that involves material and geometry discontinuations [15]. It is most important to have the vibration characteristics of structures investigated prior to be applied in order to improve the design parameters [16]. A considerable study, on the behavior of composite structures like beams, shells and plates are available however studies on the influence of stacking sequences and delamination size parameters on the delaminated composite plates are scarce. In following paragraphs, we will do aerial exploration of the work already carried out on the composite structures subjected to delamination and without delamination along with the methodologies used to analyze the effect on the vibration characteristics of different structures [17, 18].

The behavior of delaminated composites under vibration has been extensively studied analytically [19-23]. It was found that delamination size and location badly affect the vibration behavior of the composite structures. Experimental results showed that fundamental frequency further decreased in the presence of matrix. However, this decrease in fundamental frequency was not significant for small delamination [24-29]. It was concluded that the values of natural frequencies for clamped square plate with delamination decreased significantly with increase in delamination size.

Finite element analysis is an effective tool for the prediction of the structural behavior under loadings like static, dynamic, thermal and vibration. The behavior of delaminated composites under vibration has been extensively studied using commercial software packages like ANSYS and ABAQUS [30-43].

From the literature, it is observed that the vibration analysis on the carbon fiber reinforced polymer composite plates subjected to CCCC boundary condition for (0/90/45/90), (0/45) and (0/90) is very limited and the availability of the vibration behavior for this specific structure is poor [44-47]. The critical applications of composite in aeroplane wings, bridges and columns are mostly used CCCC. Therefore, it is utmost important to investigate the vibration characteristics of CFRP composite plate under these constraints. In this paper, the vibration investigation of delaminated and non-delaminated under CCCC constraint is performed using analytical, experimental and finite element analysis techniques. To study the effect of delamination size on the vibration properties of CFRP composite plate, delamination of 6.25%, 25% and 56.25% of the total plate area were incorporated at the middle of the rectangular plate. Three stacking sequences (0/45/90/45), (0/90) and (0/45) are investigated experimentally and FEA with the above delamination sizes.

The following section provides the detailed investigations which is followed by results and discussion.

## 2. ANLAYTICAL ANALYSIS

It is found in the first-order displacement theory as follows. An 8-nodded plate or sheet element & 5-degree of self-determination at every node is obtainable here for finite element model.

$$\boldsymbol{u} = \sum_{i=1}^{8} u_i N_i \; v = \sum_{i=1}^{8} u_i N_i \; v = \sum_{i=1}^{8} v_i N_i \; w = \sum_{i=1}^{8} w_i N_i \; \theta_x = \sum_{i=1}^{8} \theta_{xi} N_i \; \theta_y = \sum_{i=1}^{8} \theta_{yi} N_i \;, \tag{1}$$

Here, Ni represents "shape function" and 'i' represents "node number".

Strain-Stress relationship in global co-ordinate axes scheme or system that was derived in earlier section and has taken the following form from equation 35.

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_x \\ M_{xy} \end{bmatrix} = \begin{bmatrix} a & b \\ b & d \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ k_x \\ k_y \\ k_{xy} \end{bmatrix}, \tag{2}$$

Extensional stiffness matrix, extensions-bending coupling matrix and bending stiffness matrix can be extracted from the equations set 37 and can be rewritten as follows,

$$\begin{aligned} [a_{ij}] &= \sum_{k=1}^{N} [Q_{ij}]_k (Z_k - Z_{k-1}) \\ [a_{ij}] &= \sum_{k=1}^{N} [Q_{ij}]_k (Z_k - Z_{k-1}) \\ [b_{ij}] &= \frac{1}{2} \sum_{k=1}^{N} [Q_{ij_k} (Z^2{}_k - Z^2{}_{k-1}) \\ [d_{ij}] &= \frac{1}{3} \sum_{k=1}^{N} [Q_{ij}]_k (Z^3{}_k - Z^3{}_{k-1}) \end{aligned} \tag{3}$$

where:

$[a_{ij}]$ = 'extensional stiffness matrix'

$[b_{ij}]$ = 'extension-bending coupling matrix'

$[d_{ij}]$ = 'bending stiffness matrix'

'Lamina Stiffness Matrix' is presented as,

$$[D] = \begin{bmatrix} a & b \\ b & d \end{bmatrix}_k, \tag{4}$$

Strain-Stress relation for 'shear forces' is given by,

$$\begin{bmatrix} Q_x \\ Q_y \end{bmatrix} = \sum_{k=1}^{N} \int_{Z_{K-1}}^{Z_K} K1 \begin{bmatrix} Q_{55} & Q_{45} \\ Q_{55} & Q_{44} \end{bmatrix} \begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} dz \tag{5}$$

Here N = layer number in coat or laminate &, k = the specific layer

- k1 represents 'shear correction factor'.
- a, b and d matrix is obtained

The 'eigenvalue' shape of governing eq. for finding 'natural frequency' of given scheme or system is expressed as,

$$([\mathrm{K}] - \omega^2 [M])\Delta = 0, \tag{6}$$

## 3. EXPERIMENTAL METHODOLOGY

The composite plates models used in the present work were made of carbon fibers. $0^o$ and $90^o$ direct rovings were used for interweaving the fabrics with epoxy matrix. Ratio of 50:50 in weight of the fiber and matrix was used for sample preparation. The individual materials used for preparation of samples were epoxy as resin, hardener as catalyst, carbon fibers rovings as reinforcement and polyvinyl alcohol as releasing agent. Samples were prepared using handy layup method and cured at room temperature. During fabrication, Teflon tape was used to generate artificial delamination of sizes 6.25%, 25% and 56.25% of the area of the rectangular plate. These delaminations were incorporated at the mid plane of the plate after each four layers as total layers were eight. All the models were subjected to free vibrations. The natural frequencies for first twelve modes of eight-layered carbon fiber reinforced polymer composite plate were determined experimentally with and without delam

ination. The experiments were performed using the analyzers, transducers and modal hammer as shown in Figure 1.

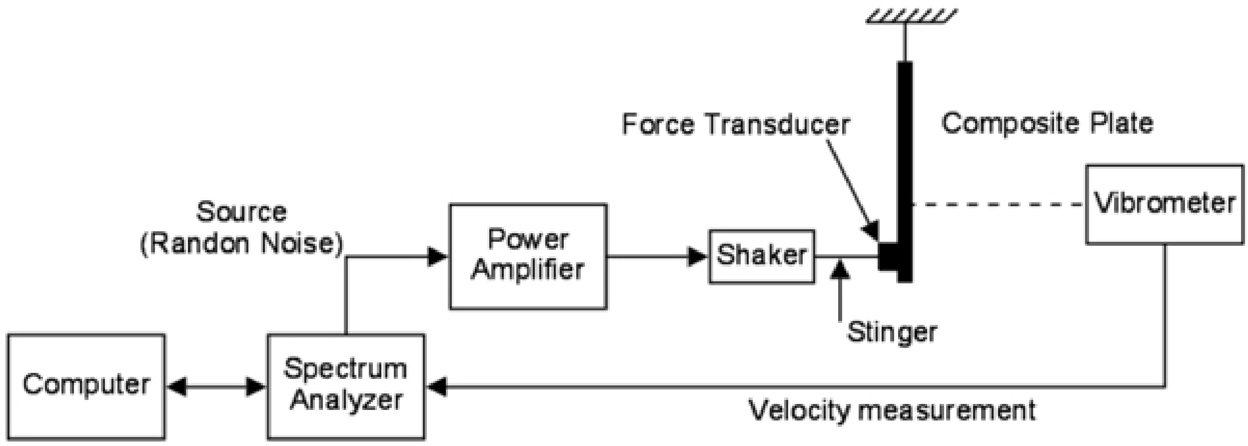


**Figure 1.** Simple layout of Experimental Setup

The LABVIEW software was used during the vibration measurements. The plates were exited using impact hammer on the random points and the required data was received through data acquisition component. Signals obtained from data acquisitions system were then processed in LABVIEW software. The acceleration signals were then processed through power spectrum module to fetch the natural frequencies along with their mode shapes.

## 4. FINITE ELEMENT ANALYSIS

Figure 2 shows the schematic diagram of delaminated composite plate with mid plane delamination. Delaminated region was modeled by merging the nodes in contact while other nodes of the area were keep unmerged considering the non-delaminated area. This methodology is extensively adapted to model the delamination in composite structures. Cohesive zone model or virtual crack closure techniques were not used because the proposed delamination were not bonded. Triangular elements were used to mesh the plate for the vibration properties of delaminated composite plate. All the modeling, meshing and simulation were done in ANSYS APDL version 17 software package. For the simulations solid elements were used in the thickness directions for each prepeg.

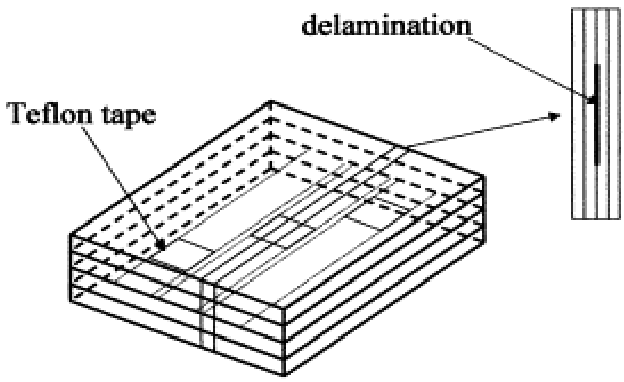


**Figure 2**. Schematic View of Delamination [25]

## 5. RESULTS AND DISCUSSION

The characterization of the vibration properties of cracked composite structures presents more difficulties than isotropic materials. This chapter presents a comparison of numerical results with experimental observations and propose a new set of estimated values based on the trends of experimental and numerical results. This method is so-called Mixed Numerical-Experimental Technique (MNET). In a

MNET, the numerical model of a Composite structure is correlated with experimental results of the composite plates. Mixed numerical-experimental technique is used for the prediction of behavior of composite structures. From an experimental side, method requires the experimental results and numerical results are required in the numerical analysis for the composite plates. The experimental and numerical results have already been concluded in earlier chapters so writing the details about plate's geometry, stacking sequences of composite plates, delamination sizes, and methodology and test procedures would not be included here again. In this chapter we will analyze the case studies for the boundary conditions CFFF, SSSS, FFFF and CCCC subjected to delamination sizes of 0%, 6.25%, 25%, 56.25% and stacking sequences of 0/90/45/90, 0/90 and 0/45. The experimental and numerical results are comprised of 48 case studies but we will study only 25% and 56.25% delamination cases here.

### 5.1 Case Study for 25% Delamination and CFFF Boundary Condition

In the Table 1, the numerical and experimental results for the first twelve natural frequencies are presented for 25% delamination size and 0/90/45/90, 0/45 and 0/90 stacking sequences subjected to CFFF boundary condition.

Change in natural frequencies is insignificant for small delamination like 0% and 6.25% delamination. Similar result was also concluded by [48-50]. Zak et al. [51] further validated this assumption through experimental technique subjected to CFFF boundary condition for the beams and plates with single edge delamination.

For the stacking sequence of 0/90/45/90, the numerically measured natural frequencies lie within 10% of the experimental results that show a great agreement. In FEM, there is significant rise in the first mode and the difference gets stable as we move towards higher modes and there is significant decrease in the natural frequencies in the twelfth mode. This predicts that the methodologies used in this research are however produced almost similar results for CFFF boundary conditions.

For the stacking sequence of 0/45, variations lie within 8% for all modes except the mode 12 where the error difference seems to be abnormal and reached 26% difference.

**Table 1.** Experimental and Numerical comparison of first 12 natural frequencies for 25% delamination and CFFF boundary condition

| | 25% Delamination | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | CFFF | | | CFFF | | | CFFF | | |
| Mode Number | 0/90/45/90 | | | 0/45 | | | 0/90 | | |
| | Experimental | FEM | % Difference | Experimental | FEM | % Difference | Experimental | FEM | % Difference |
| **1** | 76.95 | 89.65 | 16% | 78.88 | 83.17 | 5% | 87.12 | 95.29 | 9% |
| **2** | 155 | 166.80 | 8% | 180 | 195.25 | 8% | 112.23 | 125.17 | 12% |
| **3** | 536.2 | 549.77 | 3% | 483.18 | 501.09 | 4% | 566.61 | 591.74 | 4% |
| **4** | 626.69 | 645.85 | 3% | 627.36 | 647.52 | 3% | 621.57 | 635.93 | 2% |
| **5** | 663.03 | 681.54 | 3% | 699.69 | 718.01 | 3% | 630.56 | 637.90 | 1% |
| **6** | 1114.4 | 1,135.60 | 2% | 1245.1 | 1,258.50 | 1% | 999.7 | 975.26 | -2% |
| **7** | 1567.5 | 1,571.90 | 0% | 1454.8 | 1,469.50 | 1% | 1636.2 | 1,666.50 | 2% |
| **8** | 1599.6 | 1,594.80 | 0% | 1503 | 1,508.90 | 0% | 1654.3 | 1,669.60 | 1% |
| **9** | 1693.7 | 1,706.80 | 1% | 1670.2 | 1,687.70 | 1% | 1702.5 | 1,720.80 | 1% |
| **10** | 2044.6 | 2,067.90 | 1% | 2166.6 | 2,086.80 | -4% | 1940.4 | 1,923.60 | -1% |
| **11** | 2090.7 | 2,086.80 | 0% | 2251 | 2,086.80 | -7% | 1959.2 | 1,939.70 | -1% |
| **12** | 2937.2 | 2,086.80 | -29% | 2825.6 | 2,086.90 | -26% | 2575.6 | 2,086.80 | -19% |

For the stacking sequence of 0/90, all errors in mixed numerical experimental comparison lie within 12% except the mode 12 where error is 19%. Broadly analyzing the results, it can be concluded that highest mode showed highest difference between experimental and numerical results. This behavior was also predicted by the Jian et al [52] for the glass fiber composite with circular delamination. This was also concluded by [53-57].Modes 2-11 almost showed similar and stable trends. Mode 1 also presented the maximum difference of 16% in case of 0/90/45/90 stacking sequence. In composite materials analysis, error of 10-20% lies within acceptable range so the current difference can be considered as the validated methodology for numerical results.

Constraints also badly affected the natural frequencies of delaminated composite plates. This is due to the fact that an increase in the number of constraints, stiffness of the structure increases. This trend was also validated by Hirwani et al. [58]. They used in-house vibration analyzer CDAQ-9178 and found that the frequency response increased with increase in the boundary constraints.

### 5.2 Case Study for 56.25% Delamination and CCCC Boundary Condition

In the Table 2, the numerical and experimental results for the first twelve natural frequencies are presented for 56.25% delamination size and 0/90/45/90, 0/45and 0/90 stacking sequences subjected to CCCC boundary condition.

For the stacking sequence of 0/90/45/90, the numerically measured natural frequencies lie within 7% of the experimental results that show a great agreement. In FEM, natural frequencies dropped for the mode 5, 7, 9 and 12. This drop may be due to intermittent error. Difference is consistent within 5% for all modes except those specified modes.

Mode numbers 5, 7, 9 and 12 also behaved differently for the stacking sequence of 0/45, where all these modes lie within 18% range. Others modes were within 6% of the error. For the stacking sequence of 0/90, all errors in mixed numerical experimental comparison lie within 5% except the modes 5, 7 and 9 where maximum error is 20%. Lowest and highest modes showed almost similar behavior and the central modes 5-9 showed highest variation in numerical and experimental results.

**Table 2**. Experimental and Numerical comparison of first 12 natural frequencies for 56.25% delamination and CCCC boundary condition

| | 56.25% Delamination | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | CCCC | | | CCCC | | | CCCC | | |
| **Mode Nr** | 0/90/45/90 | | | 0/45 | | | 0/90 | | |
| | **Experimental** | **FEM** | **% Difference** | **Experimental** | **FEM** | **% Difference** | **Experimental** | **FEM** | **% Difference** |
| **1** | 848.38 | 887.85 | 5% | 841.28 | 882.36 | 5% | 852.39 | 892.61 | 5% |
| **2** | 1760.2 | 1,817.10 | 3% | 1733.7 | 1,795.50 | 4% | 1775 | 1,836.80 | 3% |
| **3** | 1760.2 | 1,819.80 | 3% | 1733.7 | 1,798.40 | 4% | 1775 | 1,839.30 | 4% |
| **4** | 2502.4 | 2,593.80 | 4% | 2565.8 | 2,622.00 | 2% | 2454.1 | 2,534.10 | 3% |
| **5** | 3232.5 | 2,624.80 | -19% | 3124.8 | 2,649.00 | -15% | 3299.5 | 2,626.40 | -20% |
| **6** | 3244.3 | 3,301.60 | 2% | 3140.5 | 3,214.20 | 2% | 3306.6 | 3,385.70 | 2% |
| **7** | 3826.6 | 3,312.80 | -13% | 3888.7 | 3,228.80 | -17% | 3777.8 | 3,390.90 | -10% |
| **8** | 3826.6 | 3,968.50 | 4% | 3888.7 | 4,026.50 | 4% | 3777.8 | 3,905.40 | 3% |
| **9** | 4981.7 | 3,980.80 | -20% | 4915.5 | 4,039.00 | -18% | 4841.5 | 3,917.30 | -19% |
| **10** | 5120.1 | 5,208.20 | 2% | 4915.5 | 5,124.00 | 4% | 5235.2 | 5,038.90 | -4% |
| **11** | 5120.1 | 5,302.60 | 4% | 5171.5 | 5,126.30 | -1% | 5235.2 | 5,394.00 | 3% |
| **12** | 5698.2 | 5,305.30 | -7% | 5688.9 | 5,323.50 | -6% | 5694.5 | 5,399.80 | -5% |

### 5.3 Case Study for 56.25% Delamination and CFFF Boundary Condition

Figure 1 presents the first 12 modes for 56.25% delamination, 0/90/45/90 stacking sequence and CFFF boundary condition. It clearly depicts that first mode showed higher trend in natural frequencies using FEM as compared to experimental results. There was a difference of 59% for the first mode, 31% for second mode and 5% for the third mode. The results were consistent for mode 3 and onward.

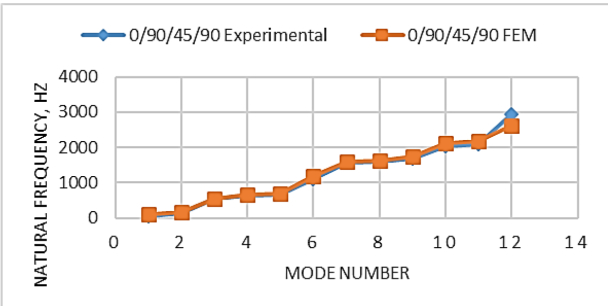


**Figure 3.** Experimental and Numerical comparison of first 12 natural frequencies for 56.25% delamination and CFFF boundary condition subjected to 0/90/45/90

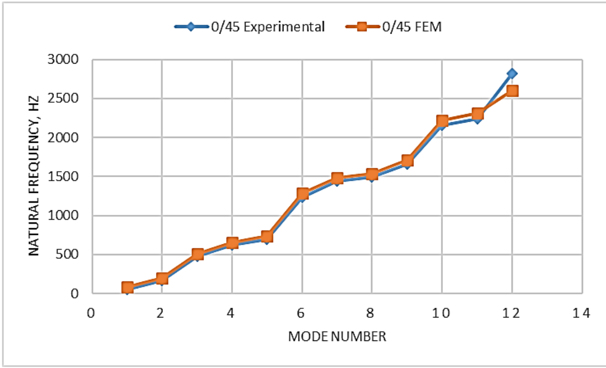


**Figure 4.** Experimental and Numerical comparison of first 12 natural frequencies for 56.25% delamination and CFFF boundary condition subjected to 0/45 stacking sequence

## 6. CONCLUSION

Overall, there was a perfect agreement between the two methods between modes 3-11. There was a difference of 11% for mode 12. Similar trend was observed for 0/45 stacking sequence as shown in Figure 2 where the first mode difference between experimental and finite element results was 70% and in second mode it reduced to 23% and in third mode it further reduced to 7% and further it became flat and difference was within 6%. So, it clearly depicts that stacking sequence has imparted significant impact on the natural frequencies of delaminated composite plate regardless of the boundary conditions.

## REFERENCES


[1] IMRAN, M., K. Rafiullah, and S. BADSHAH, Vibration Analysis of Cracked Composite Laminated Plate and Beam Structures. *Romanian Journal of Acoustics and Vibration*, 2018. **15**(1): p. 3-13.

[2] Kamar, N.T., et al., Nanoscale toughening of carbon fiber reinforced/epoxy polymer composites (CFRPs) using a triblock copolymer. *Polymer*, 2017. **111**: p. 36-47.

[3] Jiang, Z., H. Wen, and S. Ren. *Modeling delamination of FRP laminates under low velocity impact*. in *IOP Conference Series: Materials Science and Engineering*. 2017. IOP Publishing.

[4] Aksencer, T. and M. Aydogdu, Vibration of a rotating composite beam with an attached point mass. *Composite Structures*, 2018. **190**: p. 1-9.

[5] Bakis, C., et al., Fiber-reinforced polymer composites for construction-state-of-the-art review. *Journal of composites for construction*, 2002. **6**(2): p. 73-87.

[6] Kumar, N., et al., Light-weight high-strength hollow glass microspheres and bamboo fiber based hybrid polypropylene composite: A strength analysis and morphological study. *Composites Part B: Engineering*, 2017. **109**: p. 277-285.

[7] Yelve, N.P., M. Mitra, and P. Mujumdar, Detection of delamination in composite laminates using Lamb wave based nonlinear method. *Composite Structures*, 2017. **159**: p. 257-266.

[8] Saghafi, H., et al., Investigation of interleaf sequence effects on impact delamination of nano-modified woven composite laminates using cohesive zone model. *Composite Structures*, 2017. **166**: p. 49-56.

[9] Kharghani, N. and C. Guedes Soares, Behavior of composite laminates with embedded delaminations. *Composite Structures*, 2016. **150**: p. 226-239.

[10] Agarwal, B.D., L.J. Broutman, and K. Chandrashekhara, Analysis and performance of fiber composites. 2017: John Wiley & Sons.

[11] Imran, M., Pre-stress and Free Vibration Optimization of Composite Ocean Current Turbine Blade. *International Journal of Science, Engineering and Innovative Research*, 2015. **3**: p. 1-5.

[12] Asmatulu, E., et al. *Sustainability of fiber reinforced laminate and honeycomb composites in manufacturing industries*. in *Behavior and Mechanics of Multifunctional Materials and Composites XII*. 2018. International Society for Optics and Photonics.

[13] Imran, M., R. Khan, and S. Badshah, Finite Element Analysis to Investigate the Influence of Delamination Size, Stacking Sequence and Boundary Conditions on the Vibration Behavior of Composite Plate. *Iranian Journal of Materials Science & Engineering*, 2018. **15**(3): p. 0-0.

[14] Venkate Gowda, C., N. Rajanna, and N.G.S. Udupa, Investigating the effects of delamination location and size on the vibration behaviour of laminated composite beams. *Materials Today: Proceedings*, 2017. **4**(10): p. 10944-10951.

[15] Muhammad Imran, R.K., Saeed Badshah, Vibrtaion Analysis of Cracked Composite Laminated Plate. *Pakistan Journal of Scientific and Industrial Research Series A: Physical Sciences* 2018. **61(A)**(2).

[16] Shao, D., et al., Free vibration of refined higher-order shear deformation composite laminated beams with general boundary conditions. *Composites Part B: Engineering*, 2017. **108**: p. 75-90.

[17] Imrana, M., R. Khana, and S. Badshaha, Investigating the Effect of Delamination Size, Stacking Sequences and Boundary Conditions on The Vibration Properties of Carbon Fiber Reinforced Polymer Composite. *Materials Research*, 2019. **22**(2): p. e20190478.

[18] Muhammad Imran, R.K., Saeed Badshah, Finite Element Analysis to Investigate the Influence of Delamination Size, Stacking Sequence and Boundary Conditions on the Vibration Behavior of Composite Plate. *Iranian Journal of Materials Science and Engineering*, 2019. **16**(1).

[19] Shu, D. and C.N. Della, Vibrations of multiple delaminated beams. *Composite Structures*, 2004. **64**(3–4): p. 467-477.

[20] Jadhav, V. and D.M. Bhoomkar, Experimental and Numerical FEM Analysis of Cracked Composite Cantilever Beam by Vibration Techniques. *International Journal of Engineering Science*, 2016. **3347**.

[21] Lee, S., T. Park, and G.Z. Voyiadjis, Free vibration analysis of axially compressed laminated composite beam-columns with multiple delaminations. *Composites Part B: Engineering*, 2002. **33**(8): p. 605-617.

[22] Kim, H.-Y. and W. Hwang, Effect of debonding on natural frequencies and frequency response functions of honeycomb sandwich beams. *Composite Structures*, 2002. **55**(1): p. 51-62.

[23] Brandinelli, L. and R. Massabò, Free vibrations of delaminated beam-type structures with crack bridging. *Composite Structures*, 2003. **61**(1–2): p. 129-142.

[24] Luo, H., et al. *Delaminated beam nonlinear dynamic response calculation and visualization*. in *38th Structures, Structural Dynamics, and Materials Conference*. 1997.

[25] Lee, S., T. Park, and G.Z. Voyiadjis, Vibration analysis of multi-delaminated beams. *Composites Part B: Engineering*, 2003. **34**(7): p. 647-659.

[26] Luo, H. and S. Hanagud, Dynamics of delaminated beams. *International Journal of Solids and Structures*, 2000. **37**(10): p. 1501-1519.

[27] Chawla, K. and S. Ray-Chaudhuri, Effect of Cut-Out and Delamination on Modal Properties of Singly-and Doubly-Curved Composite Plates. *Procedia Engineering*, 2017. **199**: p. 1982-1987.

[28] Thornburgh, R. and A. Chattopadhyay. *Combined delamination and matrix cracking in adaptive composite laminates*. in *44th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference*. 2003.

[29] Thornburgh, R. and A. Chattopadhyay, Unified approach to modeling matrix cracking and delamination in laminated composite structures. *AIAA journal*, 2001. **39**(1): p. 153-160.

[30] Yashavantha Kumar, G.A. and K.M. Sathish Kumar, Free vibration analysis of smart composite beam. *Materials Today: Proceedings*, 2017. **4**(2, Part A): p. 2487-2491.

[31] Mohammed, D., Effect of Fiber Angles on Dynamic Response of Cantilever Composite Beams. *ZANCO Journal of Pure and Applied Sciences*, 2017. **29**(1): p. 157-163.

[32] Mallik, P.K.S. and D.S. Rao, VIBRATION CONTROL ON COMPOSITE BEAMS WITH MULTIPLE PIEZOELECTRIC PATCHES USING FINITE ELEMENT ANALYSIS. 2017.

[33] Shukla, A. and S.P. Harsha, Vibration Response Analysis of Last Stage LP Turbine Blades for Variable Size of Crack in Root. *Procedia Technology*, 2016. **23**: p. 232-239.

[34] Yurddaskal, M., et al., The effect of foam properties on vibration response of curved sandwich composite panels. *Composite Structures*, 2018. **183**: p. 278-285.

[35] Juhász, Z., et al., Sensitivity analysis for frequency based prediction of crack size in composite plates with through-the-width delamination. *International Journal of Damage Mechanics*, 2017: p. 1056789517709893.

[36] Hirwani, C.K., et al., Experimental and numerical analysis of free vibration of delaminated curved panel. *Aerospace Science and Technology*, 2016. **54**: p. 353-370.

[37] Sadeghpour, E., M. Sadighi, and A. Ohadi, Free vibration analysis of a debonded curved sandwich beam. *European Journal of Mechanics-A/Solids*, 2016. **57**: p. 71-84.

[38] Zhang, Z., et al., Vibration-based assessment of delaminations in FRP composite plates. *Composites Part B: Engineering*, 2018. **144**: p. 254-266.

[39] Hirwani, C., S. Sahoo, and S. Panda. *Effect of delamination on vibration behaviour of woven Glass/Epoxy composite plate-An experimental study*. in *IOP Conference Series: Materials Science and Engineering*. 2016. IOP Publishing.

[40] Vo, T.P., H.-T. Thai, and M. Aydogdu, Free vibration of axially loaded composite beams using a four-unknown shear and normal deformation theory. *Composite Structures*, 2017. **178**: p. 406-414.

[41] Zhu, P., Z.X. Lei, and K.M. Liew, Static and free vibration analyses of carbon nanotube-reinforced composite plates using finite element method with first order shear deformation plate theory. *Composite Structures*, 2012. **94**(4): p. 1450-1460.

[42] Imran, M., R. Khan, and S. Badshah, Experimental, Numerical and Finite Element Vibration Analysis of Delaminated Composite Plate. *Scientia Iranica*, 2019: p. -.

[43] Imran, M., R. Khan, and S. Badshah, Investigating the Effect of Delamination Size, Stacking Sequences and Boundary Conditions on The Vibration Properties of Carbon Fiber Reinforced Polymer Composite. *Materials Research*, 2019. **22**(2).

[44] Vescovini, R., et al., On the application of the Ritz method to free vibration and buckling analysis of highly anisotropic plates. *Composite Structures*, 2018. **192**: p. 460-474.

[45] Oliveri, V. and A. Milazzo, A Rayleigh-Ritz approach for postbuckling analysis of variable angle tow composite stiffened panels. *Computers & Structures*, 2018. **196**: p. 263-276.

[46] Sayyad, A.S. and Y.M. Ghugal, Bending, buckling and free vibration of laminated composite and sandwich beams: A critical review of literature. *Composite Structures*, 2017. **171**: p. 486-504.

[47] Ardestani, M.M., L. Zhang, and K. Liew, Isogeometric analysis of the effect of CNT orientation on the static and vibration behaviors of CNT-reinforced skew composite plates. *Computer Methods in Applied Mechanics and Engineering*, 2017. **317**: p. 341-379.

[48] Pardoen, G.C., Effect of delamination on the natural frequencies of composite laminates. *Journal of composite materials*, 1989. **23**(12): p. 1200-1215.

[49] Shen, M.-H. and J. Grady, Free vibrations of delaminated beams. *AIAA journal*, 1992. **30**(5): p. 1361-1370.

[50] Saravanos, D. and D. Hopkins, Effects of delaminations on the damped dynamic characteristics of composite laminates: analysis and experiments. *Journal of Sound and Vibration*, 1996. **192**(5): p. 977-993.

[51] Żak, A., M. Krawczuk, and W. Ostachowicz, Numerical and experimental investigation of free vibration of multilayer delaminated composite beams and plates. *Computational Mechanics*, 2000. **26**(3): p. 309-315.

[52] Jian, X., et al., Damage detection by piezoelectric patches in a free vibration method. *Journal of composite materials*, 1997. **31**(4): p. 345-359.

[53] Tenek, L.H., E.G. Henneke II, and M.D. Gunzburger, Vibration of delaminated composite plates and some applications to non-destructive testing. *Composite Structures*, 1993. **23**(3): p. 253-262.

[54] Kim, H.S., A. Chattopadhyay, and A. Ghoshal, Dynamic analysis of composite laminates with multiple delamination using improved layerwise theory. *AIAA journal*, 2003. **41**(9): p. 1771-1779.

[55] Paolozzi, A. and I. Peroni, Experimental assessment of debonding damage in a carbon-fibre reinforced plastic sandwich panel by frequency variations. *Composite structures*, 1996. **35**(4): p. 435-444.

[56] Valdes, S.D. and C. Soutis, Delamination detection in composite laminates from variations of their modal characteristics. *Journal of sound and vibration*, 1999. **228**(1): p. 1-9.

[57] Kessler, S.S., et al., Damage detection in composite materials using frequency response methods. *Composites Part B: Engineering*, 2002. **33**(1): p. 87-95.

[58] Hirwani, C.K., S.K. Panda, and T.R. Mahapatra, Nonlinear finite element analysis of transient behavior of delaminated composite plate. *Journal of Vibration and Acoustics*, 2018. **140**(2): p. 021001.