
A Cost Function to Assess Cracks in Simply Supported Beams with Artificial Intelligence

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Abstract: - In this paper, we propose a model for detecting transverse cracks in simply supported beams, which can be part of more complex structural systems. The relative frequency shifts of the structure are considered a basis for damage identification. An original method developed by the authors is employed to evaluate the required modal parameters. A multi-stage optimization approach based on the rigidity loss suffered by the affected structure is employed to accurately recognize the locations of potential cracks. The outcome presented in this research shows the computational ability of the proposed model to indicate the presence and location of damages in beam-like structures.

Keywords: - frequency analysis, damage detection, simply supported beam, relative frequency shift curve

1. INTRODUCTION

The integrity of structural elements used in chemical installations can be affected during the operation by overloads, environmental factors, corrosion, or improper production methods [1]. To prevent possible catastrophes or long maintenance periods to replace the affected structures, it is necessary to monitor them during operation [2]. The aim to detect damages that can occur timely, thus preventing their spread. In this way, maintenance costs and time will be reduced, and the possibility of injuries and loss of life to be avoided.

Transverse cracks locally modify the geometry of a structure due to the existence of discontinuities affecting the global stiffness of the structure. Cracks lead to changes in modal parameters, among which the most affected are the eigenfrequencies, modal shapes, and curvatures [3], because the stiffness loss

decreases the quantity of energy that a structure can store [4]. If a dynamic analysis is performed, the effect of a crack is best observable by the eigenfrequency drop, whose value depends on the position [5] and the severity [6] of the crack. Numerous techniques based on modal parameter changes have been developed for detecting damage in real structures, see for instance [7-10]. Recently, the most of them involve artificial intelligence [11-13] but they still present a few shortcomings, the incorrect results achieved results leading to false damage detection [14].

This research defines a reliable cost function that considers the eigenfrequencies of the healthy and damaged beam. The existence of the cost function transforms damage detection in an optimization problem, and artificial intelligence can so be easily involved.

2. PROBLEM FORMULATION

The study is made by involving a simply supported beam which has the normalized length $L=1$. The crack is located at a normalized distance x and has the depth a . The relation to calculate the frequency of the healthy beam f_{i-U} , for any mode i , is

$$f_{i-U} = \frac{\lambda_i^2}{2\pi} \sqrt{\frac{EI}{\rho AL^4}} \quad (1)$$

where λ_i is the eigenvalue for i -th mode, E is the Young modulus, I is the moment of inertia, ρ is the density and A is the cross-section of the beam.

The relation to calculate the frequency of the damaged beam f_{i-D} is given in [15] as:

$$f_{i-D}(x, a) = f_i \left\{ 1 - \gamma(0, a) \cdot [\bar{\phi}_i''(x)]^2 \right\} \quad (2)$$

The two terms in the bracket in Eq.(2) are the severity $\gamma(0, a)$ and the modal normalized curvature $\bar{\phi}_i''(x)$. In order to evaluate the results, for the damaged beam cases, the term relative frequency shift (RFS) is introduced, defined with the relation contrived by our research group [15]:

$$\Delta \bar{f}_i(x, a) = \frac{f_{i-U} - f_{i-D}(x, a)}{f_{i-U}} = \gamma(0, a) \cdot [\bar{\phi}_i''(x)]^2 \quad (3)$$

The modal curvature for the simply supported beam is given by the relation:

$$\bar{\phi}_i''(x) = -\sin(\alpha x) \quad (4)$$

Our research team developed a simple algorithm for determining the severity $\gamma(a)$ for different transversal cracks of depth a , that takes into consideration undamaged and damaged beam deflections. Following relation applies [16]:

$$\gamma(0, a) = \frac{\sqrt{\delta_D(0, a)} - \sqrt{\delta_U}}{\sqrt{\delta_D(0, a)}} \quad (5)$$

In Eq.(5) we denoted with δ_U the beam's free-end deflection under dead mass and $\delta_D(0, a)$ is the same deflection for the beam having a crack with depth a located at the position where the biggest curvature is achieved, e.g. the mid-span for the simply supported beam. The damage severity $\gamma(0, a)$ is the function representing the highest stiffness decrease due to a crack of depth a .

3. PROBLEM SOLUTION

The model presented here centers around the modifications of global or individual situations in order to detect the presence, severity and location of cracks along the evaluated structure.

The cost function for global damage detection is based on the eigenfrequencies. First, we calculate the RFSs for numerous damage locations and depths for a given number of vibration modes and obtain plots like the ones represented in Fig.1 at the left column. The calculus is made involving Eq. (3), the data used are the severity and beam curvature. Also, by involving Eq. (3), but with the measured frequencies, we calculate the measured RFS, which contains one value per vibration mode. Finally, we calculate a distance between the numerous calculated RFSs and the measured one, with the mathematical relation:

$$D_i(x, a) = \frac{f_{i-U} - f_{i-D} - \gamma(0, a) \cdot [\bar{\phi}_i''(x)]^2}{f_{i-U}} \quad (6)$$

The calculus is made for all combination of locations x and depths a . The results are graphically represented in the right column in Fig 1.

Further, we calculate the sum of $D_i(x, a)$ for all modes for a given combination of crack location and depth, with the relation:

$$D(x, a) = \left(\sum_{i=1}^n [D_i(x, a)]^j \right)^{1/k} \quad (7)$$

In this way, we obtain a value that indicates how well the measured RFSs fit with a set of calculated RFSs for location x and depth a . By setting the power j and k , we can define if we search a minimum or a maximum. So, adjusting these two constants, we can control the shape of the cost function to present relevant minima or maxima.

The distance we imagined is derived from the Euclidean distance, but the power order j and k is not necessarily equal with two, therefore is it a generalization of it.

The values calculated using Eq. (7) are presented in a graphical form in Figs. 2 and 3. The constants j and k are set as follows:

- Euclidean distance: $j=2$ and $k=2$, Fig. 2 and 3;
- Proposed distance: $j=4$ and $k=-4$, Fig. 4.

The normalized damage parameters are $x=0.25$ and $a=0.09$, so the RFSs considered as being measured exactly fit a calculated RFS. The case is marked with yellow in Table 1.

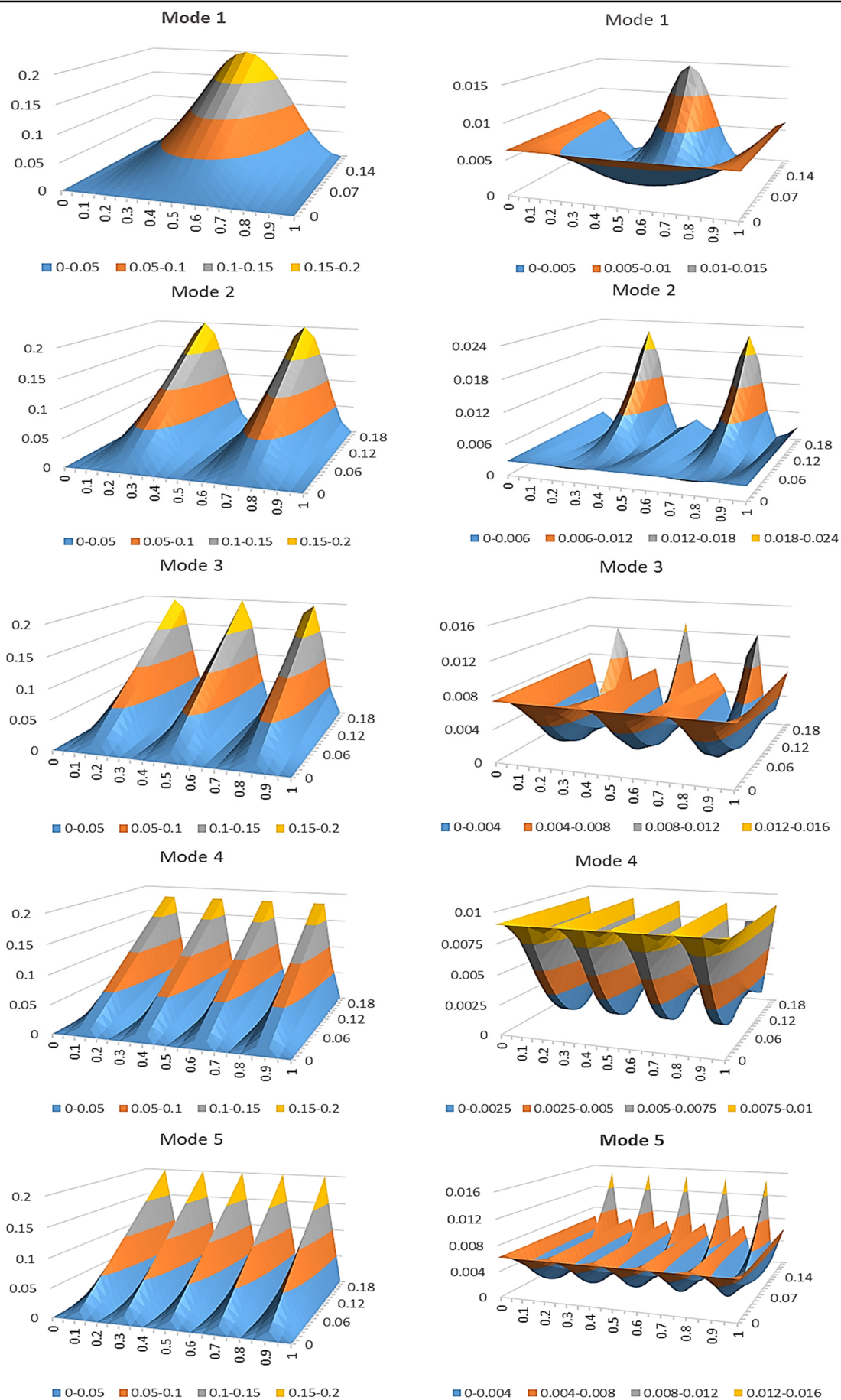


Figure 1. RFS differences calculated with Eq.(6) for $j=2$.

Table 1 presents the calculated values for the distances $D(x,a)$ for all combination of locations x and all possible depths a . Due to symmetry, it is sufficient to consider the half-beam. The highlighted values in the table represent the distance calculated with the measured RFSs, as follows:

- yellow marker is used for the case the measurement fit a calculated scenario;
- pink marker shows the limits in which frames the measured RFSs relative to the calculated one.

Table 1. Calculated values for multiple damage combinations of x and a for mode 1

$x \backslash \gamma(a)$	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
0	0	0	0	0	0	0	0	0	0	0	0
0.01	0	0.000245	0.000955	0.002061	0.003455	0.005	0.006545	0.007939	0.009045	0.009755	0.01
0.02	0	0.000489	0.00191	0.004122	0.00691	0.01	0.01309	0.015878	0.01809	0.019511	0.02
0.03	0	0.000734	0.002865	0.006183	0.010365	0.015	0.019635	0.023817	0.027135	0.029266	0.03
0.04	0	0.000979	0.00382	0.008244	0.01382	0.02	0.02618	0.031756	0.03618	0.039021	0.04
0.05	0	0.001224	0.004775	0.010305	0.017275	0.025	0.032725	0.039695	0.045225	0.048776	0.05
0.06	0	0.001468	0.005729	0.012366	0.020729	0.03	0.039271	0.047634	0.054271	0.058532	0.06
0.07	0	0.001713	0.006684	0.014428	0.024184	0.035	0.045816	0.055572	0.063316	0.068287	0.07
0.08	0	0.001958	0.007639	0.016489	0.027639	0.04	0.052361	0.063511	0.072361	0.078042	0.08
0.09	0	0.002202	0.008594	0.01855	0.031094	0.045	0.058906	0.07145	0.081406	0.087798	0.09
0.1	0	0.002447	0.009549	0.020611	0.034549	0.05	0.065451	0.079389	0.090451	0.097553	0.1
0.11	0	0.002692	0.010504	0.022672	0.038004	0.055	0.071996	0.087328	0.099496	0.107308	0.11
0.12	0	0.002937	0.011459	0.024733	0.041459	0.06	0.078541	0.095267	0.108541	0.117063	0.12
0.13	0	0.003181	0.012414	0.026794	0.044914	0.065	0.085086	0.103206	0.117586	0.126819	0.13
0.14	0	0.003426	0.013369	0.028855	0.048369	0.07	0.091631	0.111145	0.126631	0.136574	0.14
0.15	0	0.003671	0.014324	0.030916	0.051824	0.075	0.098176	0.119084	0.135676	0.146329	0.15
0.16	0	0.003915	0.015279	0.032977	0.055279	0.08	0.104721	0.127023	0.144721	0.156085	0.16
0.17	0	0.00416	0.016234	0.035038	0.058734	0.085	0.111266	0.134962	0.153766	0.16584	0.17
0.18	0	0.004405	0.017188	0.037099	0.062188	0.09	0.117812	0.142901	0.162812	0.175595	0.18
0.19	0	0.00465	0.018143	0.03916	0.065643	0.095	0.124357	0.15084	0.171857	0.18535	0.19
0.2	0	0.004894	0.019098	0.041221	0.069098	0.1	0.130902	0.158779	0.180902	0.195106	0.2

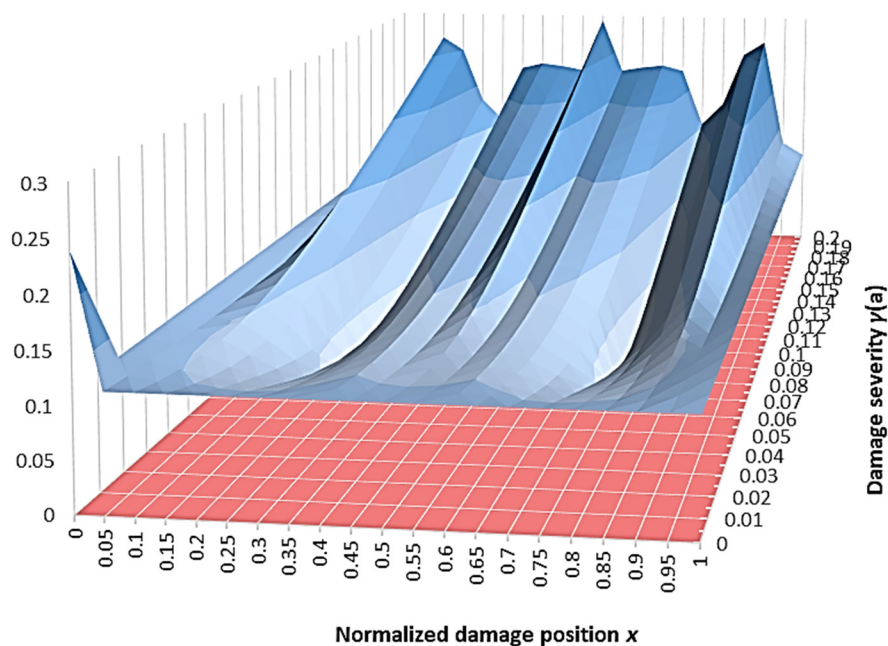


Figure 2. Cost function (3D view) for damage parameters $x=0.25$ and $a=0.09$ considering constants $j=2$ and $k=2$

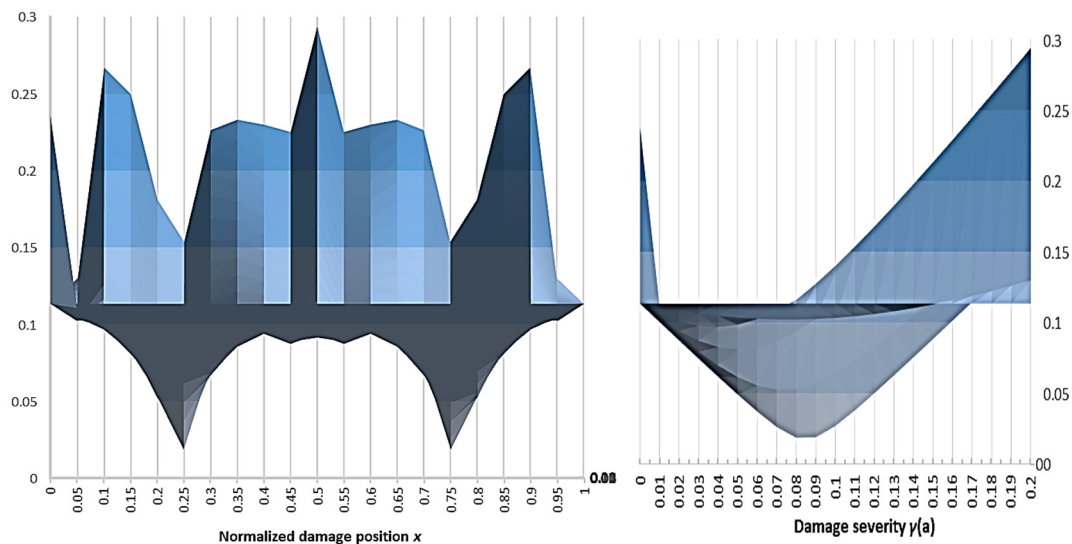


Figure 3. Cost function (lateral views) for damage parameters $x=0.25$ and $a=0.09$ considering constants $j=2$ and $k=2$

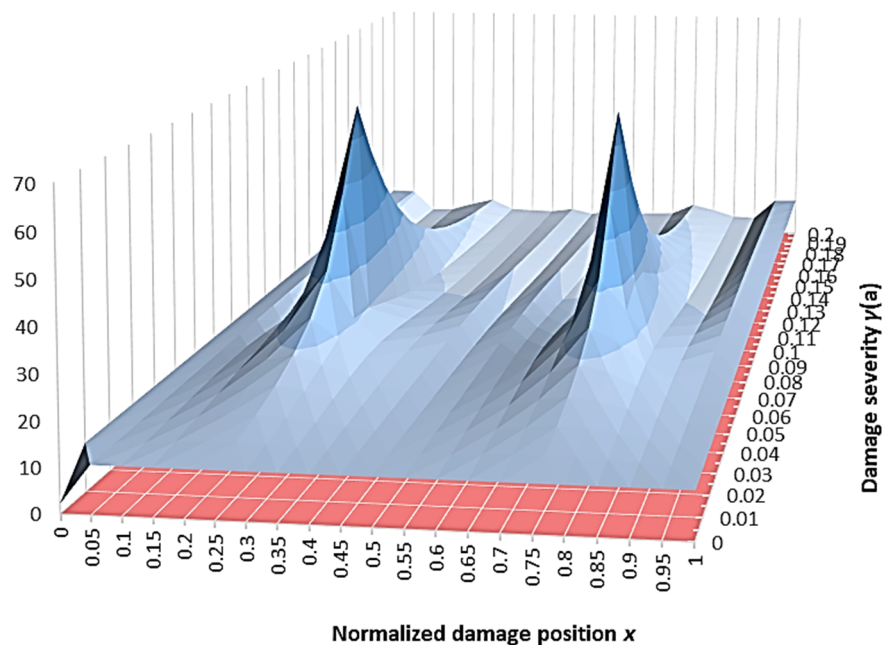


Figure 4. Cost function (3D view) for damage parameters $x=0.25$ and $a=0.09$ considering constants $j=4$ and $k=-4$

In Fig. 2 to 4, due to symmetry of the structure, we obtained two minima or maxima at mirrored positions. One can observe that, for the same damage parameters, by adjusting the constants k and j , we can control the shape of the cost function in order to present a more relevant minima or maxima. For instance, considering the Euclidean distance, we obtain approximately the same values of $D(0.25,0.08)$ and $D(0.25,0.09)$, which makes difficult finding the point of extrema. This can be observed in the lateral views in Fig. 3. Dissimilar, for $j=4$ and $k=-4$ we obtain a clear maximum, observable in Fig. 4.

To determine if the cost function provides a surface with a clear minimum or maximum, we also

tested it for measured RFSs which differ from the calculated RFSs because the measured one are for a crack that has different x and a as those considered to calculate RFSs. The considered measured values are the ones highlighted with magenta in Table 1. The damage parameters are x between 0.35 and 0.4, respectively a between 0.1 and 0.11. Thus, the measured RFSs do not fit exactly a calculated one. The values of the cost function calculated with Eq. (7), for $j=4$ and $k=-4$, are presented in Fig. 5.

After numerous simulations with different values of the constants, we observed that the higher the value of j , the clearer the diagram. This demonstrates that the method must be further studied in order to permit achieving best results.

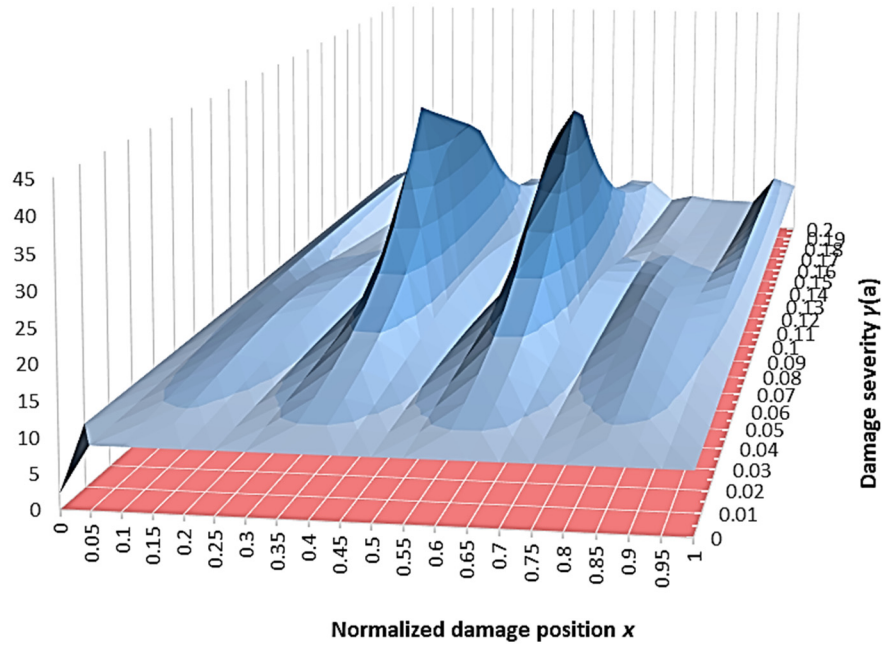


Figure 5. Cost function (3D view) for damage located between $x=0.35$ and 0.4 and having the depth between $a=0.1$ and 0.11 , calculated with the constants $j=4$ and $k=-4$

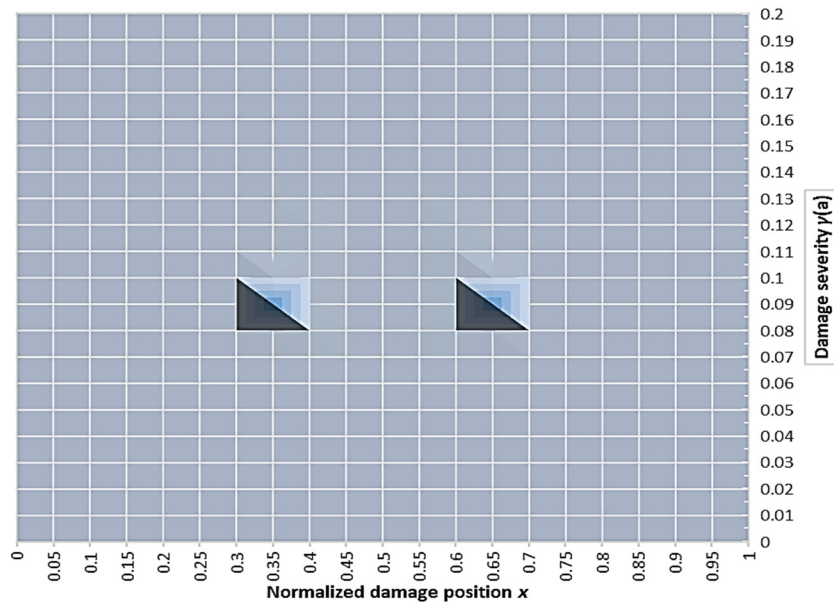


Figure 6. Top view of the cost function for a damage located between $x=0.35$ and 0.4 and having the depth between $a=0.1$ and 0.11 , calculated with the constants $j=50$ and $k=-4$

One of the simulation results, when we consider the power $j=50$, is represented in Fig. 6. This top view demonstrates that it is possible to find a damage even if its location and depth are not identical to any of the positions and depths of the RFSs used in the calculation of the cost function. The assumed location and depth of damage are indicated by the rectangle in Fig. 6, where the cost function has the highest values. In this case, we can observe that the location is accurately found, while the damage depth is underestimated.

It is the intention of the authors to find out the accuracy of the method for any possible damage cases that include simple damages [17], branched cracks [18], and multiple cracks [19]. We also intend to approach by this method the case of continuous beams [20], i.e. beams with multiple supports. Another special case we target, and which presents particularities different to all other before-mentioned cases, is that of structures with non-ideal boundary conditions [21].

4. CONCLUSIONS

In the current paper, the RFSs of simply supported beams affected by cracks of different depths and positions are calculated for the first five transversal modes of vibration. The calculus is made by involving the severity and the beam curvature. We propose a new distance and use it to calculate the distance between the numerous calculated RFSs and the measured one, for the five considered vibration modes. This is actually a cost function, which can be used to estimate the damage location and depth.

We have found that, the damage index $D(x,a)$ can be made more evident by setting the power j and k in a proper manner, so that a more relevant minimum or maximum is achieved. From our studies resulted that the negative power j produces a maximum in the cost function which is always more clearly the minimum obtained when using a positive power j .

In our further research, we intend to determine the optimal setting for the power j and k and to estimate the accuracy of the assessment method.

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