An Analytical Solution of Dynamic Vibration Absorber to Suppress the Vibration of a Pendulum Structure Subjected to Moving Loads Caused by A Hanging Point

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Abstract: - In reality, there are many real structures are shaped like a pendulum structure, such as ropeway carriers, cranes, balloon baskets, boats, etc. These pendulum structures are often hung on moving points such as cable, balloons, water surface, etc. It is this movement of this hanging point that generates an inertial force acting on the pendulum structure and produces vibrations. Therefore, this study proposes a new approach, in which a pendulum structure installs a dynamic vibration absorber (DVA) subjected to moving loads caused by a hanging point. The new studies are performed as follows: In the first step, the differential equations of motions for the pendulum structure and DVA are established, this is an extremely important step for designing the DVA's optimum parameters to suppress vibration of the pendulum structure. In the next step, the minimum quadratic torque method is used to determine the DVA's optimum parameters. The DVA's optimum parameters are obtained explicit analytical solutions. In order for the scientist can advantage to find the DVA's optimum parameters to suppress vibration of the pendulum structure. The last step, the vibrations of the system are simulated by using Maple software in order to evaluate the effect of reducing vibration for the pendulum structure. Simulation results show that the vibration of the pendulum structure is efficient suppression by using optimum parameters of the DVA.

Keywords: -dynamic vibration absorber, inertial force, optimal parameters, pendulum structure, hanging point

1. INTRODUCTION

One of the most popular methods of dissipating vibration energy in main structures is to use a dynamic vibration absorber (DVA) or a tuned mass damper (TMD), which consists of masses attached to main structures through springs and dampers, they have been widely used to eliminate vibration for many mechanical devices and engineering structures.

Wong and Cheung [1] have designed the optimal DVA to eliminate vibration for the structure subjected to moving loads caused by a ground. Chung et al. [2] studied an optimal design theory for a friction pendulum tuned mass. Morga and Marano [3] proposed the reduction of vibrations for the slender structures. Elias and Matsagar [4] introduced single tuned mass damper for high-rise building. Xiang and Nishitani [5] presented a pendulum-type nontraditional tuned mass damper for a system equipped.

Love et al. [6] have designed the DVA to reduce vibration for the tall buildings. Chen et al. [7] studied bridge-based designed TMD on the trains. Chinh [8, 12] have introduced a symmetric TMD to eliminate torsional vibration of the machine shaft.

Zhang [9] has provided explicit formulas for optimal tuning of the TMD for wind turbine blades. Chinh [10] gave the optimal parameters of DVA to control torsional vibration of a rotating shaft. The optimal parameters of TMD have been found by Chinh [11] to suppress vibrations of an inverted pendulum has two degrees of freedom.

Figure 1. Model real structures of a ropeway carrier

In reality, there are many real structures are shaped like a pendulum structure, such as ropeway carriers, cranes, balloon baskets, boats, etc. For example, the model real structures of a ropeway carrie is shown in Figure 1.

These pendulum structures are often hung on moving points such as cable, balloons, water surface, etc. It is this movement of this hanging point that generates an inertial force acting on the pendulum structure and produces vibrations. Therefore, this study proposes a new approach, in which a pendulum structure installs a dynamic vibration absorber (DVA) subjected to moving loads caused by a hanging point.

Vibration equations are formulated for the system; this is an extremely important step for designing the DVA's optimum parameters to suppress vibration of the pendulum structure. In particular, the DVA's optimum parameters are obtained explicit analytical solutions.

In order for the scientist can advantage to find the DVA's optimum parameters to suppress vibration of the pendulum structure subjected to moving loads caused by a hanging point.

Finally, the vibrations of the system are simulated by using Maple software in order to evaluate the effect of reducing vibration for the pendulum structure.

Simulation results show that the vibration of the pendulum structure is efficient suppression by using optimum parameters of the DVA.

2. MODELING OF A PENDULUM STRUCTURE AND VIBRATION EQUATIONS

2.1. Modeling of a pendulum structure attached with DVA

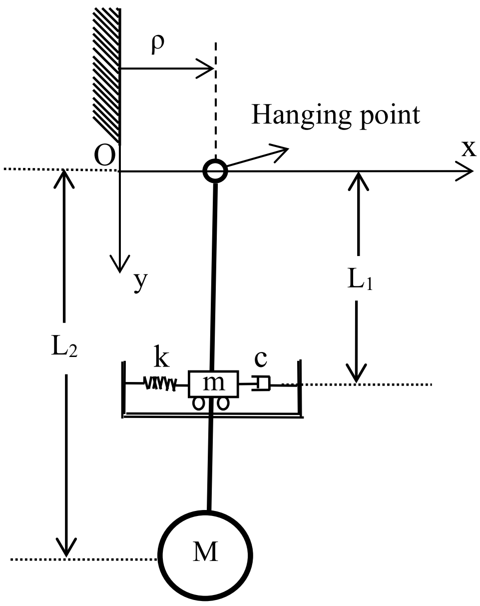

Figure 2. Modelling of a pendulum structure attached with DVA

Figure 2 shows a pendulum structure attached with a dynamic vibration absorber (DVA). The DVA with the mass of m, the spring constant of k, and the damping constant of c. The position of the DVA with respect to a hanging point as L_1. The mass and length of the pendulum structure are M and L_2, respectively. Let ρ denote the displacement of the hanging point.

Figure 3 shows the system has 2 degrees of freedom. Denote the angular variation of the pendulum structure as φ. Denote the displacement of DVA with respect to the pendulum structure as s.

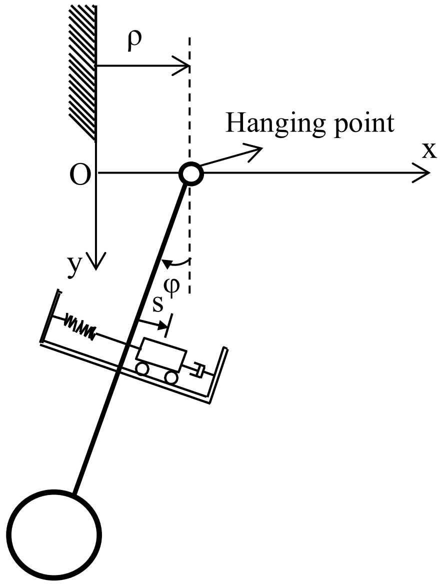

Figure 3. The pendulum structure and the DVA with two degrees of freedom

2.2. Vibration equations

The pendulum structure with DVA has 2 degrees of freedom: S and φ. The Lagrange equations are given by

$$\begin{aligned} &\frac{d}{dt}\left(\frac{\partial(T-V)}{\partial\dot{S}}\right)-\frac{\partial(T-V)}{\partial S}+\frac{\partial\Phi}{\partial\dot{S}}=0 \\ &\frac{d}{dt}\left(\frac{\partial(T-V)}{\partial\dot{\varphi}}\right)-\frac{\partial(T-V)}{\partial\varphi}+\frac{\partial\Phi}{\partial\dot{\varphi}}=0 \end{aligned} \tag{1}$$

Consider the coordinate system as shown in Figure 2, the positions of the pendulum structure (x_M, y_M) and the DVA (x_m, y_m) are determined as

$$\begin{aligned} &x_M=\rho-L_2\sin\varphi;\ x_m=\rho-L_1\sin\varphi+S\cos\varphi \\ &y_M=L_2\cos\varphi;\ y_m=L_1\cos\varphi+S\sin\varphi \end{aligned} \tag{2}$$

* The kinetic energy of the pendulum structure with DVA is

$$T=\frac{1}{2}M(\dot{x}_M^2+\dot{y}_M^2)+\frac{1}{2}m(\dot{x}_m^2+\dot{y}_m^2) \tag{3}$$

* The potential energy of the pendulum structure with DVA is

$$V=\frac{1}{2}kS^2+Mg(L_2-y_M)+mg(L_1-y_m) \tag{4}$$

* The energy dissipation function of the pendulum structure with DVA is

$$\Phi = \frac{1}{2} c \dot{S}^2 \quad (5)$$

Using (1) to (5) and ignoring the high power terms, the motion equations of the pendulum structure with DVA are written as follows

$$\begin{aligned} &(ML_2^2 + mL_1^2)\ddot{\varphi} + mL_1\ddot{S} + (ML_2 + mL_1)g\varphi \\ &\qquad + mgS = -(ML_2 + mL_1)\ddot{\rho} \\ &mL_1\ddot{\varphi} + m\ddot{S} + mg\varphi + kS + c\dot{S} = -m\ddot{\rho} \end{aligned} \quad (6)$$

The right-hand side in the Expressions of equation (6) represent the inertial forces caused by the angular acceleration of the hanging point as $\ddot{\rho}$, it is this inertial force that causes vibrations for the pendulum structure. Therefore, the equation (6) is used to design the DVA.

2.3. The vibrations of the pendulum structure attached with normal DVA

Vibration simulation with the data of the pendulum structure are summarized in Table 1. In this section, the normal DVA is attached to the pendulum structure to reduce vibration of the system. The normal DVA's parameters are randomly selected, is not optimally designed, as shown in Table 2.

Table 1. The input parameters for the pendulum structure

Description	Parameters	Value	Unit
Mass of the pendulum structure	M	800.0	kg
Length of the pendulum structure	L_2	4.0	m

Table 2. The input parameters for the normal DVA

Description	Parameters	Value	Unit
Mass of DVA	m	16	kg
The position of DVA	L_1	3.5	m
The damping constant of DVA	c	3.5	Ns/m
The spring constant of DVA	k	30	N/m

The vibrations of the pendulum structure with and without DVA are performed. Each numerical simulation is considered under three cases: The pendulum structure with initial deflections, initial velocities and with both the initial deflections and initial velocities.

Case 1. The pendulum structure attached with the normal DVA with initial deflection

The pendulum structure's parameters and the normal DVA's parameters are identified in Tables 1, 2 and equations (6). The initial conditions of the pendulum structure are setup with $\dot{\varphi}_0 = 0.0(rad/s)$ and $\varphi_0 = 5.5\times10^{-3}(rad)$. The vibrations of the pendulum structure and the normal DVA are expressed in the Figures (4)-(5).

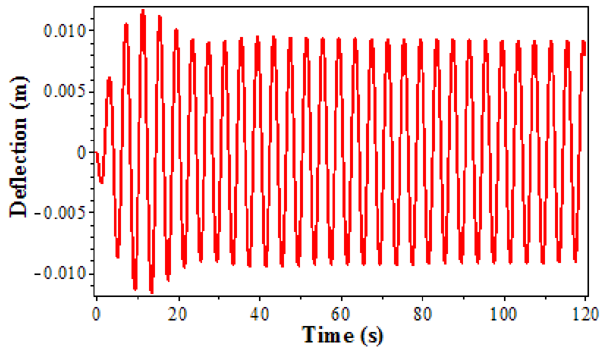

Figure 4. The normal DVA's vibration with case

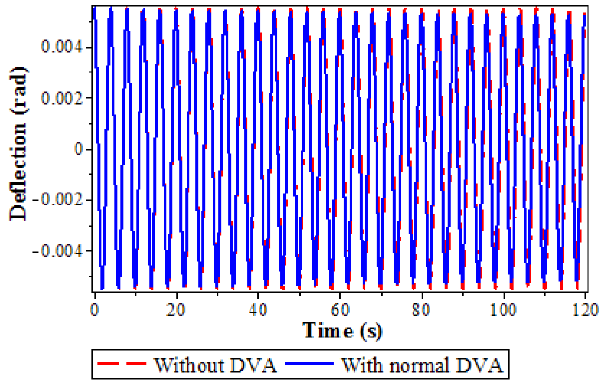

Figure 5. The pendulum structure's vibration with case 1

Case 2. The pendulum structure attached with the normal DVA with initial velocity

The pendulum structure's parameters and the normal DVA's parameters are identified in Tables 1, 2 and equations (6). The initial conditions of the pendulum structure are setup with $\dot{\varphi}_0 = 6.5\times10^{-3}(rad/s)$ and $\varphi_0 = 0.0(rad)$. The vibrations of the pendulum structure and the normal DVA are simulated in the Figures (6)-(7).

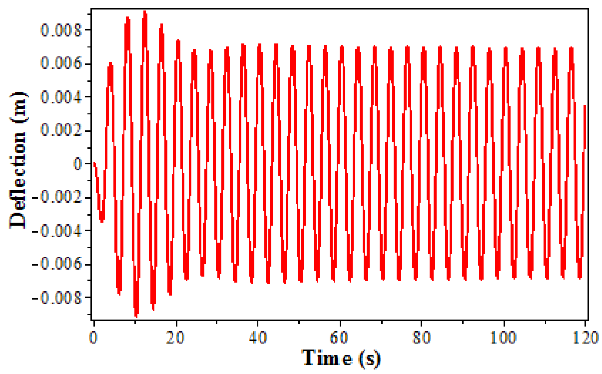

Figure 6. The normal DVA's vibration with case 1

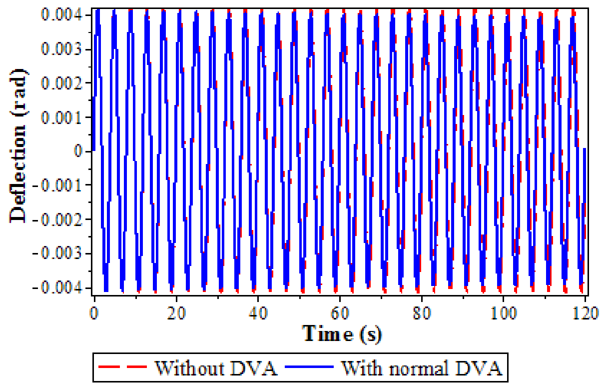

Figure 7. The pendulum structure's vibration with case 2

Case 3. The pendulum structure attached with the normal DVA with initial deflection and initial velocity

The pendulum structure's parameters and the normal DVA's parameters are identified in Tables 1, 2 and equations (6). The initial conditions of the pendulum structure are setup with $\varphi_0 = 5.5\times10^{-3}(rad)$ and $\dot{\varphi}_0 = 6.5\times10^{-3}(rad/s)$. The vibrations of the pendulum structure and the normal DVA are represented in the Figures (8)-(9).

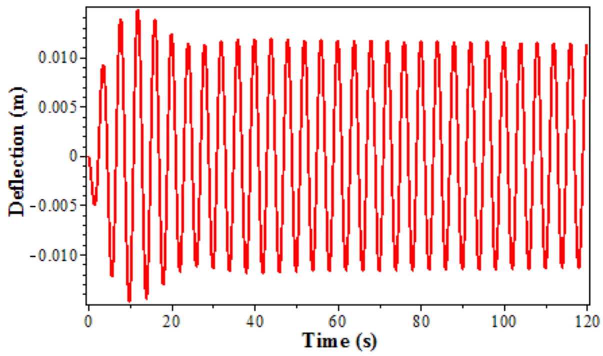

Figure 8. The normal DVA's vibration with case 3

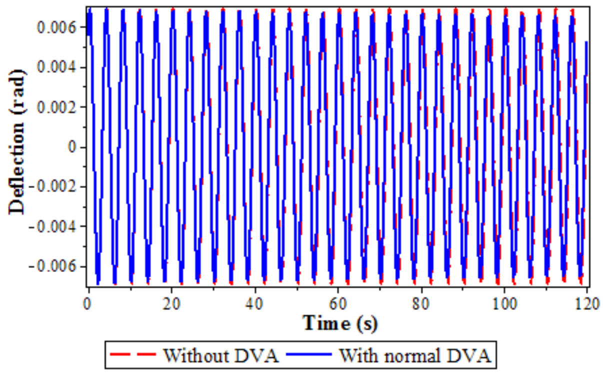

Figure 9. The pendulum structure's vibration with case 3

Figures (5), (7), (9) show that the vibrations of the pendulum structure have not reduced significantly by using the normal DVA under different initial conditions. This confirms that the determination of the DVA's optimum parameters to eliminate the vibration of the pendulum structure is necessary.

3. OPTIMIZATION OF THE DVA

3.1. Determination of optimal parameters of the DVA

To write the non-dimensional equations, introducing parameters

$$\mu = \frac{m}{M};\eta = \frac{L_1}{L_2};\omega_s^2 = \frac{k}{m};u = L_2\varphi;$$
$$\zeta = \frac{c}{2m\omega_s};\ \omega_\varphi^2 = \frac{g}{L_2};\alpha = \frac{\omega_s}{\omega_\varphi}. \quad (7)$$

Table 3 describes the parameters in the expressions of equation (7)

Table 3. Parameters of the pendulum structure and DVA

Parameters	Description
μ	The mass ratio of DVA and the pendulum structure
η	The installed position ratio of DVA
ω_φ	The natural frequency of the pendulum structure
ω_s	The natural frequency of DVA
ξ	The damping ratio of DVA
α	The natural frequency ratio

Therefore, equation (6) can be rewritten in the matrix form

$$\mathbf{M}\ddot{\Upsilon} + \mathbf{C}\dot{\Upsilon} + \mathbf{K}\Upsilon = \mathbf{F} \quad (8)$$

Where

$$\Upsilon = [u \quad s]^T \quad (9)$$

$$\mathbf{F} = [-(1+\mu\eta)\ddot{\rho} \quad -\mu\ddot{\rho}]^T \quad (10)$$

$$\mathbf{M} = \begin{bmatrix} 1+\mu\eta^2 & \mu\eta \\ \mu\eta & \mu \end{bmatrix};\mathbf{C} = \begin{bmatrix} 0 & 0 \\ 0 & 2\xi\alpha\mu\omega_\varphi \end{bmatrix} \quad (11)$$

$$\mathbf{K} = \begin{bmatrix} (1+\mu\eta)\omega_\varphi^2 & \mu\omega_\varphi^2 \\ \mu\omega_\varphi^2 & \mu\alpha^2\omega_\varphi^2 \end{bmatrix} \quad (12)$$

From equations (8)-(12), the equations of state are expressed as

$$\dot{\mathbf{y}}(t) = \mathbf{Z}\mathbf{y}(t) + \mathbf{\Psi}\,\ddot{\rho} \quad (13)$$

where

$$\mathbf{y} = \{s \quad u \quad \dot{s} \quad \dot{u}\} \tag{14}$$

$$\mathbf{\Psi} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1-\eta \end{bmatrix} \tag{15}$$

$$\mathbf{Z} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ Z_{31} & Z_{32} & 0 & Z_{34} \\ Z_{41} & Z_{42} & 0 & Z_{44} \end{bmatrix} \tag{16}$$

$$\begin{aligned} Z_{31} &= -\omega_\varphi^2(1+\mu\eta) + \omega_\varphi^2\mu\eta) \\ Z_{32} &= \omega_\varphi^2\mu(\eta\alpha^2 - 1) \\ Z_{34} &= 2\eta\xi\alpha\omega_\varphi\mu \\ Z_{41} &= \omega_\varphi^2\eta(1+\mu\eta) - \omega_\varphi^2(1+\mu\eta^2) \\ Z_{42} &= \omega_\varphi^2[\eta\mu - \alpha^2(1+\mu\eta^2)] \\ Z_{44} &= -2\xi\alpha\omega_\varphi(1+\mu\eta^2) \end{aligned} \tag{17}$$

The quadratic torque matrix **E** is a solution of the Lyapunov equation as follows

$$\mathbf{ZE} + \mathbf{EZ}^T + \sigma\mathbf{\Psi\Psi}^T = 0 \tag{18}$$

where σ is the white noise spectrum of the acceleration of the hanging point.

By solving the system of equations (15)-(18), the matrix E is defined as

$$\mathbf{E} = \begin{bmatrix} E_{11} & E_{12} & E_{13} & E_{14} \\ E_{21} & E_{22} & E_{23} & E_{24} \\ E_{31} & E_{32} & E_{33} & E_{34} \\ E_{41} & E_{42} & E_{43} & E_{44} \end{bmatrix} \tag{19}$$

where

$$\begin{aligned} E_{11} = &-\frac{1}{\alpha\mu\xi\omega_\varphi^3(\eta-1)^2}[\sigma((\frac{1}{4}+\frac{1}{4}\eta^2\alpha^4 \\ &+\alpha^2(\xi^2-\frac{1}{2})\eta)\eta^3\mu^3 + 2\eta(\frac{1}{8}\eta^2(\eta+2)\alpha^4 \\ &+((\xi^2-\frac{1}{4})\eta+\frac{1}{2}\xi^2-\frac{1}{2})\eta\alpha^2+\frac{1}{8}\eta+\frac{1}{4})\mu^2 \\ &+((\frac{1}{2}\eta^2+\frac{1}{4}\eta)\alpha^4+(-\frac{1}{4}-\frac{1}{4}\eta^2-\eta+\xi^2\eta^2+2\xi^2\eta)\alpha^2 \\ &+\frac{1}{2}+\frac{1}{4}\eta)\mu+\frac{1}{4}+\frac{1}{4}\alpha^4+\alpha^2(-\frac{1}{2}+\xi^2))] \end{aligned} \tag{20}$$

$$E_{12} = \frac{[\sigma(\alpha^2\eta^3-\eta^2)\mu^2+(\eta+1)(\eta\alpha^2-1)\mu+\alpha^2-1]}{4\alpha\mu\xi\omega_\varphi^3(\eta-1)} \tag{21}$$

$$E_{13} = 0 \tag{22}$$

$$E_{14} = \frac{\sigma(\eta\mu+1)^2}{2\mu\omega_\varphi^2(\eta-1)} \tag{23}$$

$$E_{21} = \frac{\sigma\begin{bmatrix}(\eta^3\alpha^2-\eta^2)\mu^2+(\eta+1)(\eta\alpha^2-1)\mu \\ +\alpha^2-1\end{bmatrix}}{\alpha\mu\xi\omega_\varphi^3(\eta-1)} \tag{24}$$

$$E_{22} = \frac{\sigma\eta\mu+1)}{4\alpha\mu\xi\omega_\varphi^3} \tag{25}$$

$$E_{23} = -\frac{\sigma(\eta\mu+1)^2)}{2\mu\omega_\varphi^2(\eta-1)} \tag{26}$$

$$E_{24} = 0 \tag{27}$$

$$E_{31} = 0 \tag{28}$$

$$E_{32} = -\frac{\sigma(\eta\mu+1)^2)}{2\mu\omega_\varphi^2(\eta-1)} \tag{29}$$

$$E_{34} = \frac{\sigma(\eta^2\alpha^2\mu^2+2\eta\alpha^2\mu-\eta\mu^2+\alpha^2-2\mu-1)}{\alpha\mu\xi\omega_\varphi(\eta-1)} \tag{30}$$

$$E_{41} = \frac{\sigma(\eta\mu+1)^2}{2\mu\omega_\varphi^2(\eta-1)} \tag{31}$$

$$E_{42} = 0 \tag{32}$$

$$E_{43} = \frac{\sigma(\eta^2\alpha^2\mu^2+2\eta\alpha^2\mu-\eta\mu^2+\alpha^2-2\mu-1)}{\alpha\mu\xi\omega_\varphi(\eta-1)} \tag{33}$$

$$\begin{aligned} E_{33} = &-\frac{1}{\alpha\mu\xi\omega_\varphi(\eta-1)^2}[\sigma((\frac{1}{4}+\frac{1}{4}\eta^2\alpha^4 \\ &+\alpha^2(\xi^2-\frac{1}{2})\eta)\eta^2\mu^3+((\frac{1}{2}\eta^3+\frac{1}{4}\eta^2)\alpha^4 \\ &+((3\xi^2-1)\eta^2-\frac{1}{2}\eta)\alpha^2+\frac{1}{2}\eta+\frac{1}{4})\mu^2 \\ &+(\frac{3}{4}+(\frac{1}{4}\eta^2+\frac{1}{2}\eta)\alpha^4+(3\xi^2\eta-\eta-\frac{1}{2})\alpha^2)\mu \\ &+\frac{1}{4}+\frac{1}{4}\alpha^4+\alpha^2(\xi^2-\frac{1}{2}))] \end{aligned} \tag{34}$$

$$E_{44}=\frac{\sigma(1+\mu)}{4\alpha\mu\xi\omega_{\varphi}} \quad (35)$$

In the quadratic torque matrix **E**, the response of the pendulum structure is E_{11}. The smaller the E_{11} is, the faster the vibration of the pendulum structure turns off. So, the conditions for the response of the pendulum structure, E_{11}, yield minimum value as follows

$$\left.\frac{\partial E_{11}}{\partial \alpha}\right|_{\alpha_{opt}=\alpha}=0 \quad (36)$$

$$\left.\frac{\partial E_{11}}{\partial \xi}\right|_{\xi_{opt}=\xi}=0 \quad (37)$$

Solving the system of equations (20), (36), (37), the optimal parameters of the DVA are obtained as

$$\alpha_{opt}=\frac{\sqrt{2}}{2(1+\eta^2\mu)}\sqrt{\frac{\left[\begin{array}{l}2\eta^4\mu^3+2\eta^3\mu^2+4\eta^2\mu^2\\+\eta^2\mu+\mu+2\end{array}\right]}{\eta\mu+1}} \quad (38)$$

$$\xi_{opt}=\frac{1}{4}\frac{|\eta-1|\sqrt{\mu(4\eta^3\mu^2+3\eta^2\mu+6\eta\mu-\mu+4)}}{\sqrt{(1+\eta^2\mu)\left[\begin{array}{l}(1+4\eta^4\mu^3+\eta^2(\eta+2)\mu^2\\+(\frac{1}{2}\eta^2+2\eta+\frac{1}{2})\mu\end{array}\right]}} \quad (39)$$

From equation (39), it is clear that $\xi_{opt}=0$ when $(\eta-1)=0$, deduce $L_1=L_2$. So, if the DVA is installed at the position of the center of gravity of the pendulum then it has no effect on reducing vibration of the system. Therefore, the DVA is installed at the position the farther from the center of gravity of the pendulum, the better the effect of eliminating vibration.

Equations (38) and (39) express optimal expressions of the DVA for the pendulum structure subjected to moving loads caused by the hanging point.

3. 2. The vibrations of the pendulum structure attached with the optimal DVA

The input parameters for the pendulum structure are listed in Table 1. The input parameters of DVA are listed in Table 4. To verify the merit of proposed method, numerical simulations are performed for determining the vibrations of the pendulum structure attached with the DVA with optimal design parameters. Each numerical simulation is still considered under three cases:

The pendulum structure with initial deflections, initial velocities and with both the initial deflections and initial velocities.

Table 4. The input parameters for the DVA

Description	Parameters	Value	Unit
Mass of DVA	M	16.0	kg
The position of DVA	L_1	2.0	m

From in Tables (1), (4) and equations (7), (38), (39), values of the optimal parameters of the DVA can be obtained as

$$\alpha_{opt}=1.01;\ \xi_{opt}=0.052 \quad (40)$$

From equations (7) and (40), the spring constant and the damping constant are determined as

$$k=39.883N/m,\ c=2.642Ns/m \quad (41)$$

Case 1. The pendulum structure attached with the optimal DVA with initial deflection

The pendulum structure's parameters and the optimal DVA's parameters are identified in Tables 1, 4 and equation (41).

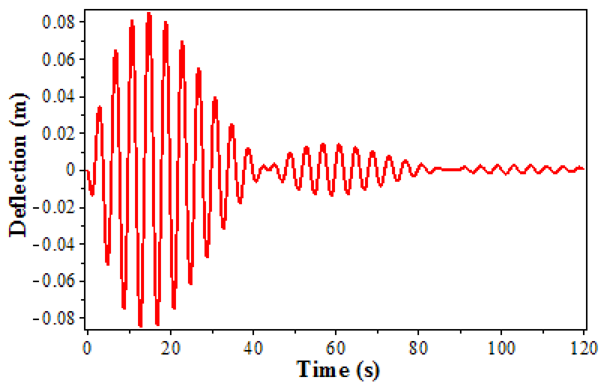

Figure 10. The optimal DVA's vibration with case 1

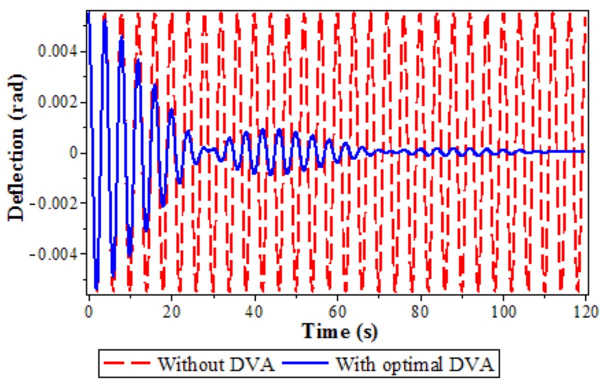

Figure 11. The pendulum structure's vibration with case 1

The initial conditions of the pendulum structure are setup with $\dot{\varphi}_0 = 0.0(rad/s)$ and $\varphi_0 = 5.5\times10^{-3}(rad)$. The Maple software is used to simulate the vibrations of the pendulum structure with the optimal DVA, as shown in the Figures (10)-(11).

Case 2. The pendulum structure attached with the optimal DVA with initial velocity

The pendulum structure's parameters and the optimal DVA's parameters are identified in Tables 1, 4 and equation (41).

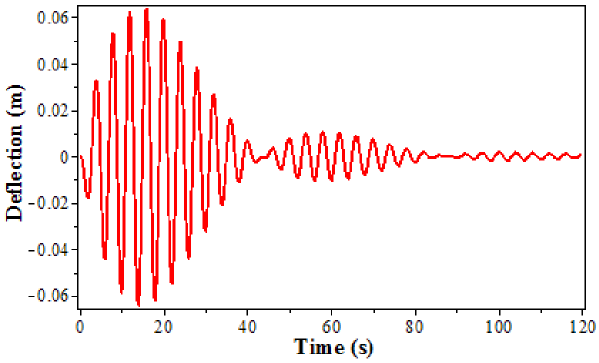

Figure 12. The optimal DVA's vibration with case 2

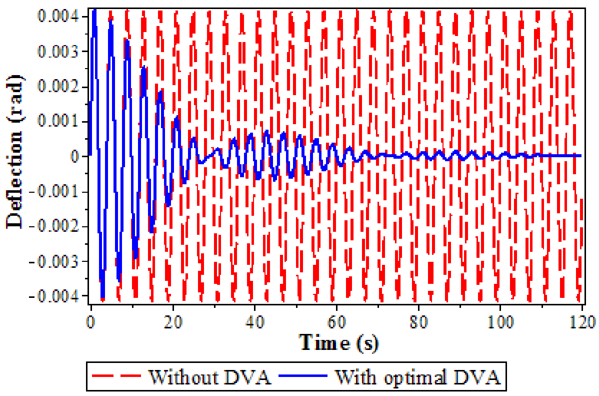

Figure 13. The pendulum structure's vibration with case 2

The initial conditions of the pendulum structure are setup with $\dot{\varphi}_0 = 6.5\times10^{-3}(rad/s)$ and $\varphi_0 = 0.0(rad)$. The vibrations of the pendulum structure and the optimal DVA are expressed in the Figures (12)-(13).

Case 3. The pendulum structure attached with the optimal DVA with initial deflection and initial velocity

The pendulum structure's parameters and the optimal DVA's parameters are identified in Tables 1, 4 and equation (41).

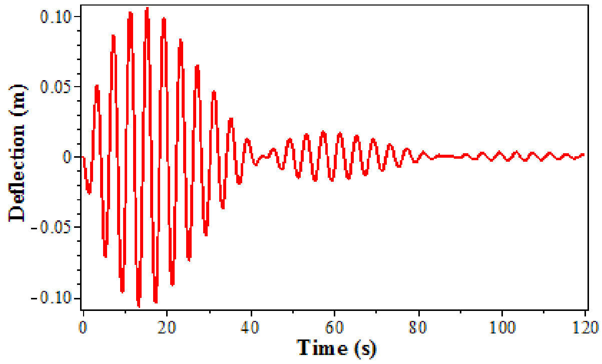

Figure 14. The optimal DVA's vibration with case 3

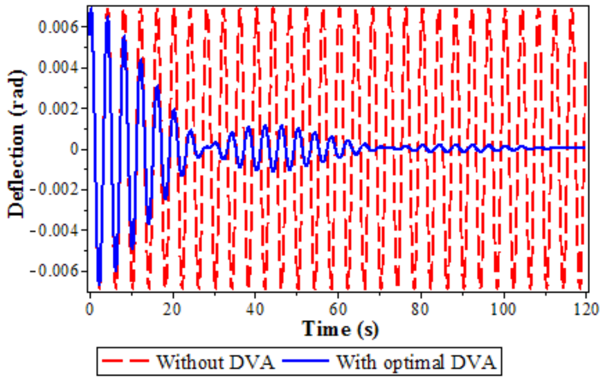

Figure 15. The pendulum structure's vibration with case 3

The initial conditions of the pendulum structure are setup with $\varphi_0 = 5.5\times10^{-3}(rad)$ and $\dot{\varphi}_0 = 6.5\times10^{-3}(rad/s)$. The vibrations of the pendulum structure and the optimal DVA are represented in the Figures (14)-(15).

Figures (11), (13) and (15) shown that the optimum parameters of DVA are determined from equations (38) and (39), which has a very good effect on vibration suppression of the pendulum structure subjected to moving loads caused by the hanging point.

4. CONCLUSIONS

A new of this study is to find explicit analytical solutions of the optimal DVA, which are used to suppress vibrations of the pendulum structure subjected to moving loads caused by the hanging point.

In this paper, a mathematical model for determining the vibration of the pendulum structure with DVA under the inertial force of accelerating hanging point is proposed. Vibration equations are formulated for the system shown in the Expressions of equation (6). This is an extremely important step for designing the DVA's optimum parameters to suppress vibration of the pendulum structure. The optimal parameters of the DVA are determined by using the minimum quadratic torque method are shown in equations (38) and (39).

The numerical simulations are constructed to evaluate the effectiveness of the optimal design parameters of the DVA. The numerical results show that the vibration of the pendulum structure is suppressed when the DVA is optimally designed.

ACKNOWLEDGMENTS

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