
The Effects of Nonlinear Viscosity of Asphalt Concrete in Vibration Compaction Process

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Abstract: - The paper evidences the results of experimental and numerical research on the dynamic compaction of asphalt mixtures applied in the road wear layer. There are studied two cases for 8 and 15 cm layer thickness of asphalt concrete, the linear and nonlinear behaviors when studying the viscosity rheological aspect at 120 ÷ 160 °C temperature range, subject to technological vibrations of self-propelled vibratory roller. As example, there are considered the required power values for vibration in the two considered cases and the obtained results prove distinct different values for the compaction process characteristics, in linear and nonlinear regimes.

Keywords: - nonlinear vibrations, dynamic compaction, asphalt concrete, active power, vibrations

1. INTRODUCTION

The research presented in this paper is based on the calculation model for one degree of freedom (1 DOF) dynamic system, subjected to a harmonic excitation force and made of both linear elastic and nonlinear viscous elements. This type of system is representative for the stationary machines with vibrating action, such as: vibrating screens, vibrating feeders, vibrating mills, etc.

Also, the studied model would be appropriate for mobile vibrating machines such as vibratory rollers used for the compaction of soils and roads.

The rheological characterization of freshly poured asphalt concrete consists in a special procedure for both laboratory and in-situ evaluation of physico-mechanical characteristics.

The vibration compaction machines (vibratory rollers, vibratory paltes) designed with maximum available power under the assumption of linear behavior for the asphalt concrete layer, do not provide the required power that should be higher (than designed) under the assumption of nonlinear behavior of the material in the asphalt concrete mixture. This is the reason that cases of good working practice have to be studied so that, further, required technological performances to be obtained [1-5].

2. NONLINEAR CHARACTERISTIC OF ASPHALT CONCRETE

Depending on the dose and the nature of mineral aggregates, the water / concrete ratio and the beating degree, the materials laid in roads construction are characterized by a third degree nonlinearity of the viscosity in the process of vibration compaction [6-9].

Thus, the global viscous force for a certain quantity of material (fresh concrete, asphalt mixture, soil) under vibratory regime, is given by relation (1)

$$F_v = \alpha \dot{x} + \gamma \dot{x}^3 \quad (1)$$

where: α is the dissipation coefficient of the surface distribution energy and γ – the dissipation coefficient of the temporal distribution energy.

The physical aspect of the dissipation coefficient α can be interpreted in the terms of energy as surface distribution energy dissipated per unit of volume, at the vibratory roller and viscoelastic material, as expressed by relation (2)

$$\alpha = \frac{E_v}{V} \cdot S = D \cdot S \quad (2)$$

with the measurement unit expressed by the dimensions relation (3)

$$[\alpha]_{SI} = \frac{J}{m^3} \cdot m^3 \Leftrightarrow \frac{Ns}{m} \quad (3)$$

The γ dissipation coefficient can be interpreted in the terms of energy as the dissipated energy per unit of volume, with the effect in the time interval τ - see relation (4).

$$\gamma = \frac{E_v}{V} \cdot \tau^2 = D \cdot \tau^2 \quad (4)$$

with the measurement unit expressed by relation (5)

$$[\gamma]_{SI} = \frac{J}{m^3} \cdot s^2 \Leftrightarrow \frac{Ns^3}{m^3} \quad (5)$$

The significance of the above mentioned symbols is presented next:

E_v is the dissipated energy in the viscous component of material;

V – the volume of material in which the vibratory motion and the dissipation of energy are noticeable;

D – dissipated specific energy;

S – the area of contact surface between the working tool (vibratory roller, vibratory plate) and the material under compaction process;

τ – the time while there are effects of material flow and relaxation, until stabilization.

Based on the experiments carried out, both in laboratory and in-situ, for a certain recipee of asphalt concrete with 15 cm layer thickness, freshly layed on, there were obtained the values for compaction process characteristics, as follows next [10-12]:

- self resonance frequency, $f_0 = 2,5$ Hz

- dissipation coefficient α (within the frequency values interval 33...50 Hz)

$$\alpha = 10^3 \frac{Jm^3}{m^3}$$

- dissipation coefficient γ (within the frequency values interval 33...50 Hz)

$$\gamma = 9 \cdot 10^6 \frac{Js^2}{m^3}$$

- the phase shift angle between inertial harmonic excitation force and displacement (in stabilized regime for frequency values interval 33...50 Hz)

$$\varphi = 2,26 \dots 2,70 \text{ rad.}$$

3. EVALUATION OF NONLINEARITY EFFECT IN VIBRATION COMPACTION REGIME

It is assumed that at the contact zone between the working tool (vibratory roller, vibratory plate) and the material under compaction process, the interaction is defined by the total force (viscous and elastic) shown by relation (6)

$$F = F_v + F_e \quad (6)$$

which expressed by instantaneous variables, $x = x(t)$ si $\dot{x} = \dot{x}(t)$, turns into relation (7)

$$F = \alpha \dot{x} + \gamma \dot{x}^3 + cx \quad (7)$$

For the exterior excitation force of the equipment, there is further considered the harmonic perturbation force, with the constant amplitude, H , as shown by relation (8):

$$Q = H \cos \omega t \quad (8)$$

where ω is the pulsation of perturbation force.

The nonlinear differential equation for the system's motion is given by relation (9) [13-15]:

$$m\ddot{x} + \alpha \dot{x} + \gamma \dot{x}^3 + cx = H \cos \omega t \quad (9)$$

or, similarly, by relation (10)

$$\ddot{x} + \frac{\alpha}{m} \dot{x} + \frac{\gamma}{m} \dot{x}^3 + p^2 x = h \cos \omega t \quad (10)$$

where: $p = \sqrt{\frac{c}{m}}$ is the self-pulsation of the linear system;

$h = \frac{H}{m}$ – the maximum perturbation force for the unit of mass.

Solving the nonlinear differential equation is done by iterative approximation method, finding the first approximation solution as shown by relation (11):

$$x = A \cos(\omega t + \varphi) \quad (11)$$

By replacing x , \dot{x} , \ddot{x} , in previous relations, results a mathematical relation that is holds true only if its two parts are identically, as seen in relation (12) [14-16]:

$$-A\omega^2 \cos(\omega t + \varphi) - \frac{\alpha}{m} A\omega \sin(\omega t + \varphi) - \frac{\gamma}{m} A^3 \omega^3 \sin^3(\omega t + \varphi) + p^2 A \cos(\omega t + \varphi) = h \cos \omega t \quad (12)$$

and by further check, it turns into relations (13) and (14):

$$A\omega^2\sin\varphi - \frac{\alpha}{m}A\omega\cos\varphi - \frac{3}{4}\cdot\frac{\gamma}{m}A^3\omega^3\cos\varphi - p^2A\sin\varphi = 0 \quad (13)$$

$$A\omega^2\sin\varphi - \frac{\alpha}{m}A\omega\cos\varphi - \frac{3}{4}\cdot\frac{\gamma}{m}A^3\omega^3\cos\varphi - p^2A\sin\varphi = 0$$

$$A\omega^2\sin\varphi - \frac{\alpha}{m}A\omega\cos\varphi - \frac{3}{4}\cdot\frac{\gamma}{m}A^3\omega^3\cos\varphi - p^2A\sin\varphi = 0$$

$$\frac{\gamma}{m}A^3\omega^3\cos\varphi - p^2A\sin\varphi = 0$$

$$\frac{\alpha}{m}A\omega\cos\varphi - \frac{3}{4}\cdot\frac{\gamma}{m}A^3\omega^3\cos\varphi - p^2A\sin\varphi = 0 \quad (14)$$

$$-A\omega^2\cos\varphi - \frac{\alpha}{m}A\omega\sin\varphi - \frac{3}{4}\cdot\frac{\gamma}{m}A^3\omega^3\sin\varphi + p^2A\cos\varphi - h = 0$$

$$(14) - A\omega^2\cos\varphi - \frac{\alpha}{m}A\omega\sin\varphi - \frac{3}{4}\cdot\frac{\gamma}{m}A^3\omega^3\sin\varphi + p^2A\cos\varphi - h = 0$$

$$A\omega^2\sin\varphi - \frac{\alpha}{m}A\omega\cos\varphi - \frac{3}{4}\cdot\frac{\gamma}{m}A^3\omega^3\cos\varphi - p^2A\sin\varphi = 0$$

If relation (13) is multiplied by $\cos\varphi$, and relation (14) is multiplied by $\sin\varphi$ and, further, part by part are added, than it is obtained relation (15):

$$\frac{\alpha}{m} = -\frac{1}{A\omega} \left[\frac{3}{4}\frac{\gamma}{m}A^3\omega^3 + h\sin\varphi \right] \quad (15)$$

From the relation (13) it is determined the formula for p^2 , see relation (16)

$$p^2 = \frac{1}{A\sin\varphi} \left[A\omega^2\sin\varphi - \frac{\alpha}{m}A\omega\cos\varphi - \frac{3}{4}\cdot\frac{\gamma}{m}A^3\omega^3\cos\varphi \right] \quad (16)$$

and, by further considering $\frac{\alpha}{m}$ formula, it results the relation (17):

$$p^2 = \frac{1}{A} [A\omega^2 + h\cos\varphi] \quad (17)$$

The functional parameters of the equipment are:

- ratio $\frac{H}{m} = h$
- vibration amplitude, A
- the required power for maintaining the vibrations in forced mode, N .

From the relation (17) it is finally obtained

$$p^2A = A\omega^2 + h\cos\varphi \quad (18)$$

and

$$h = \frac{H}{m} = (p^2 - \omega^2) \frac{A}{\cos\varphi} \quad (19)$$

Starting from relation (15) and further considering the other above mentioned relations, it is determined the second order amplitude relation, as shown by relation (20) [17-20]:

$$A^2 = -\frac{4}{3} \frac{m}{\gamma\omega^3} \left[\frac{\alpha}{m} \omega - (p^2 - \omega^2) \right] \tan\varphi \quad (20)$$

where $\gamma < 0$.

The required power for maintaining the vibrations in stabilization regime, N , only for internal dissipation of energy into the material is given by relation (21):

$$N = \frac{1}{2} HA\omega\sin\varphi \quad (21)$$

Let us consider, for example, the case study of a vibratory roller, with mass $m = 1000$ Kg, with vibration force $Q = 2 \cdot 10^4 \cos 100\pi t$, being in contact with asphalt concrete layer of 15 cm thickness. The angle between perturbation force direction and instantaneous displacement is $\varphi = 150^\circ$ and thus, the $\tan 150^\circ = -0,6$. There are studied two different cases as follows next [21-23]:

a) linear viscous material with the viscous force $F'_v = \alpha \dot{x}$

b) nonlinear viscous material with the viscous force $F''_v = \alpha \dot{x} + \gamma \dot{x}^3$

The values α and γ , previously mentioned, have been easily determined and resulted in:

a) linear case

- amplitude $A_{lin} = 0,2$ mm

- power $N_{lin} = 376$ W

b) nonlinear case

- amplitude $A_{nelin} = 9,3$ mm

- power $N_{nelin} = 17\,521$ W

4. CONCLUSIONS

The nonlinear behavior of asphalt concrete in viscous state determines a careful selection of the vibration compaction machine so that it could ensure the dynamic and energetic performances technologically required.

The viscous nonlinearity of asphalt concrete requires higher power values compared to the ones in linear behavior. The vibration compaction machines designed for linear behavior materials can not be used in technological procedures applied for nonlinear materials because they can not ensure enough required power and thus their functionality is not possible.

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