Subharmonic Vibrations in Asymmetric Nonlinear Elastic Systems in The Case of Dynamic Equipment Within Special Buildings (nuclear, hospitals, energy)

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Abstract: - There are presented the results of the research conducted by the authors on the response of the dynamic system composed of dynamic equipment with rotating inertial force excitation and with elastic asymmetric connections. In this case, it is highlighted that the dynamic system generates subharmonic vibrations of order ½ which can be damaging to buildings as well as to the serving staff.

Keywords: - asymmetric elasticity, subharmonics, stationary excitations, inertial-rotating force.

1. INTRODUCTION

The adopted model highlights the fact that the elastic force is of second order in relation to the instantaneous deformation.

In this case it is demonstrated both theoretically and experimentally that at a harmonic excitation given with the pulsation ω subharmonics of order ½ ω can appear with specific influences both on the construction and on the staff in the building [1-5].

2. ANALYSIS OF THE OCCURRENCE OF SUBHARMONIC VIBRATIONS OF ORDER 1/2Ω

The system is characterized by rigidity [6-9]:

$$k=k_0(1+\gamma q) \tag{1}$$

where $\gamma > 0$, which shows that the elastic force in the system increases in one direction and decreases in the opposite direction, where γ is measured in m^{-1} . The differential equation of motion for natural vibrations, without amortization, is:

$$a\ddot{q} + k_0(1+\gamma q) = Q_0 cos\omega t \tag{2}$$

which inserted in equation (2) leads to the relation $q = A\sin\Omega\tau + C$, which :

$$(-A\Omega^2 + A + 2Ac\gamma)\sin\Omega\tau + \gamma C^2 + C + \frac{1}{2}\gamma A^2(1-\cos 2\Omega\tau) = 0 \tag{3}$$

For Ω = 1 the constant term must be canceled, that is we have:

$$\gamma C^2 + C + \frac{1}{2}\gamma A^2 = 0$$

from where

$$C = \frac{1}{2\gamma}\left(1 \pm \sqrt{1-2\gamma^2 A^2}\right) \tag{4}$$

or, approximating the radical $\sqrt{1-2x^2} \cong 1+x^2$, eventually, we have:

$$C = -\frac{1}{2}\gamma A^2 \qquad (5)$$

Also, in (3), the coefficient of sinΩτ must be null, that is:

$$1\text{-}\Omega^2 + 2C\gamma = 0$$

which, with (4) and (5) becomes

$$\Omega^2 = 1\text{-}\gamma^2 A^2$$

For the second approximation, the solution is adopted:

$$Q = A\sin\Omega\tau + B\sin 2\Omega\tau + C,$$

which, inserted in equation (2) leads to equation

$$A(1-\Omega^2 - B\gamma + 2C\gamma)\sin\,\Omega\tau + \left(B - 4B\Omega^2 - \frac{1}{2}\gamma A^3 + 2BC\gamma\right)\cos\,2\Omega\tau + \\ + \left(C + \gamma C^2 + \frac{1}{2}\gamma A^2 + \frac{1}{2}\gamma B^2\right) + AB\gamma\sin\,3\Omega\tau + \frac{1}{2}\gamma B^2 \cos\,4\Omega\tau = 0 \qquad (6)$$

In equation (6) there are canceled the coefficients of the functions sinΩτ, cosΩτ and the free term, obtaining:

$$\begin{cases} 1-\Omega^2 - B\gamma + 2C\gamma = 0 \\ B - 4B\Omega^2 - \frac{1}{2}\gamma A^2 + 2BC\gamma = 0 \\ C + \gamma C^2 + \frac{1}{2}\gamma A^2 + \frac{1}{2}\gamma B = 0 \end{cases} \qquad (7)$$

Taking into account (4) from the relations (7) emerges:[10-14]

$$B \cong -\frac{1}{6}\gamma A^2;\ \Omega^2 = 1 - \frac{5}{6}\gamma^2 A^2$$

For the occurrence of subharmonics of order ½ the mechanical work of the excitation absorbed at the displacement q_2 =Bcos2Ωτ is determined, in resonance Ω = 1. Thus, we have

$$L_2 = \int_0^{2\pi} Q_0 \sin\,2\Omega\pi dq_2 \qquad (8)$$

or, taking into account dq_2 =-2Ωbsin2Ωτdτ, we obtain

$$L_2 = -2BQ_0 \int_0^{2\pi} \sin^2\,2\pi \mathrm{d}\tau = -2\pi B Q_0. \qquad (9)$$

The mechanical work absorbed by the dissipative system by the force $F_1 = c\dot{q}_1$, with q_1 =AsinΩτ, for Ω = 1, is $L_1 = pb\pi A^2$.
The condition for the occurrence of subharmonic vibrations of order ½ is L_2 > L_1, that is $-2BQ_0\pi \rangle cp\pi A^2$

In order to prevent the occurrence of subharmonic vibrations of order ½ it is necessary that $r > \frac{1}{3}\gamma q_0$.

In conclusion, in order to eliminate the excitation of subharmonic vibrations in systems with nonlinear elastic force, the total amortization must meet the following conditions:[15-16]

a) in order to avoid subharmonic vibrations of order 1/3, the relative amortization r must meet the condition $r > \frac{3}{32}\beta q_0 A$

b) in order to avoid subharmonic vibrations of order 1/2, the relative amortization r must meet the condition $r > \frac{1}{3}\gamma q_0$

It should be noted that, in practice, the same system can have symmetrical or asymmetrical elasticity, depending on whether it operates in no load or in load. Usually, crankshafts are asymmetric systems in which subharmonic vibrations of order 1/2 can occur. [17]

When operating compressors with hydrodynamic bearings, due to the change in the viscosity of the oil, that is the change in parameter r, subharmonic vibrations of order 1/3 or 1/2 may occur, also known as subharmonic "vortices".

3. CONCLUSIONS

In the case of nonlinear elastic connections with asymmetric rigidity of order one, the following is highlighted:

a) the appearance of subharmonic vibrations of order 1/2 when the hysteretic amortization r is related to the degree of asymmetry and the inertial deformation so that $32r < 3q_0 A\beta$

b) in order to avoid the appearance of 1/2 order subharmonics, amortization b must meet the requirement $r = \frac{b}{m\omega_n} > \frac{3}{32} A q_0 \beta$

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