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# Free Vibration Analysis of Square Structure Plate with Different Boundary Conditions

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*Abstract:* - A mathematical model is presented to analyse the free vibration of viscoelastic square structure plate by employing a Rayleigh–Ritz method. Visco-elastic plates find a large range of applications, mainly in weight responsive structures like aircraft and spaceship due to their high strength-to-weight ratio and high stiffness-to-weight ratio. This paper deals with free vibration of visco-elastic square plate using different boundary conditions and thickness of the plate is assumed to be varying circularly in x-direction and temperature variation in visco-elastic plate is also taken linearly in x-direction. Boundary conditions for square structure plate is supposed to be clamped (C-C-C-C) and simply supported (SS-SS-SS-SS). A High-level Mathematical computational software MAPLE does all the calculations. The first two modes of vibration are calculated to see the effects of two factors taper constants and thermal gradient, and the findings of the present analysis are presented in tables and graphs.

*Keywords:* - Visco-elastic, vibration, non-homogeneity, clamped, and simply supported.

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## 1. INTRODUCTION

In recent years, tapered plates have been increasingly used in modern engineering structures or space vehicles due to their wide technical importance. Consideration of tapered plates not only helps to reduce the weight of structural elements but also improves the utilisation of the material. Almost every structure or vehicle works under the influence of elevated temperature field due to which creates a heat flux that directly affects the efficiency of a vehicle or structure. In this paper, we have taken a plate of visco-elastic material. Duralumin, an aluminium alloy, is used to make visco-elastic plates. It contains 95% aluminium, 4% copper, 0.5% magnesium and 0.5% manganese.

Plates with thickness variations are extremely important in the engineering field. It has been used in jet structures for many years and is recurrently used in order to lighten the plates, mainly when used in wings for high-speed and performance.

In modern times, scientists and researchers are interested to know about the effect of temperature on the vibration of plates, as it has great importance in space technology, high speed atmospheric flights, and in nuclear energy applications. Due to variations in temperature, non-homogeneity occurs in materials, which cannot be neglected. Therefore, it becomes necessary to study the impact of temperature variation.

In the literature review, Leissa [1] gave numerical data about the vibration of plates in his monograph. Gupta and Khanna [2] studied the vibrational effect on a rectangular plate. They assumed that plates have linearly varying thickness in both directions. Khanna and Sharma [3] also studied the vibrational analysis of non-homogeneous tapered square plate with bi-parabolic temperature variations. Tufoi M. et al. [4] discussed the dynamic analysis of Thin Plates with Defects by Experimental and FEM Methods. Sharma et al. [5] have discussed about natural vibration of square plate with circular variation in thickness. Sharma et al. [6] discussed the mathematical study of vibration of a non-homogeneous parallelogram plate with thermal effect.

Wu and Lu [7] worked on a rectangular plate with internal column and uniform elastic edge. Quintana and Nallima [8] presented a general Ritz formulation for the free vibration analysis of thick trapezoidal and triangular laminated plates resting on elastic supports. Hatiegan C. et al. [9] have present a improved mathematical relation of the modal shapes of thin rectangular plates. Khanna et al. [10] study the effect of Poisson ratio on a non-homogeneous rectangular plate with varying thermal effect for First two modes of vibration. Sharma S.K. and Sharma A.K. [11] applied the Rayleigh-Ritz method to solve the equation of frequency for free vibration of orthotropic rectangular plate with 2D thickness and temperature variation. Hatiegan L.B. et al. [12] studied the natural

frequencies of thin rectangular plates clamped on contour using the Finite Element Method. Sharma et al. [13] also described the effect of vibration on an orthotropic visco-elastic rectangular plate with two-dimensional temperature and thickness variation.

In this research paper, the free vibration problem of visco-elastic square structure plate with different boundary conditions has been calculated and equation of frequency is solved with the help of a Rayleigh-Ritz method for two different boundary conditions. Results are given in the form of tables and graphs.

## 2. GOVERNING EQUATION OF MOTION

A visco-elastic isotropic square plate's differential equation of motion can be written as[1]:

$$\frac{\partial^2 M_x}{\partial x^2} + 2 \frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} = \rho h \frac{\partial^2 w}{\partial t^2} \quad (1)$$

where

$$\left. \begin{aligned} M_x &= \tilde{D} D_1 \left( \frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2} \right) \\ M_y &= -\tilde{D} D_1 \left( \frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2} \right) \\ M_{xy} &= -\tilde{D} D_1 \left( (1-\nu) \frac{\partial^2 w}{\partial x \partial y} \right) \end{aligned} \right\}$$

Here  $x$  and  $y$  are the plate coordinates,  $M_x$  and  $M_y$  are the bending moments,  $M_{xy}$  is the twisting moment per unit length,  $\rho$  is the mass density,  $h$  is the thickness of plate,  $w$  is the displacement at time  $t$  and  $\tilde{D}$  is visco-elastic operator.

Here  $D_1$  is the flexural rigidity of the plate and derived as

$$D_1 = \frac{Eh^3}{12(1-\nu^2)} \quad (2)$$

here  $E$  represent Young's modulus and  $\nu$  is Poisson ratio.

Putting all values of  $M_x$ ,  $M_y$  and  $M_{xy}$  in equation (1), we get fourth order differential equation of motion

$$\tilde{D} \left[ \begin{aligned} &D_1 \left( \frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} \right) + 2 \frac{\partial D_1}{\partial x} \left( \frac{\partial^3 w}{\partial x^3} + 2 \frac{\partial^3 w}{\partial x \partial y^2} \right) \\ &+ 2 \frac{\partial D_1}{\partial y} \left( \frac{\partial^3 w}{\partial y^3} + 2 \frac{\partial^3 w}{\partial y \partial x^2} \right) + \frac{\partial^2 D_1}{\partial x^2} \left( \frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2} \right) \\ &+ \frac{\partial^2 D_1}{\partial y^2} \left( \frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2} \right) + 2(1-\nu) \frac{\partial^2 D_1}{\partial x \partial y} \frac{\partial^2 w}{\partial x \partial y} \end{aligned} \right] + \rho h \frac{\partial^2 w}{\partial t^2} = 0 \quad (3)$$

The deflection of the plate can be written as product of deflection and time function as:

$$w(x, y, t) = W(x, y) \times T(t) \quad (4)$$

where  $W(x, y)$  is the deflection function in  $x$  and  $y$  and  $T(t)$  is a time function.

Putting the value of  $w(x, y, t)$  from equation (4) into equation (3), we get

$$\left[ \begin{aligned} &D_1 \left( \frac{\partial^4 W(x, y)}{\partial x^4} + 2 \frac{\partial^4 W(x, y)}{\partial x^2 \partial y^2} + \frac{\partial^4 W(x, y)}{\partial y^4} \right) \\ &+ 2 \frac{\partial D_1}{\partial x} \left( \frac{\partial^3 W(x, y)}{\partial x^3} + 2 \frac{\partial^3 W(x, y)}{\partial x \partial y^2} \right) \\ &+ 2 \frac{\partial D_1}{\partial y} \left( \frac{\partial^3 W(x, y)}{\partial y^3} + 2 \frac{\partial^3 W(x, y)}{\partial y \partial x^2} \right) \\ &+ \frac{\partial^2 D_1}{\partial x^2} \left( \frac{\partial^2 W(x, y)}{\partial x^2} + \nu \frac{\partial^2 W(x, y)}{\partial y^2} \right) \\ &+ \frac{\partial^2 D_1}{\partial y^2} \left( \frac{\partial^2 W(x, y)}{\partial y^2} + \nu \frac{\partial^2 W(x, y)}{\partial x^2} \right) \\ &+ 2(1-\nu) \frac{\partial^2 D_1}{\partial x \partial y} \frac{\partial^2 W(x, y)}{\partial x \partial y} \end{aligned} \right] = \frac{1}{DT(t)} \frac{\partial^2 T(t)}{\partial t^2} \rho h W(x, y) \quad (5)$$

Equation (5) is satisfied if both of its sides are equal to a constant  $p^2$ , i.e.

$$\left[ \begin{aligned} &D_1 \left( \frac{\partial^4 W(x, y)}{\partial x^4} + 2 \frac{\partial^4 W(x, y)}{\partial x^2 \partial y^2} + \frac{\partial^4 W(x, y)}{\partial y^4} \right) \\ &+ 2 \frac{\partial D_1}{\partial x} \left( \frac{\partial^3 W(x, y)}{\partial x^3} + 2 \frac{\partial^3 W(x, y)}{\partial x \partial y^2} \right) \\ &+ 2 \frac{\partial D_1}{\partial y} \left( \frac{\partial^3 W(x, y)}{\partial y^3} + 2 \frac{\partial^3 W(x, y)}{\partial y \partial x^2} \right) \\ &+ \frac{\partial^2 D_1}{\partial x^2} \left( \frac{\partial^2 W(x, y)}{\partial x^2} + \nu \frac{\partial^2 W(x, y)}{\partial y^2} \right) \\ &+ \frac{\partial^2 D_1}{\partial y^2} \left( \frac{\partial^2 W(x, y)}{\partial y^2} + \nu \frac{\partial^2 W(x, y)}{\partial x^2} \right) \\ &+ 2(1-\nu) \frac{\partial^2 D_1}{\partial x \partial y} \frac{\partial^2 W(x, y)}{\partial x \partial y} \end{aligned} \right] - \rho p^2 h W(x, y) = 0 \quad (6)$$

and

$$\frac{\partial^2 T(t)}{\partial t^2} + p^2 DT(t) = 0 \quad (7)$$

Equations (6) and (7) are differential equations of motion and time function for a visco-elastic square plate.

### 2.1. Assumption for thickness variation

In this paper, the thickness of the square structure plate vary circularly in the x- direction. i.e.

$$h = h_0 \left[ 1 + \beta \left( 1 - \sqrt{1 - \frac{x}{a}} \right) \right] \quad (8)$$

where  $a$  is the length of the square plate,  $\beta$  ( $0 \leq \beta \leq 1$ ) is the taper parameter in x- direction,  $h_0$  is the thickness of the plate at  $x = 0$  and thickness of plate become constant i.e.  $h = h_0$  if  $x = x_0$ .

### 2.2. Assumption for temperature variation

We assumed that for square structure plate the temperature vary linearly in one dimensional. i.e.

$$\tau = \tau_0 \left( 1 - \frac{x}{a} \right) \quad (9)$$

where  $\tau$  denotes the temperature excess above the reference temperature at any point on the plate, and  $\tau_0$  denotes the temperature excess above the reference temperature at  $x = 0$ .

The temperature dependence of the modulus of elasticity can be expressed as follows:

$$E = E_0 (1 - \gamma \tau) \quad (10)$$

where  $E_0$  denotes the value of Young's modulus at reference temperature, i.e.  $\tau = 0$  and  $\gamma$  is slope of variation.

Using equation (9) and (10), we get

$$E = E_0 \left( 1 - \alpha \left( 1 - \frac{x}{a} \right) \right) \quad (11)$$

where  $\alpha = \gamma \tau_0$  ( $0 \leq \alpha \leq 1$ ) is the thermal gradient.

Using equations (8) and (11) in equation (2), we get

$$D_1 = \frac{E_0 \left( 1 - \alpha \left( 1 - \frac{x}{a} \right) \right) h_0^3 \left( 1 + \beta \left( 1 - \sqrt{1 - \frac{x^2}{a^2}} \right) \right)}{12(1 - \nu^2)} \quad (12)$$

### 2.3. Boundary Conditions

All boundary conditions of plate are satisfied deflection function  $W(x, y)$  is taken:

$$W(x, y) = \left( \frac{x}{a} \right)^p \left( 1 - \frac{x}{a} \right)^q \left( \frac{y}{b} \right)^r \left( 1 - \frac{y}{b} \right)^s \times \left[ A + B \left( \frac{x}{a} \right) \left( \frac{y}{b} \right) \left( 1 - \frac{x}{a} \right) \left( 1 - \frac{y}{b} \right) \right]$$

The parameter p, q, r and s can take any of the values 0, 1 and 2 indicating the free, simply supported or clamped boundary conditions, respectively.

In this paper, we discussed the vibration of a square plate for two different boundary conditions, i.e. C-C-C-C and SS-SS-SS-SS.

**For Clamped Square Plate (C-C-C-C):** Fig 1 represent the boundary condition where deflections at all edges are zero, slope at all edges is zero. With these conditions, we obtain displacement function as:

$$W(x, y) = \left( \frac{x}{a} \right)^2 \left( 1 - \frac{x}{a} \right)^2 \left( \frac{y}{a} \right)^2 \left( 1 - \frac{y}{a} \right)^2 \times \left[ A + B \left( \frac{x}{a} \right) \left( \frac{y}{a} \right) \left( 1 - \frac{x}{a} \right) \left( 1 - \frac{y}{a} \right) \right]$$

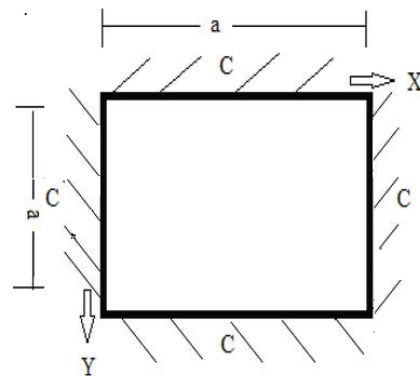


Figure 1. Clamped Square Plate (C-C-C-C)

**For Simply Supported Square Plate (SS-SS-SS-SS):** Fig 2 represent the boundary condition where deflections and moments at all edges are zero, slope at all edges except  $Y=b$  is zero. With these conditions, we obtained displacement function as:

$$W(x, y) = \left( \frac{x}{a} \right) \left( 1 - \frac{x}{a} \right) \left( \frac{y}{a} \right) \left( 1 - \frac{y}{a} \right) \times \left[ A + B \left( \frac{x}{a} \right) \left( \frac{y}{a} \right) \left( 1 - \frac{x}{a} \right) \left( 1 - \frac{y}{a} \right) \right]$$

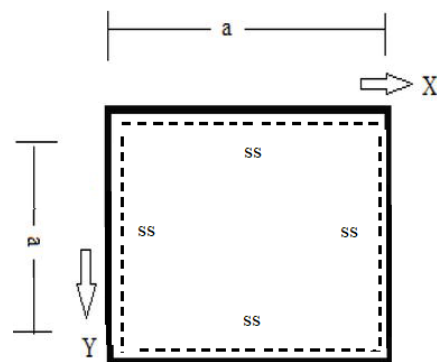


Figure 2. Simply Supported Square Plate (SS-SS-SS-SS)

### 3. SOLUTIONS OF FREQUENCY EQUATION

The Rayleigh–Ritz method is used for finding the solution of frequency equation. In this method, maximum kinetic energy ( $K_E$ ) is equal to maximum strain energy ( $S_E$ ).  
i.e.

$$\delta(S_E - K_E) = 0 \quad (13)$$

where

$$K_E = \frac{1}{2} \rho P^2 \int_0^a \int_0^a h W^2 dy dx \quad (14)$$

and

$$S_E = \frac{1}{2} \int_0^a \int_0^a D_1 \left[ (W_{xx})^2 + (W_{yy})^2 + 2\nu W_{xx} W_{yy} + 2(1-\nu)(W_{xy})^2 \right] dy dx \quad (15)$$

Now, we introduced non-dimensional variables as

$$X = \frac{x}{a}, Y = \frac{y}{b}, \bar{W} = \frac{w}{a}, \bar{h} = \frac{h}{a} \quad (16)$$

Using equation (16) in (14) and (15), we get

$$K_E' = \frac{1}{2} \rho P^2 \int_0^a \int_0^a \bar{h} \bar{W}^2 dY dX \quad (17)$$

and

$$S_E' = \frac{1}{2} \int_0^a \int_0^a D_1 \left[ (\bar{W}_{xx})^2 + (\bar{W}_{yy})^2 + 2\nu \bar{W}_{xx} \bar{W}_{yy} + 2(1-\nu)(\bar{W}_{xy})^2 \right] dY dX \quad (18)$$

Put the modified values of kinetic energy and strain energy from equations (17) and (18) in equation (13), we get

$$\delta(S_E' - \lambda^2 K_E') = 0 \quad (19)$$

where

$$K_E' = \int_0^a \int_0^a \left( 1 + \beta \left( 1 - \sqrt{1 - \frac{x^2}{a^2}} \right) \right) \bar{W}^2 dY dX \quad (20)$$

and

$$S_E' = \frac{1}{2} \int_0^a \int_0^a \left( 1 - \alpha \left( 1 - \frac{x}{a} \right) \right) \left( 1 + \beta \left( 1 - \sqrt{1 - \frac{x^2}{a^2}} \right) \right) \times \left[ (\bar{W}_{xx})^2 + (\bar{W}_{yy})^2 + 2\nu \bar{W}_{xx} \bar{W}_{yy} + 2(1-\nu)(\bar{W}_{xy})^2 \right] dY dX \quad (21)$$

Here  $\lambda^2 = \frac{12\rho P^2 a^2 (1-\nu^2)}{E_0 h_0}$  is the frequency

parameter.

Equation (19) consists of two unknown constants A and B. They are generating due to the substitution of deflection function W (x, y) to the clamped and simply supported boundary conditions.

Now, two constants, A and B, are to be determined as follows:

$$\left. \begin{aligned} \frac{\partial}{\partial A} (S_E' - \lambda^2 K_E') &= 0, \\ \frac{\partial}{\partial B} (S_E' - \lambda^2 K_E') &= 0 \end{aligned} \right\} \quad (22)$$

On simplification equation (22), we get

$$d_{n1} A + d_{n2} B = 0, \quad n = 1, 2 \quad (23)$$

where  $d_{n1}$  and  $d_{n2}$  include structural parametric constants. For a non-trivial solution, the determinant of the coefficient matrix must be equal to zero.

$$\begin{vmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{vmatrix} = 0 \quad (24)$$

### 4. RESULT AND DISCUSSION

All calculations are carried out with the help of latest computer software (MAPPLE). To study the first two modes of vibrations, computation has been done for frequency of visco-elastic square structure plate with mixed boundary condition (C-C-C-C and SS-SS-SS-SS) for different values of taper constants  $\beta$  and thermal gradient  $\alpha$ .

In **Table 1**, the value of frequency decreases as value of thermal gradient  $\alpha$  increases from 0.0 to 1.0 for  $\beta = 0.0$ ,  $\beta = 0.4$ , and  $\beta = 0.8$  for both modes of vibration.

In **Table 2**, the value of frequency increases sharply as value of taper constant  $\beta$  increases from 0.0 to 1.0 at fixed value of thermal gradient  $\alpha$  for both modes of vibration.

In **Table 3**, here, value of frequency decreases as value of thermal gradient  $\alpha$  increases from 0.0 to 1.0 for  $\beta = 0.0$ ,  $\beta = 0.4$ , and  $\beta = 0.8$  for both modes of vibration.

In **Table 4**, we observed that the value of frequency increases sharply as value of taper constant  $\beta$  increases from 0.0 to 1.0 at fixed value of thermal gradient  $\alpha$ .

**Table 1.** Frequency parameter ( $\lambda$ ) of visco-elastic square plate for both Modes on different values of thermal gradient ( $\alpha$ ) with clamped boundary condition

$\beta \rightarrow$		0.0		0.4		0.8	
$\alpha \downarrow$	Sides 1-2-3-4	Mode 1	Mode 2	Mode 1	Mode 2	Mode 1	Mode 2
0	C-C-C-C	35.9997	140.8842	39.4346	153.8729	43.3509	169.1401
0.2	C-C-C-C	34.15241	133.6545	37.6372	146.9016	41.57819	162.4041
0.4	C-C-C-C	32.1991	126.0106	35.7476	139.583	39.72	155.3778
0.6	C-C-C-C	30.11959	117.8722	33.74975	131.8595	37.76199	148.0206
0.8	C-C-C-C	27.8853	109.1284	31.6218	123.6555	35.6844	140.2811
1.0	C-C-C-C	25.4557	99.62018	29.33373	114.8688	33.4601	132.0936

**Table 2.** Frequency parameter ( $\lambda$ ) of visco-elastic square plate for both Modes on different values of taper constant ( $\beta$ ) with clamped boundary condition

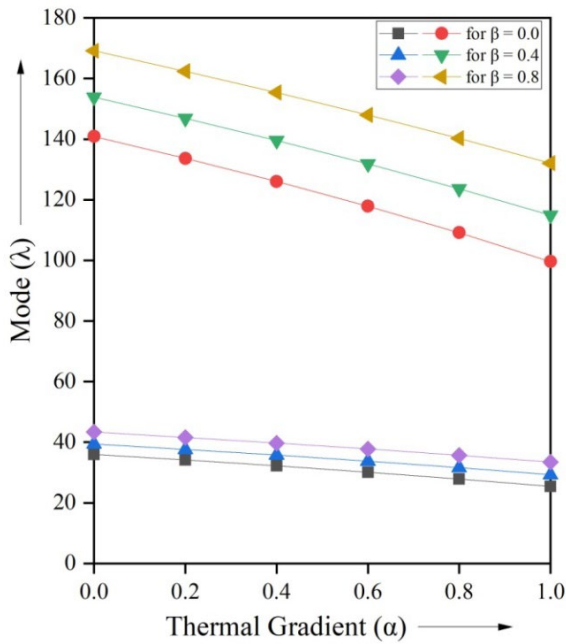
$\alpha \rightarrow$		0.0		$\alpha=0.4$		$\alpha=0.8$	
$\beta \downarrow$	Sides 1-2-3-4	Mode 1	Mode 2	Mode 1	Mode 2	Mode 1	Mode 2
0	C-C-C-C	35.9997	140.8842	32.1991	126.0106	27.8853	109.1284
0.2	C-C-C-C	37.64798	147.0709	33.91064	132.4935	29.70353	116.1007
0.4	C-C-C-C	39.4346	153.8729	35.7476	139.583	31.6218	123.6555
0.6	C-C-C-C	41.34142	161.245	37.68992	147.2283	33.6206	131.7345
0.8	C-C-C-C	43.3509	169.1401	39.72007	155.3778	35.6844	140.2811
1.0	C-C-C-C	45.44814	177.512	41.82359	163.982	37.80191	149.2434

**Table 3.** Frequency parameter ( $\lambda$ ) of visco-elastic square plate for both Modes on different values of thermal gradient ( $\alpha$ ) with simply-supported boundary condition

$\beta \rightarrow$		0.0		0.4		0.8	
$\alpha \downarrow$	Sides 1-2-3-4	Mode 1	Mode 2	Mode 1	Mode 2	Mode 1	Mode 2
0	SS-SS-SS-SS	19.78581	140.4847	21.55784	155.7269	23.5801	174.0061
0.2	SS-SS-SS-SS	18.77047	133.2755	20.56522	149.0322	22.60192	167.7517
0.4	SS-SS-SS-SS	17.69697	125.6533	19.52188	142.0224	21.57857	161.2549
0.6	SS-SS-SS-SS	16.554	117.5379	18.4191	134.6481	20.50302	154.4853
0.8	SS-SS-SS-SS	15.32602	108.819	17.24529	126.846	19.36623	147.4054
1.0	SS-SS-SS-SS	13.99068	99.33766	15.98452	118.5315	18.15612	139.9681

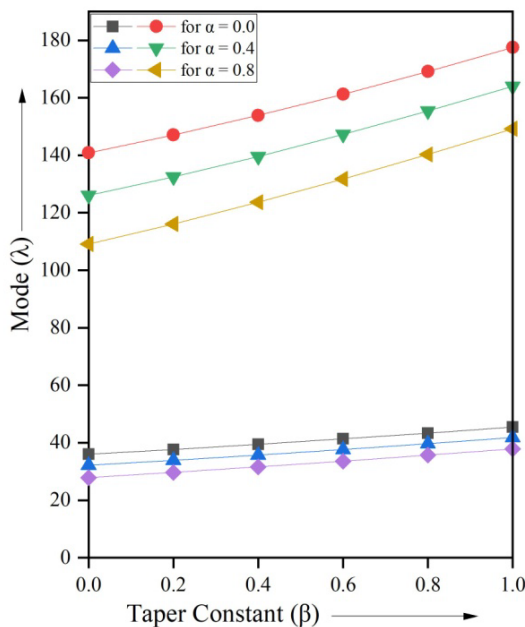
**Table 4.** Frequency parameter ( $\lambda$ ) of visco-elastic square plate for both Modes on different values of taper constant ( $\beta$ ) with simply-supported boundary condition

$\alpha \rightarrow$		0.0		0.4		0.8	
$\beta \downarrow$	Sides 1-2-3-4	Mode 1	Mode 2	Mode 1	Mode 2	Mode 1	Mode 2
0	SS-SS-SS-SS	19.78581	140.4847	17.69697	125.6533	15.32602	108.819
0.2	SS-SS-SS-SS	20.63732	147.6848	18.57716	133.4344	16.25743	117.4678
0.4	SS-SS-SS-SS	21.55784	155.7269	19.52188	142.0224	17.24529	126.846
0.6	SS-SS-SS-SS	22.54082	164.5279	20.52437	151.3248	18.28335	136.8543
0.8	SS-SS-SS-SS	23.5801	174.0061	21.57857	161.2549	19.36623	147.4054
1.0	SS-SS-SS-SS	24.67005	184.0845	22.67909	171.7339	20.48924	158.4243



**Figure 3.** Frequency (mode I and mode II) v/s thermal gradient for CCCC.

From **Table 1** and **Figure 3**, it is clearly seen that value of frequency decreases as value of thermal gradient  $\alpha$  increases from 0.0 to 1.0 for  $\beta = 0.0$ ,  $\beta = 0.4$ , and  $\beta = 0.8$  for both modes of vibration with clamped boundary condition.

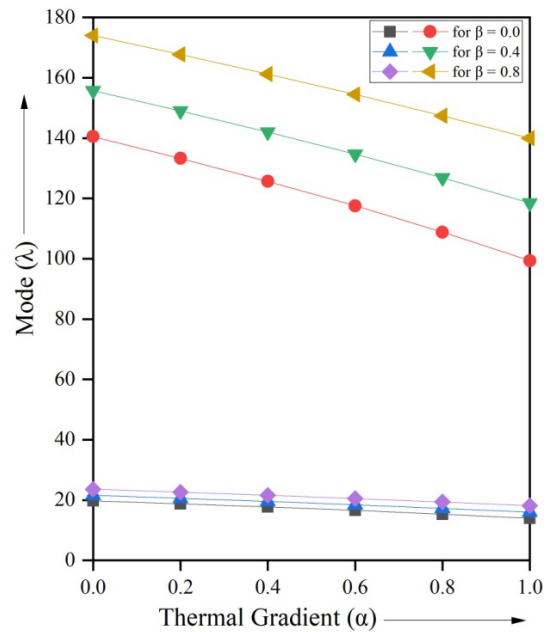


**Figure 4.** Frequency (mode I and mode II) v/s taper constant for CCCC

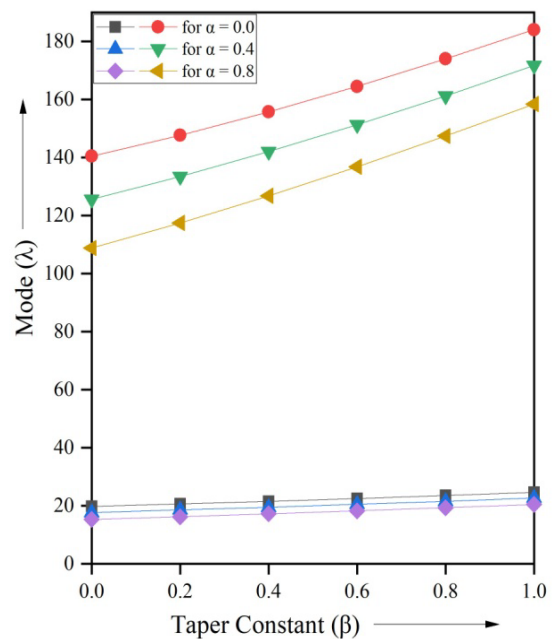
In **Table 2** and **Figure 4**, we observed that the value of frequency increases sharply as value of taper constant  $\beta$  increases from 0.0 to 1.0 at fixed value of

thermal gradient  $\alpha$  for both modes of vibration for clamped boundary condition.

From **Table 3** and **Figure 5**, it is clearly seen that value of frequency decreases as value of thermal gradient  $\alpha$  increases from 0.0 to 1.0 for  $\beta = 0.0$ ,  $\beta = 0.4$ , and  $\beta = 0.8$  for both modes of vibration for simply-supported boundary condition.



**Figure 5.** Frequency (mode I and mode II) v/s thermal gradient for SS-SS-SS



**Figure 6.** Frequency (mode I and mode II) v/s taper constant for SS-SS-SS

In **Table 4** and **Figure 6**, we observed that the value of frequency increases sharply as value of taper constant  $\beta$  increases from 0.0 to 1.0 at fixed value of thermal gradient  $\alpha$  for simply-supported boundary condition.

## 5. CONCLUSION

In conclusion, we conclude that for both boundary conditions (clamped, simply supported):

- i). The value of frequency is decreases when the thermal gradient increases throughout the study.
- ii). In the overall research, the frequency value increases as the taper constant increases.
- iii). The current paper provides some appropriate numerical data of frequency modes, which will be very useful to researchers, engineers, and scientists when designing final mechanical structures.
- iv). Therefore, engineers and technocrats are advised to see and used the findings of this paper before finalizing any design of a machine or mechanical equipments so that they can produce more effective and reliable structures.

## ABBREVIATIONS

Symbol	Description
$x, y$	Cartesian coordinates of the plate
$M_x$	Bending moment in x-direction
$M_y$	Bending moment in y-direction
$M_{xy}$	Twisting moment per unit length
$a$	Side of the square plate
$w(x, y, t)$	Displacement
$D_I$	Flexural rigidity.
$\mathcal{D}$	Visco-elastic operator
$\nu$	Poisson ratio of the plate material
$\rho$	Mass density per unit volume of the plate material
$h$	Thickness of the plate at $(x, y)$
$W(x, y)$	Deflection function
$T(t)$	Time function of triangular plate
$t$	Time
$p^2$	Frequency constant
$h_0$	Thickness of the plate at $(x, y) = (0, 0)$
$E$	Young's modulus
$K_E$	Kinetic energy
$S_E$	Strain energy
$\beta$	Taper constant
$\tau$	Temperature excess above the reference temperature at any point

$\tau_0$	Temperature excess above the reference temperature at $\xi = 0$
$\alpha$	Thermal gradient
$A, B$	Two unknown constants of deflection function
$E_0$	Young's modulus at reference temperature
$\lambda$	Frequency parameter

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