
Modal Analysis of the Inertial Platform of the Laser ELI-NP Facility in Magurele-Bucharest

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Abstract: - Starting from the need to determine the deformability of the inertial platform from the Research Facility of ELI-NP in Magurele-Bucharest in case of changing the position of some very heavy partition walls, a vibration study of the entire inertial platform was made. The research team studied various aspects of this issue and some of the results were published. The paper presented the obtained results of the dynamic analysis, in a unitary approach and with the conclusions that emerge from these researches. The results may be useful in the study of problems of anti-vibration isolation of very large masses, generally used in research infrastructure.

Keywords: - inertial mass, modal analysis, vibration, shock absorber, nuclear physics

1. INTRODUCTION

In the project it was designed and built in a system for transforming optical photons into energy photons with gamma rays up to 20 MeV. The latter will have an unprecedented quality, especially in terms of low bandwidth and very high spectral density, representing a substantial improvement of existing systems. ELI-NP was funded by the European Commission and consists, at Măgurele-Bucharest, in a modern research infrastructure. It is hoped that this will enable advanced applications in the field of

national security, nuclear medicine, nuclear residues processing and fundamental studies in nuclear physics. This infrastructure aims to achieve the following objectives: adjustable photon energy in the range of 1-20 MeV, rms bandwidth less than 0.5% and spectral density greater than 104 photons / sec. eV, with a source spot size of less than 100 microns and a linear polarization of a gamma-ray beam greater than 95%. Such ambitious objectives also require a very good design and a special execution precision. Is important to see if the stability and vibration isolation is achieved. The whole project, with all the objectives

that have been achieved are presented in [1-5] (Figure 1). Extremely precise experiments to be carried out on the platform cannot be performed if the platform is not perfectly isolated from the vibrations coming from human activities or as a result of some natural phenomena (earthquakes). Consequently, special attention must be paid to the platform on which the research infrastructure will be installed. The particularly high weight of the platform can help to isolate it. In essence, this platform is a special engineering achievement both due to its very large size and weight but also due to the anti-vibration qualities it can provide.

For the platform, first of all, an insulation to the usual vibrations must be provided, due to the current human activities, which propagate from the outside of the infrastructure to the inertial mass. In order to ensure this insulation in the project, batteries with springs and viscous dampers were used. These batteries are in large numbers and support the entire weight of the platform. They are distributed approximately homogeneously, to ensure a uniform weight distribution on the supports. This also ensures a uniform distribution of the entire weight on the ground. A second protection must be provided against the phenomena that occur less frequently, due to natural causes, namely earthquakes. The damage due to such phenomena can be very great. We note that

the region in which the research infrastructure was built is subject, historically, to such phenomena with a periodicity of approx. 40 years. It is an economic problem of great importance, the price of the equipment found on the platform being extremely high. Thirdly, the other equipment on the platform is also sources of vibration that should not be transmitted. The last source of vibrations can be isolated easily, by carefully designing the spring batteries and by judiciously placing the equipment on the platform. Research to isolate large masses from external vibrations has long been conducted [5,6]. Following these researches and the experience gained following the development of the construction field, at present the support of large masses is made using metal springs and associated shock absorbers ensuring the shock response and the necessary damping [7]. In this case, if we take into account the high costs as well as the performances that must be met, it is necessary to develop the best possible model at the lowest costs.

If the deformability of the platform is neglected and it is considered to be modeled as a rigid with 6 degrees of freedom (DOF), the mechanical response has been studied by numerous researchers in papers summarized in [8]. They have been used from classical methods to unconventional methods such as genetic sorting algorithms [9].

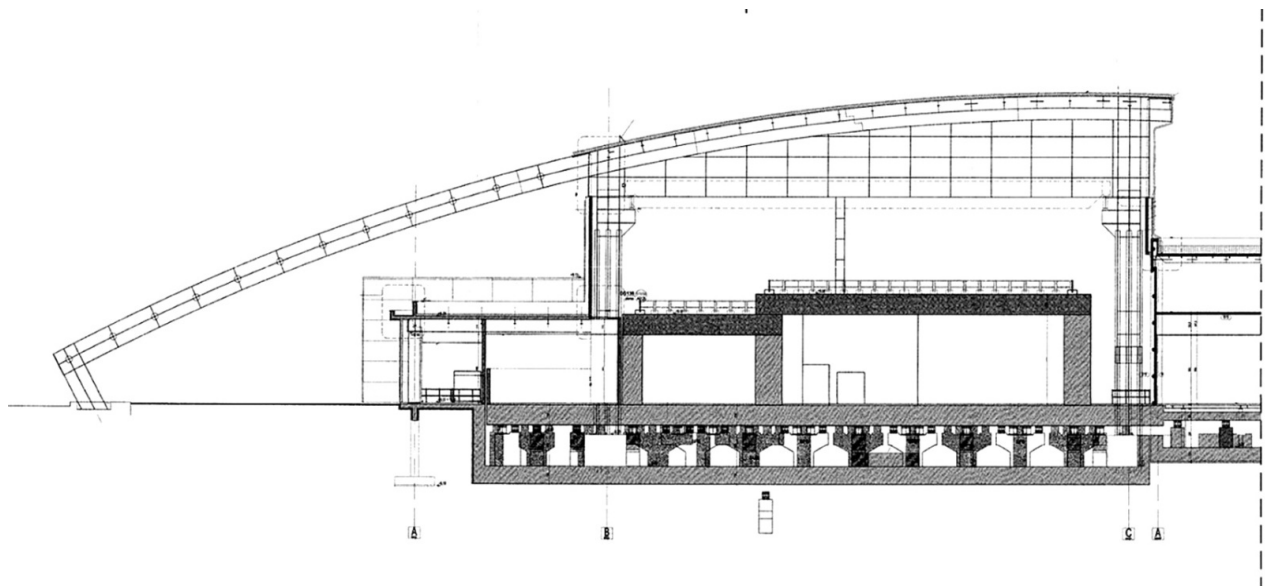


Figure 1. Platform. Lateral view [1,2]

Other more complex models of vibration isolation or active isolation methods are presented in [10], [11]. Such systems are generally robust and provide very good performance.

External sources of vibration are mainly due to transport activities. These activities take place constantly in a city. The analysis for this type of

systems is presented in [12]. Analytical models for vibration isolation for this type of excitation are described in [13]. Vibrations due to natural phenomena (earthquakes) are generally low frequency and their effect has been studied [14]. They can be horizontal or vertical and a way of isolation in front of these phenomena is proposed in the mentioned paper.

Another method of reducing the effect of vibrations is proposed in [15]. In our case of application in nuclear physics, some results are described in the literature [16–19].

2. PLATFORM DESCRIPTION

The inertial platform on which the research instruments and equipment will be positioned represents a massive concrete block, made of two

concrete slabs, one main, with a thickness of 1.6 m and the other, secondary, with a thickness of 0.6 m, rigidly connected to each other.

The block has remarkable dimensions and a very high weight. The geometric and mass data of this ensemble are presented in the paper. This massive block is supported on spring batteries and dampers. In a first approximation, this block can be considered a rigid, suspended on elastic elements and having a number of 6 degrees of freedom.

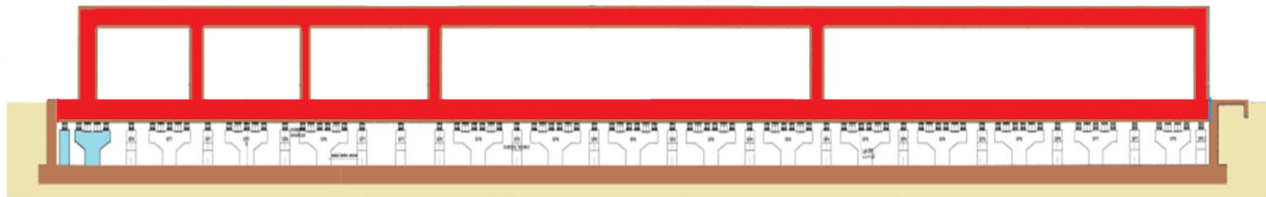


Figure 2. The platform suspended on the batteries of springs [20]

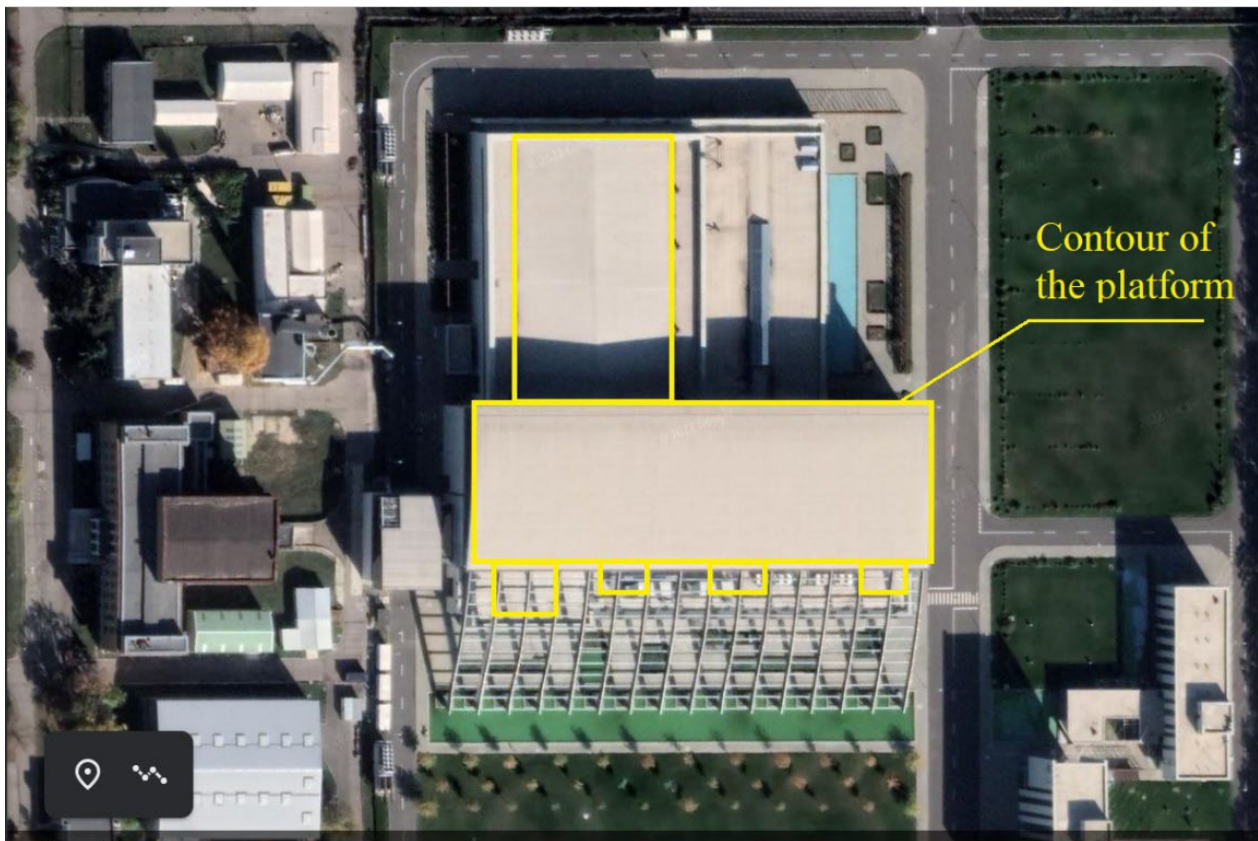


Figure 3. The platform suspended on the batteries of springs [21]

However, due to its very large size, the block can also have an elastic behavior which, in a more careful study, must be taken into account. In this case, concrete is considered as an elastic material. The study of the vibrations of this platform was done in both cases, in first case considering it as a rigid body elastic suspended on springs and in the second case considering the platform as a block consisting of an

elastic concrete plate, suspended on spring batteries, which may have a deformability, perpendicular to the plane of block, due to the elasticity of the concrete.

Following such an analysis of the fully equipped inertial platform, predictions can be made regarding the behavior of the equipment installed on the platform, in case of external vibrations. A conclusion resulting from the research is that the natural

frequencies due to the elasticity of the concrete must be considered in a study of such system and that the elasticity cannot be neglected.

Figure 2 shows the concrete platform suspended on the spring batteries. There is a distance of 10-15 cm between the concrete block and the ground, thus ensuring a lateral insulation of the concrete block.

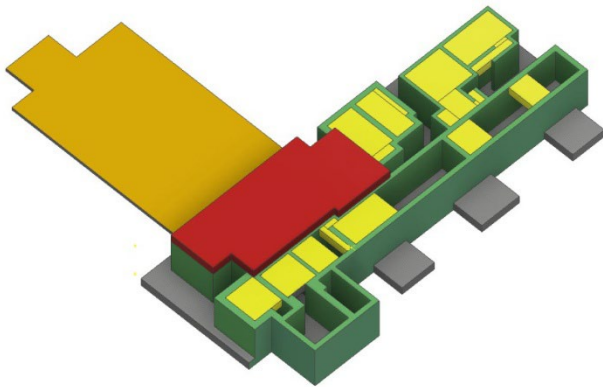


Figure 4. The 3D model of the platform used in analysis [20]

The concrete platform is located inside the Research Infrastructure building. The research equipment is placed on the suspended platform. Also on the platform are mounted massive partition walls and roofs that insulate the research rooms. Figure 3 shows the positioning of the platform within the Research Infrastructure building.

The body of the platform modeled for the studies that are done in the paper is presented in Figure 4, where the partitions and the roof of the platform are positioned.

The machinery and equipment that will be used to the Research Infrastructure are very varied, most of them having to work in strict isolation conditions and to ensure a high precision, in specific operating conditions. They are located on the inertial platform which, with all the equipment mounted on it, in operation, can reach 54,000 tons. The role of the platform is to allow the installation of all the necessary research equipment and at the same time to ensure a proper isolation in terms of shocks and vibrations, both vibrations from the outside and the transmission of vibrations from one machine to another.

The platform is placed on a number of approximately 1,000 batteries equipped with elastic springs and shock absorbers [21-24]. In this way the weight of the platform is distributed approximately evenly on the floor of the assembly. The batteries with elastic springs can be activated or not, this being done at the recommendation of the designers, depending on the research equipment that will be

installed above the platform. The mechanical properties of springs and shock absorbers are known.

The concrete platform has a high rigidity and can be treated as a rigid with 6 degrees of freedom. The dimension of the concrete platform can be seen in Figure 5.

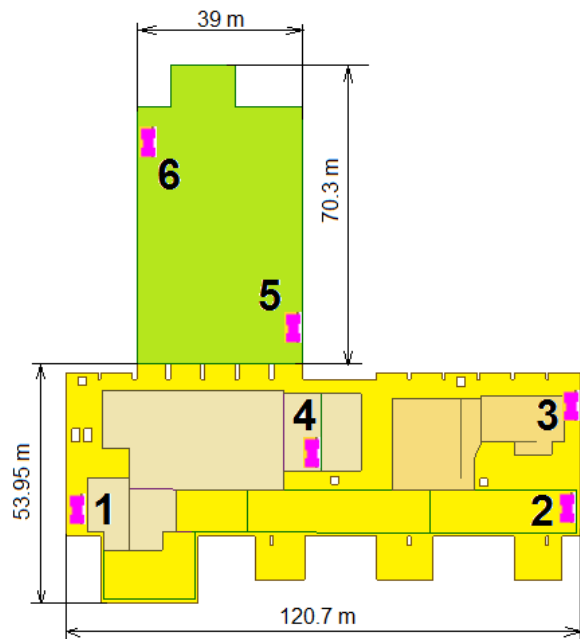


Figure 5. The dimension of the concrete platform

3. MODAL ANALYSIS OF THE RIGID CONCRETE PLATFORM

The platform is supported on the ground by means of arches. The stiffness of a single spring is $k_z = 1.85 \times 10^6 \text{ N/m}$ (in vertical direction) and $k_x = k_y = 0.55 \times 10^6 \text{ N/m}$ for the other two directions (in plane). The shock absorbers have the damping coefficient $c_x = c_y = c_z = 2 \times 10^6 \text{ Ns/m}$.

The platform is rigid enough to support the location of partitions, roofs and research equipment. There are no problems with the breaking strength of the platform material.

In a first approximation, for the study of the vibrations that the very heavy platform can have, placed on elastic springs and shock absorbers, a model can be used that considers the whole platform as a rigid one with 6 DOF. This model has the advantage of simplicity, the calculations to be made being easy to perform.

The platform, as a whole, is reinforced, both at the bottom and at the top, by a network of profiled steel bars of concrete type B500 (with resistance class 5, and flow limit of at least 500 N/mm²), ductility category C (elongation at breaking > 16%, elongation at maximum force > 7.5%, the ratio between the breaking strength and the yield strength R_m/R_e ($R_p = 0.2$)).

The support pillars of the anti-vibration plate have two branches each, arranged symmetrically with respect to the vertical axis. The perimeter of the vertical walls afferent to the infrastructure of the two buildings, is separated from the anti-vibration plate, along the entire length of an open, continuous joint, having at least 100 mm.

The concrete used is class C35/45, made with CEM III / A cement 42.5 N-LH (reduced hydration with resistance class $42.5 N/mm^2$) and the water/cement ratio is 0.2. A layer of cement-based screed reinforced with polypropylene fibers type CT-C35, having a compressive strength of at least $35 N/mm^2$, is placed on top of the plate. The thickness of this screed is 10 cm and its role is to allow the fixing of the various components of the research equipment.

There is an elastic platform support system consisting of a system of Maurer Schone springs and shock absorbers, attached to the ends of the reinforced concrete columns. The system is equipped with a system of screws. By tightening it activates the springs and in the opposite direction deactivates the springs. The height of the battery is 540 mm. The viscous dissipative system attached to the spring battery consists of cylindrical metal containers filled with a high viscosity material of the Gerb Schwingungsisolierungen type.

The floor ensures the conditions of installation / assembly of equipment. Following the evaluation of the design project of the anti-vibration plate (upper platform) related to the Gamma building, it is found that the requirement of mechanical resistance is ensured.

The thickness of the plate is appropriate, ensuring both rigidity and mechanical strength, the number of support points is high ensuring a uniform weight distribution on the foundation and the free distance between the columns of no more than 3.00 m (usually 2.80 m). The stability requirements and the normal deformation limit state are thus met. The cement-based screed floor reinforced with polypropylene fibers has the necessary strength and rigidity to the demands given by permanent loads / due to operation.

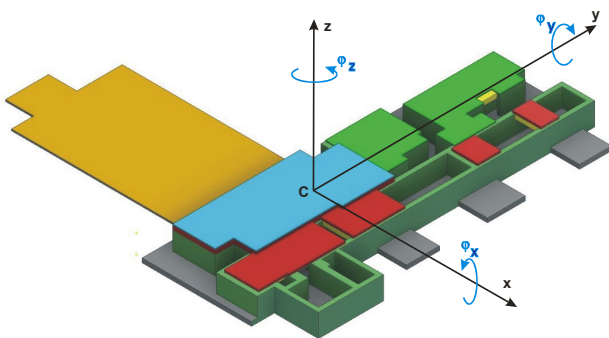


Figure 6. The central system of axis and the coordinates of the concrete platform

The coordinates used for the concrete platform modeled as a rigid body with 6 degrees of freedom are (Figure 6):

- x - lateral translation of the center of mass C;
- y - longitudinal translation of the center of mass C;
- z - vertical translation of the center of mass C;
- φ_x - rotation of the platform around the axis Cx;
- φ_y - rotation of the platform around the axis Cy;
- φ_z - rotation of the platform around the axis Cz

In order to determine the inertial characteristics, a geometric model was considered, which considered that the material of the platform is homogeneous, made of reinforced concrete, with a density $\rho = 2600 kg/m^3$. The total mass, as well as the second order moments of inertia, were determined using CAD in Autodesk Inventor® 2019.

The main inertial characteristics of the Gamma+Laser monolithic platform with radiation protection walls and ceilings and reinforced concrete roofs are presented as follows:

$$\begin{aligned}
 m &= 54,503,232 \text{ kg} \\
 J_{xx} &= 60,862,012,628 \text{ kgm}^2 \\
 J_{yy} &= 22,345,573,263 \text{ kgm}^2 \\
 J_{zz} &= 82,095,330,277 \text{ kgm}^2 \\
 J_{xy} &= J_{yx} = -1,588,948,899 \text{ kgm}^2 \\
 J_{yz} &= J_{zy} = 364,851,662 \text{ kgm}^2 \\
 J_{zx} &= J_{xz} = -534,258,413 \text{ kgm}^2,
 \end{aligned}$$

where m is the total mass of the platform and J_{xx} , J_{yy} , J_{zz} , $J_{xy}=J_{yx}$, $J_{yz}=J_{zy}$ and $J_{zx}=J_{xz}$ are the moments of inertia of the solid-rigid platform.

Considering the central axis system **Cxyz** as in Figure 6, the mass/inertia matrix can be written as follows [25-27]:

$$[M] = \begin{bmatrix} m & 0 & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & 0 & 0 & J_{xx} & -J_{xy} & -J_{xz} \\ 0 & 0 & 0 & -J_{yx} & J_{yy} & -J_{yz} \\ 0 & 0 & 0 & -J_{zx} & -J_{zy} & J_{zz} \end{bmatrix} \dots (1)$$

The inertia matrix can be partitioned in the form [27]:

$$[M] = \begin{bmatrix} [M_{11}] & [M_{12}] \\ [M_{21}] & [M_{22}] \end{bmatrix}, \quad (2)$$

where:

$$[M_{11}] = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix} \text{ kg} \quad (3a)$$

$$[M_{12}] = [M_{21}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \equiv [0_3] \text{ kgm} \dots (3b)$$

$$[M_{22}] = \begin{bmatrix} J_{xx} & -J_{xy} & -J_{xz} \\ -J_{yx} & J_{xx} & -J_{yz} \\ -J_{zx} & -J_{zy} & J_{xx} \end{bmatrix} kgm^2 \quad (3c)$$

After some calculations the inertia matrix becomes:

$$[M] = 10^6 \begin{bmatrix} 54.5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 54.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 54.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 60862 & 1588.9 & 534.26 \\ 0 & 0 & 0 & 1588.9 & 22346 & -364.9 \\ 0 & 0 & 0 & 534.26 & -364.9 & 82095 \end{bmatrix} \quad (4)$$

The model of the elastic support system with steel helical springs (Maurer) was performed considering the geometric model of the Gamma+Laser concrete platform. For this the linear elastic characteristics from Table 1 were taken into account.

Table 1. The elastic characteristic of the springs

Springs Box (Type)	k_z [kN/mm]	$k_x = k_y$ [kN/mm]
B6-5	9.25	2.75
B6-6	11.1	3.3
B9-9	16.7	4.9

The 6×6 stiffness matrix can be written as follows:

$$[K] = \begin{bmatrix} [K_{11}] & [K_{12}] \\ [K_{21}] & [K_{22}] \end{bmatrix}, \quad (5)$$

where $[K_{ij}]$ $i, j = \overline{1,2}$ are 3×3 sub matrices:

$$\begin{aligned} [K_{11}] & N/m \\ [K_{12}] & N \\ [K_{21}] & N \\ [K_{22}] & Nm \end{aligned}$$

The stiffness matrix that characterizes in terms of the elastic connections of the Gamma + Laser platform (with walls, ceilings and roofs) was determined based on the positioning plan of the 468 supports of the platform (Gamma — 355 supports, Laser — 109 supports, Monolithic area — 4 supports) and the coordinates of the action points of the elastic elements and is:

$$[K] = 10^9 \begin{bmatrix} 7.424 & 0 & 0 & 0 & -26.933 & -22.607 \\ & 7.424 & 0 & 26.933 & 0 & -6.704 \\ & & 25.153 & 76.865 & 22.119 & 0 \\ & & & 27786 & -3130.4 & -6.566 \\ & SYMM. & & & 14128 & 89.138 \\ & & & & & 12322 \end{bmatrix} \quad (6)$$

For modeling the viscous damping system with Gerb devices, the linear damping coefficients in Table 2 were taken into account.

Table 2. GERB shock absorber characteristics.

Damping Coefficient	c_x	c_y	c_z
c [kNs/m]	2000	2000	2000

The 6×6 damping matrix for a rigid body with viscous damping supports can be written as follows:

$$[C] = \begin{bmatrix} [C_{11}] & [C_{12}] \\ [C_{21}] & [C_{22}] \end{bmatrix}, \quad (7)$$

where $[C_{ij}]$ $i, j = \overline{1,2}$ are 3×3 sub matrices:

$$\begin{aligned} [C_{11}] & Ns/m \\ [C_{12}] & Ns \\ [C_{21}] & Ns \\ [C_{22}] & Nsm \end{aligned}$$

After some calculus it results for damping matrix $[C]$:

$$[C] = 10^9 \begin{bmatrix} 1.486 & 0 & 0 & 0 & -5.385 & -3.089 \\ & 1.486 & 0 & 5.385 & 0 & -5.381 \\ & & 1.486 & 3.089 & 5.3812 & 0 \\ & & & 1866.5 & -162.31 & -15797 \\ & SYMM. & & & 896.16 & 12.678 \\ & & & & & 2602.9 \end{bmatrix} \quad (8)$$

The following results are obtained in the study of the eigenfrequencies of the platform, which is considered to be a rigid structure suspended on elastic supports, with 6 DOF (Table 3).

Table 3. The eigenfrequencies of the rigid platform with uncoupled movements [Hz]

DOF	x	y	z	φ_x	φ_y	φ_z
f_i [Hz]	1.857	1.857	3.419	3.401	4.002	1.950

The 6×6 dynamic matrix $[D]$ of the system can be calculated as follows:

$$[D] = [M]^{-1} \times [K] s^{-2} \quad (9)$$

After some calculus, dynamic matrix becomes:

$$[D] = \begin{bmatrix} 136.21 & 0 & 0 & 0 & -494.15 & -414.78 \\ 0 & 136.21 & 0 & 494.15 & 0 & -123.00 \\ 0 & 0 & 136.21 & 1410.3 & 405.83 & 0 \\ 0.341 & 0.4441 & 1.2394 & 461.1 & -68.107 & -1.597 \\ -1.212 & -0.033 & 0.9017 & -172.9 & 637.16 & 6.554 \\ -0.281 & -0.085 & -0.004 & -3.849 & 4.3607 & 150.14 \end{bmatrix} \quad (10)$$

Eigenvalues λ_i $i = \overline{1,6}$ and eigenpulsations $p_i = \sqrt{\lambda_i}$ $i = \overline{1,6}$ of the system can be calculated from the characteristic equation as follows:

$$\det([D] - \lambda[I_6]) = 0 \quad (11)$$

Solving the characteristic equation (11), result the eigenvalues, eigenpulsations and eigenfrequencies of the system with coupled movements (Table 4).

Table 4. The eigenfrequencies of the rigid platform with coupled movements [Hz]

Mode	1	2	3	4	5	6
λ [s ⁻²]	129.284	135.322	155.869	383.427	488.479	689.928
p [rad/s]	11.370	11.633	12.485	19.581	22.102	26.266
f [Hz]	1.810	1.851	1.987	3.116	3.518	4.180

The eigenvectors/Eigenshapes of the system can be obtained solving the matrix equations

$$[D]\{\phi_i\} = \lambda_i\{\phi_i\} \quad i = \overline{1,6} \quad (12)$$

Table 5. The eigenvectors of the rigid body platform

	Φ_1 for $f_1=1.810$	Φ_2 for $f_2=1.851$	Φ_3 for $f_3=1.987$	Φ_4 for $f_4=3.116$	Φ_5 for $f_5=3.518$	Φ_6 for $f_6=4.180$
x	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
y	0.2399	-3.9196	0.3176	-1.3301	-1.3054	0.2987
z	-0.0033	-0.0326	-0.0037	14.6218	-59.3530	0.0752
φ_x	0.0001	0.0063	-0.0001	-0.6653	-0.9302	0.3318
φ_y	0.0023	0.0044	0.0032	-0.5008	-0.7142	-1.1106
φ_z	0.0140	-0.0031	-0.0512	0.0006	0.0016	-0.0119

The corresponding eigenvectors are presented in Table 5 (normalized after the translational x).

4. DISCUSSIONS AND CONCLUSIONS

As already mentioned, the platform block consists of two plates, one main, where most instruments and devices are arranged and where the heavy, dividing walls and roofs are located, and the second smaller and thinner having about 1/6 of the total weight of the platform.

The paper summarized the main results obtained in the study of the response of the inertial platform suspended on sets of spring batteries, to see to what extent vibrations from the outside environment can influence the operation of the equipment within Research Infrastructure at ELI-NP Magurele-Bucharest. Anthropoc vibrations, in this case mainly from vehicles are non-stationary vibrations, transmitted through the ground. In order to isolate the platform from these vibrations, it is suspended on sets of batteries with elastic elements and shock absorbers. These batteries may or may not be activated, with or without an effect on the behavior of the platform.

The described system is a validated and useful tool for reducing the transfer of unwanted vibrations to the platform.

If we study the vibrations of the system consisting of the two platforms, main and secondary, rigidly connected, we find that the components of the modes corresponding to the secondary platform are significantly higher than the amplitudes corresponding to the main platform. It turns out that in the case of external excitations with frequencies close to natural frequencies, we can expect an amplification of these excitations at the secondary platform. For the designed construction the situation can be solved by activating additional springs from the spring batteries.

The modeling of the mechanical system of the platform placed on elastic supports considers in a first approach, that we have a rigid solid, placed on elastic supports, having 6 degrees of freedom. This modeling method does not take into account the elasticity of the concrete from which the platform is made. Following the calculation performed, in a first approximation, six eigenfrequencies and six eigenmodes were obtained. With the help of these results, which describe the rigid movement of the platform, a first study of the vibrations of this system can be made. At first glance these results may be useful to designers for a preliminary configuration of the platform.

In reality the platform is made of concrete that is deformable. This deformability can no longer be neglected, taking into account the dimensions and mass involved. A more accurate model of the platform, which also considers the elasticity of the concrete, must use a FEM model. In addition to the natural frequencies determined by considering the platform as a rigid body, there are expecting a number of new modes of vibration due to the elasticity of the concrete.

An important issue to note is that the platform's low eigenfrequencies are in the range in which periodic earthquakes (the periodicity for a large earthquake is about 40 years) have excitation frequencies. The modal analysis shows that there are a large number of natural frequencies below 5 Hz. In the event of a catastrophic event (earthquake), the movement of the platform can be amplified enough to destroy the system. The ELI-NP project has an earthquake decoupling system but it is necessary to study whether an additional whole mass isolation system is needed.

Let us further analyze what can happen if the platform is subjected to an excitation with an evolution of the accelerations shown in Figure 7 and Figure 8. This distribution of accelerations was obtained experimentally, following the earthquake in Vrancea-Romania of March 4 1977. An FFT analysis obtained the acceleration graph by frequency (Figure 9).

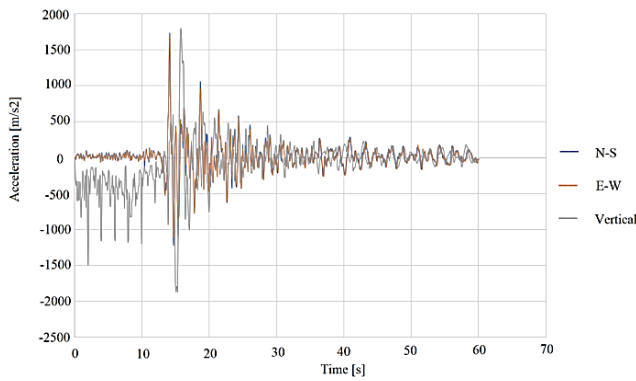


Figure 7. Spectrum of accelerations versus time during the 4 March 1977 Vrancea-Romania earthquake [28]

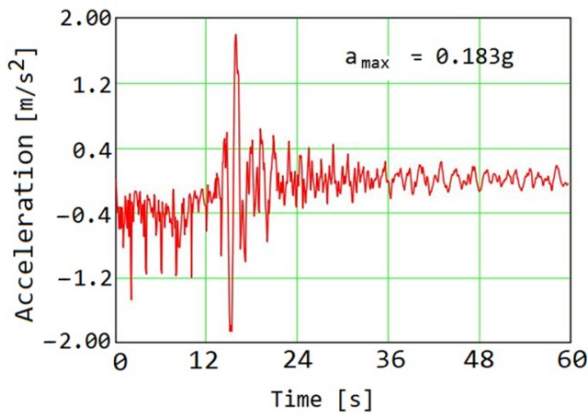


Figure 8. Vertical acceleration versus time 04.03.1977 Vrancea-Romania earthquake [28]

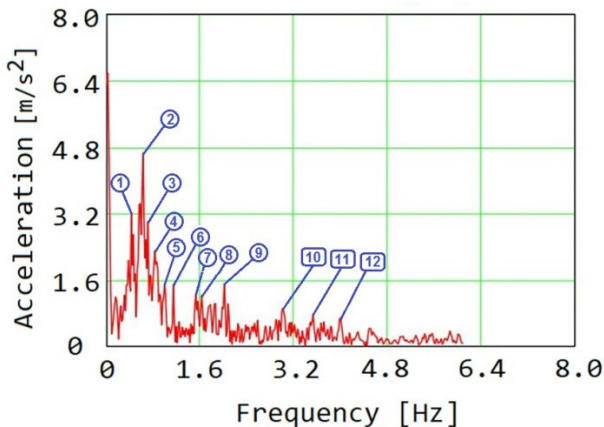


Figure 9. Vertical acceleration versus time 04.03.1977 Vrancea-Romania earthquake [28]

Table 6. The dominant frequencies of vertical acceleration (FFT analysis of Vrancea-Romania earthquake)

Point	1	2	3	4	5	6	7	8	9	10	11	12
f_i [Hz]	0.39	0.62	0.73	0.85	1.01	1.17	1.53	1.67	2.06	2.83	3.59	4.11
a [m/s ²]	3.26	4.70	3.04	2.37	1.50	1.47	1.28	1.22	1.47	0.90	0.77	0.70

Table 6 shows the values of vertical acceleration amplitudes for the dominant frequencies of the Romania-Vrancea 04.03.1977 earthquake obtained from the FFT analysis (Figure 9). The significant vertical frequencies of the earthquake are $f_1 = 0.39\text{Hz} \dots f_4 = 0.85\text{Hz}$ and the major dominant

frequency is $f_2 = 0.62\text{Hz}$ (yellow highlight). Taking into account the eigenfrequencies and the eigenvectors of the platform (Table 4, Table 5), dangerous earthquake frequencies are located in the domain $f_7 = 1.53\text{Hz} \dots f_{12} = 4.11\text{Hz}$ (red figures).

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