Magnetorheological Damper and Its Applications: Current Scenario and Future Prospects

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Abstract: - In recent years, magnetorheological dampers have been paid more attention because of their smart nature. This paper explains in detail the mathematical background of magnetorheological fluids, their use in MR dampers, and its applications in vibration control. The purpose of this paper is to delineate how to derive equations of motion for dynamics of MR dampers for different degrees of freedom and how to find their responses for various damping forces. In this paper, the performance of a MR damper are compared by MATLAB simulation.

Keywords: - Magnetorheological Fluid, Magnetorheological Damper, Vibration Control.

1. INTRODUCTION

Even though magnetorheological fluids have existed for over 65 years, there has been generally little enthusiasm for the innovation as of not long ago. Amid the late 1940s and early 1950s, there was a flurry of interest in magnetorheological fluids and dependent devices.

As of late, there has been a renewed enthusiasm for magnetorheological fluid-based gadgets. For example, Lord Corporation has been creating magnetorheological fluids and magnetorheological fluid-based gadgets since the mid-1990s. In the mid-1990s, Lord Corporation started to develop and fabricate magnetorheological dampers. Magnetorheological dampers are among the most encouraging SAS system utilized these days in prosthetic legs and vehicle suspension systems [1, 2]. In addition, Lord Corporation also creates a rotary magnetorheological brake system that has been used for various applications.

MR dampers are not, in any case, confined to vehicle applications. Recently, the U.S. military has indicated enthusiasm for utilizing magnetorheological dampers to adjust gun recoil on Naval gun turrets and field artillery. Magnetorheological dampers are also utilized in the stabilization of buildings during earthquakes [1, 3].

We expect that the mechanical properties of magnetorheological fluids make them appropriate for some advanced engineering applications [2]. Notwithstanding dampers and brakes, magnetorheological fluid can be utilized in a variety of other applications, including clutches, prosthetic devices, polishing and controlling devices. The performance of magnetorheological fluid-based devices is based upon the shear strength of magnetorheological fluid. Magnetorheological fluids and magnetorheological fluid devices have been enormously advanced in the most recent decade and there some commercial types of equipment have been created.

The damper is a device that decreases the amplitude of electronic, mechanical, acoustical, or aerodynamics vibrations or oscillations. A vehicle damper is usually known inside the automotive industry as a shock absorber. Vehicle dampers dissipate energy, where springs and tires absorb shocks in a suspension [1, 3, 4]. The damper system is more popular for vibration control of structures, because of its safe, effective, and economical design.

As of late, much consideration has been paid to the research and development of structural control techniques, for example, passive control framework, active control framework, and semi-active control framework giving unique significance to enhancement of suspension and vibration control.

Passive control frameworks do not require any external power supply. Active control frameworks require an external power supply and work dependent on sensors that are attached inside the structures. A semi-active control framework is a combination of both passive and active control frameworks, which require an external power supply and they work dependent on sensors attached to inside the structures. However, when there is no external power supply on the framework, passive control systems control the vibration of structures [5].

The magnetorheological damper is one of the advanced semi-active framework device, which contains magnetorheological fluid and an electrical circuit for the magnetic field. The overall performance and damping force of the magnetorheological damper can be adjusted by changing the viscosity of the magnetorheological fluid. The main advantage of MR damper is that it is sensitive and guaranteed to work under heavy loads.

2. MAGNETORHEOLOGICAL (MR) DAMPER

A Magnetorheological damper is one of the most popular and common applications of MR fluids. The magnetorheological dampers are not altogether different from the conventional fluid dampers the only difference being the magnetorheological fluid and the system that produces dynamic magnetic field strength. MR damper commonly comprises a cylinder, piston, magnetic system, bearing, seal, and cylinder filled with magnetorheological fluid. The fluid variable viscosity makes it perfect and ideal for use in dampers for suspension and vibration control. The continuously adjustable systems can be created to adjust damping coefficient based on certain physical estimations, for example, velocity or acceleration, in order to better oppose and control the system movement. Magnetorheological dampers are much similar to regular viscous fluid dampers in construction, but the regular damper valves system is replaced with an electrical circuit or magnet to adjust the art of magnetorheological fluid.

Magnetorheological dampers are used for a variety of different applications in science and engineering. The popularity of these dampers has led to extensive research and development to produce very controllable dampers with long life spans. The MR damper is one of the most important parts of a vehicle suspension system.

There are three types of MR dampers utilized in suspension and control systems [3]; Monotube, Twintube and Double-ended. Of the three kinds, the monotube damper is the most widely recognized it tends to be set in any direction and is compact in size.

3. CONSTRUCTION AND WORKING OF MR DAMPER

The mechanical arrangement of the cylindrical type MR damper proposed in this paper is shown in Figure 1. This damper consists of a piston, piston rod, cylinder, gas chamber, and magnetorheological fluid. The moving piston between the cylinder and the gas chamber is associated to make up for the volume induced by the movement of the piston. In addition, the gas chamber which is filled with nitrogen gas works as an accumulator for absorbing sudden force variation of the lower chamber of the damper induced by the rapid movement of the piston.

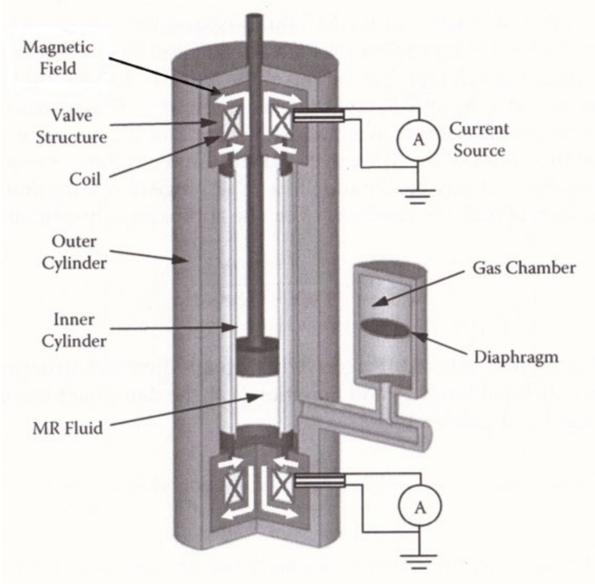

Figure 1. Schematic diagram of MR damper

In this damper system, upper and lower chambers are separated by a moving piston, and it is filled with magnetorheological fluid. The moving piston controls the flow of fluid between the adjacent fluid chambers. The magnetic poles in the piston head are utilized to adjust the yield stress of the magnetorheological fluid by providing magnetic force. In order to create and control the magnetic force in the magnetic pole. Without external magnetic force, the magnetorheological damper produces a damping force due to the resistance of the base fluid. Nonetheless, if a specific level of magnetic force is provided to the piston head, the MR damper produces an extra damping force. The variable damping force of this damper can be continuously controlled and tuned by adjusting the strength of the applied magnetic force [6, 7].

Magnetorheological dampers have been produced in different forms, sizes, and designs. The structure and designs of the MR damper were developed based on three main aspects a) Mode of operation, b) cylinder and c) piston structure.

For analysis and modeling of the magnetorheological damper, it is considered that the magnetorheological fluid is incompressible and that pressure of the fluid is uniformly distributed in one chamber. In addition, the pressure drops are ignored because the geometric structure of the annular duct and the fluid inertia are considered to be very small.

By considering the quasi-static behavior of the magnetorheological fluid, the damping force of the magnetorheological damper can be represented as [8];

$$F_d = P_l A_p - P_u\left(A_p - A_s\right) \quad (1)$$

where

A_p = Piston Area,

A_s = Piston − shaft area,

P_u = Pressure in the upper chamber of the MR damper and

P_l = Pressure in the lower chamber of the MR damper

The relation between P_u, P_l and the total pressure on the gas chamber P_g, can be written as [4];

$$P_l = P_g + \Delta P_2; \quad P_u = P_g - \Delta P_1 - \Delta P_3 \quad (2)$$

where, ΔP_1 is the pressure drop of fluid flow through the lower MR valve orifice, ΔP_2 is pressure drop of fluid flow through the upper MR valve orifice, and ΔP_3 is the pressure drop of fluid flow through the annular duct between the inner and outer cylinder.

The total pressure on the gas chamber determined by using the formula;

$$P_g = P_0\left(\frac{V_0}{V_0 + A_s x_p}\right)^{\gamma} \quad (3)$$

where,

P_0 is the initial pressure of the accumulator,

V_0 is the initial volume of the accumulator,

γ is the coefficient of thermal expansion,

x_p is the piston movement.

By ignoring the minor losses, the pressure drops ΔP_1 , ΔP_2 , ΔP_3 can be calculated as;

$$\Delta P_1 = \frac{12\eta L_m}{\pi t_m^3 R_1}(A_p - A_s)\dot{x}_p + 2c\frac{L_m}{t_m}\tau_y \quad (4)$$

$$\Delta P_2 = \frac{12\eta L_m}{\pi t_m^3 R_1}A_p\dot{x}_p + 2c\frac{L_m}{t_m}\tau_y \quad (5)$$

$$\Delta P_3 = \frac{6\eta L}{\pi t_g^3 R_2}(A_p - A_s)\dot{x}_p \quad (6)$$

where,

τ_y = Yield stress of MR fluid induced by magnetic force,

n = Plastic viscosity of Base fluid,

L = Length of the inner cylinder,

t_g = Gap between the inner and outer cylinder of damper,

R_1, R_2 = Average radii of the intermediate pole and the annular duct,

L_m = Total length of the magnetic pole, t_m = Gap of the orifice of the MR valve structure,

c = Coefficient which depends upon flow velocity profile.

In this way, the damping force of the magnetorheological damper can be represented as [9–12]:

$$F_d = KX + C_{MR}\dot{X} + F_{MR} \quad (7)$$

where F_{MR} is the damping force produced by the pressure drop due to the applied magnetic force.

For better performance of MR damper, damping force F_{MR} of the magnetorheological damper can be continuously controlled by adjusting the strength of the magnetic force [4].

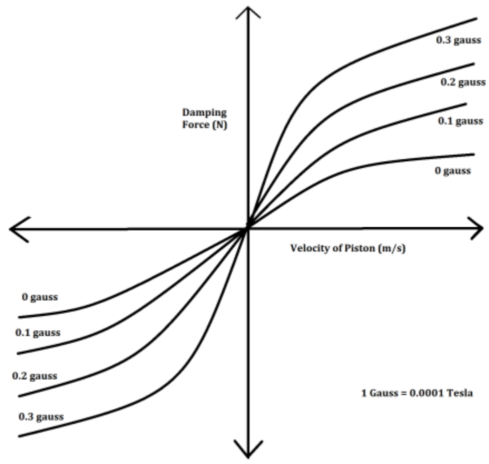

Figure 2. Velocity of piston rod vs damping force

From figure 2, it is seen that the damping force of the magnetorheological damper increases as the applied magnetic force increases [11]. This directly shows that a particular desired damping force can be achieved by just adjusting the applied magnetic force.

In MR damper control system, many control laws have been developed, for example groundhook control, fuzzy logic control, hybrid control, skyhook control and neural network [10].

4. MECHANICAL MODEL FOR MR DAMPER

Figure 3 shows the mechanical configuration of the MR damper used in this study. In this damper system, a spring and damper are suspended vertically from fixed-point support. Let a mass m attached to the upper end A of the spring and damper and spring compress the spring by a length e and come to rest at B. This position is called a static equilibrium position.

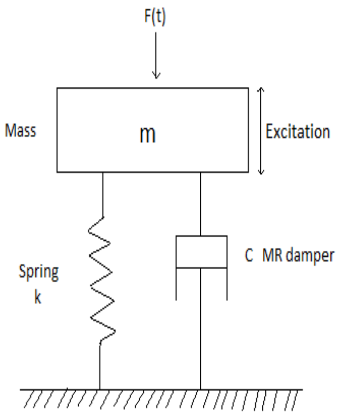

Figure 3. Mechanical model for MR damper

Now mass m is set in motion from its equilibrium position. Let at any time t, the mass is at P such that $BP = x$. The mass m experience the following forces,

1. Gravitational force mg acting downwards
2. Restoring force $k(e+x)$ due to displacement of spring acting upwards
3. Damping (frictional or resistance) force $c\frac{dx}{dt}$ of the medium opposing the motion (acting upwards)
4. External force $F(t)$ considering downward direction

By Newton's second law of motion, the ordinary differential equation for the motion of the mass m is;

$$m\frac{d^2x}{dt^2} = mg - k(e+x) - c\frac{dx}{dt} + F(t) \qquad (8)$$

At equilibrium position B,

$$mg = ke \qquad (9)$$

Hence,

$$m\frac{d^2x}{dt^2} = -kx - c\frac{dx}{dt} + F(t) \qquad (10)$$

$$\frac{d^2x}{dt^2} + \frac{c}{m}\frac{dx}{dt} + \frac{k}{m}x = \frac{F(t)}{m} \qquad (11)$$

Let

$$\frac{c}{m} = 2\lambda \quad and \quad \frac{k}{m} = w^2 \qquad (12)$$

Above equation becomes,

$$\frac{d^2x}{dt^2} + 2\lambda\frac{dx}{dt} + w^2x = \frac{F(t)}{m} \qquad (13)$$

The equation (13) represents the equation of motion and its solution gives the displacement x of the mass m at any instant t. This equation is a ordinary differential equation of order two.

5. MATHEMATICAL MODELING

The mathematical equations governing the motion of MR damper are derived by simplifying the system by considering the following assumptions. All the components of MR damper are considered as linear, all displacements are measured with respect to equilibrium conditions and hence gravity is ignored.

Mathematical modeling consists of converting real world problem into mathematical problems, solving the mathematical problems by mathematical methods and interpreting these solutions in the language of the real world problem. In short, mathematical model is a description of a given system using mathematical concepts and ideas. The process of construction of a mathematical model is termed as mathematical modeling. Such mathematical models are utilized in the many areas of engineering disciplines.

In this section, analytical (physical) modeling approach for magnetorheological damper is presented mathematically. A physical model for MR damper is depends on a detailed description of its internal structure, and the processes inside the magnetorheological damper. From the theoretical point of view, the physical model of MR damper represents the behaviour of the damper for a variable damping force. In many cases, the damper model is somewhat complex, which requests tedious subroutines.

In general, the MR damper is a hysteretic and non-linear device because of non-linear characteristic of the magnetorheological fluid. This type of non-linearity of MR damper represents the relationship between the input and output. In other words, the output of MR damper is the non-linear function of the electrical input and the mechanical input. Also in this system, the output of the damper is not only dependent on the instantaneous values of the inputs, but also on the previous history of the output. Hence a non-linear model is required to study behaviour of MR fluid. The non-linear model also represents the Bingham plastic nature of the fluid flow.

For One degree of Freedom (1DOF)

Figure 3 shows 1DOF model of MR damper, which is comprised of mass m, spring stiffness k, damping coefficient c_{MR} and x is the displacement of mass m.

According to Newton's second law of motion the dynamic motion of the mass m is;

$$m\ddot{x} + c_{MR}\dot{x} + kx = F(t) \qquad (14)$$

where, x = Vertical displacement of mass m,
$m\ddot{x}$ = Inertia force,
$c_{MR}\dot{x}$ = Viscous damping force,
kx = Spring force and
$F(t)$ = External force.

Hence general form of the ordinary differential equation governing the vibrations of a 1-dof system is;

$$m\ddot{x} + c_{MR}\dot{x} + kx = F(t) \quad (15)$$

The above model represents one degree of freedom. In case of two degree of freedom the equation of motion works in the following way,

For two degree of freedom (2DOF)

Consider a damped 2-dof MR damper system which is shown in Figure 4. The motion of MR damper system is described by the coordinates $x_1(t)$ and $x_2(t)$, which defines the positions of the masses m_1 and m_2 at time t from the equilibrium position.

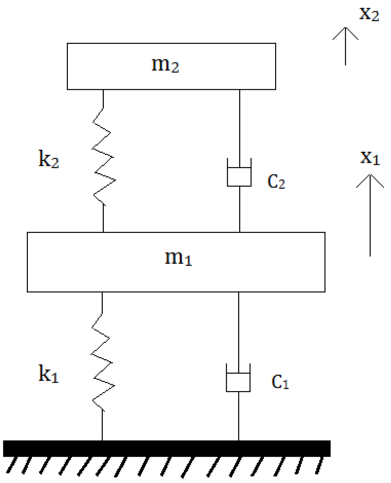

Figure 4. 2-DOF MR damper

This structure is coupled with two different structures of MR damper having mass load m_1 and m_2 on each damper with their corresponding displacement $x_1(t)$ and $x_2(t)$ respectively. The spring k_2 and MR damper c_2 mounted on the mass m_1 and this complete system is placed on spring k_1 and MR damper c_1.

The external forces $F_1(t)$ and $F_2(t)$ acts on the masses m_1 and m_2 respectively. The free body structure of the masses m_1 and m_2 are shown in the Figure 4.

According to Newton's 2nd law of motion the equation for motion of the mass m_2 is;

$$m_2\ddot{x}_2 - c_2\dot{x}_1 + c_2\dot{x}_2 - k_2x_1 + k_2x_2 = F_2(t) \quad (16)$$

According to Newton's 2nd law of motion the equation for motion of the mass $m_1$is;

$$m_1\ddot{x}_1 + (c_1 + c_2)\dot{x}_1 - c_2\dot{x}_2 + (k_1 + k_2)x_1 - k_2x_2 = F_1(t) \quad (17)$$

Thus the equations for governing the motion of the 2-dof MR dampers are [13];

$$m_1\ddot{x}_1 + (c_1 + c_2)\dot{x}_1 - c_2\dot{x}_2 + (k_1 + k_2)x_1 - k_2x_2 = F_1(t) \quad (18)$$

$$m_2\ddot{x}_2 - c_2\dot{x}_1 + c_2\dot{x}_2 - k_2x_1 + k_2x_2 = F_2(t) \quad (19)$$

Then the above set of differential equations can be expressed in matrix form as;

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} F_1(t) & 0 \\ 0 & F_2(t) \end{bmatrix} \quad (20)$$

Or

$$M\ddot{X} + C_{MR}\dot{X} + KX = F(t) \quad (21)$$

where, M is mass or inertia matrix,
C_{MR} is damping matrix,
K is stiffness matrix,
X is position (displacements) vector,
F is input of force vector.

The 2-DOF MR Damper governs a system of two coupled 2nd order ordinary differential equations.

In magnetorheological damper, springs take part of external applied load and give smooth suspension against vibrations. It also works as media for taking the damper to its original position so that for every next vibration, it will function very effectively and in efficient way.

When external force known as excitation disturbs the mass, vibrations get produced. In conventional damper, the vibration control depends upon the damping force. Due to damping, energy is dissipated in the form of heat. In the system containing MR damper, control over vibrations relies on strength of applied magnetic force. At the point when magnetic force applied to magnetorheological fluid its viscosity changes. Magnetorheological fluid acts as Bingham plastic when subjected to magnetic force. The magnetorheological fluid also satisfies Bingham plastic equation which is given as follows;

$$\tau = \tau_y(H) + \eta\,\dot{\gamma} \quad (22)$$

where, τ = Shear stress,

τ_0 = Yield stress caused by applied magnetic force[pa],

H = Strength of Magnetic field [A/m],

$\dot{\gamma}$ = Shear rate [1 /s] and

η = Plastic Viscosity [pa . s].

In MR damper, the control valve system changes damping forces as per the need. In the development of MR damper the structure of the flow channel as well as the strength of magnetic force are adjusted in order to meet particular application needs.

According to Bingham plastic equation, the damping force of the magnetorheological damper can be expressed as [7, 9];

$$F_d = KX + C_{MR}\dot{X} + F_{MR}sgn(X) \quad (23)$$

where F_{MR} is the damping force generated by the pressure drop due to the applied magnetic force strength.

6. MATHEMATICAL SOLUTION

The development of mathematical modeling of a magnetorheological damper results in the construction of a mathematical framework for motion of magnetorheological damper. The mathematical modeling of a magnetorheological damper is not complete until the proper mathematics is applied, and a system response obtained. Mathematical modeling of magnetorheological damper system leads only to system of ordinary differential equations.

The use of Laplace transform (LT) method is a appropriate method for finding the responses of a magnetorheological damper due to any type of excitation. The fundamental method is to use definition and properties of the Laplace transform (LT) to convert a system of ordinary differential equations into a set of algebraic equations in s with the help of initial conditions. Then the set of algebraic equations are solved to find the responses of the MR damper system. The responses of the MR damper is obtained by using fundamental properties of the Inverse Laplace transform (ILT).

The Laplace transform can be used to solve system of ordinary differential equations with constant or polynomial coefficients. The main disadvantage of the Laplace Transform (LT) method is the difficulty to find appropriate Inverse Laplace Transform (ILT). Some standard inversion theorems are available, involving contour integration in the complex plane system, but it is beyond the extent of this content.

Suppose $X(s)$ be the Laplace transform (LT) of the generalized coordinate system for a 1-degrees of freedom framework.
This means that,

$$X(s) = \int_0^\infty x(t)\, e^{-st} dt \quad (24)$$

Let $F(s)$ be the Laplace transform of the applied forces $F(t)$ on a MR damper system, for a particular form of $F_{EQ}(t)$ which is determined from the transform definition and properties of Laplace Transform.

By taking the Laplace transform of differential equation governing the vibrations of a 1-dof system that is $m\ddot{x} + c_{MR}\dot{x} + kx = F(t)$ and applying linearity property of the Laplace Transform;

$$mL\{\ddot{x}\} + c_{MR}\, L\{\dot{x}\} + kL\{x\} = L\{F(t)\} \quad (25)$$

$$L\{\ddot{x}\} + \left(\frac{c_{MR}}{m}\right) L\{\dot{x}\} + \left(\frac{k}{m}\right) L\{x\} = \frac{F(s)}{m} \quad (26)$$

Let,

$$\frac{c_{MR}}{m} = 2\lambda w, \qquad \frac{k}{m} = w^2$$

$$and\ \frac{F(s)}{m} = F_{re}(s)$$

$$L\{\ddot{x}\} + 2\lambda w\, L\{\dot{x}\} + w^2 L\{x\} = F_{re}(s) \quad (27)$$

The Laplace transform of a derivative permits to transform of derivatives of $x(t)$ into algebraic equation $X(s)$. By applying this property to Equation (27) we get,

$$[s^2X(s) - sx(0) - \dot{x}(0)] + 2\lambda w\,[sX(s) - x(0)] + w^2X(s) = F_{re}(s)$$

which rearranges to,

$$X(s) = \frac{F_{re}(s) + (s + 2\lambda w)x(0) + \dot{x}(0)}{s^2 + 2\lambda ws + w^2} \quad (28)$$

Then the definition and linearity property of the inverse Laplace transform is applied to determine $x(t)$,

$$x(t) = L^{-1}\left\{\frac{F_{re}(s)}{s^2 + 2\lambda ws + w^2}\right\} + L^{-1}\left\{\frac{(s + 2\lambda w)x(0) + \dot{x}(0)}{s^2 + 2\lambda ws + w^2}\right\} \quad (29)$$

The inverse Laplace transform of right hand side of Equation (29) depends on the roots of the equation $s^2 + 2\lambda ws + w^2$, which, in turn, depend on the certain value of λ. For a given certain value of λ, the inverse Laplace transform of the term $\frac{(s+2\lambda w)x(0)+\dot{x}(0)}{s^2+2\lambda ws+w^2}$ of Equation (29) is directly determined. The inverse Laplace transform of the term $\frac{F_{re}(s)}{s^2+2\lambda ws+w^2}$ is determined only after specifying $F_{EQ}(t)$ and taking its inverse Laplace transform.

Suppose the MR damper system is undamped, $\lambda = 0$, and the inverse Laplace transform of the term $\frac{(s+2\lambda w)x(0)+\dot{x}(0)}{s^2+2\lambda ws+w^2}$ becomes;

$$L^{-1}\left\{\frac{(s + 2\lambda w)x(0) + \dot{x}(0)}{s^2 + 2\lambda ws + w^2}\right\} = L^{-1}\left\{\frac{sx(0) + \dot{x}(0)}{s^2 + w^2}\right\}$$

$$L^{-1}\left\{\frac{(s + 2\lambda w)x(0) + \dot{x}(0)}{s^2 + 2\lambda ws + w^2}\right\} = x(0)L^{-1}\left\{\frac{s}{s^2 + w^2}\right\} + \dot{x}(0)L^{-1}\left\{\frac{1}{s^2 + w^2}\right\}$$

$$L^{-1}\left\{\frac{(s+2\lambda w)x(0)+\dot{x}(0)}{s^2+2\lambda ws+w^2}\right\} = x(0)\cos wt + \frac{\dot{x}(0)}{w}\sin wt \quad (30)$$

Suppose the free vibrations of the MR damper system are underdamped, then the equation $s^2 + 2\lambda ws + w^2$ has two conjugate complex roots. In such situation, it is useful to write the equation $s^2 + 2\lambda ws + w^2$ as;

$$s^2 + 2\lambda ws + w^2 = (s+\lambda w)^2 + w^2(1-\lambda^2) \quad (31)$$

Substituting Equation (31) into the last term of Equation (29) which yields,

$$L^{-1}\left\{\frac{(s+2\lambda w)x(0)+\dot{x}(0)}{s^2+2\lambda ws+w^2}\right\} = L^{-1}\left\{\frac{(s+2\lambda w)x(0)+\dot{x}(0)}{(s+\lambda w)^2+w^2(1-\lambda^2)}\right\} \quad (32)$$

By using first shifting property and linearity of the inverse Laplace transform to write Equation (32) as

$$L^{-1}\left\{\frac{(s+2\lambda w)x(0)+\dot{x}(0)}{s^2+2\lambda ws+w^2}\right\} = x(0)L^{-1}\left\{\frac{(s+\lambda w)+\dot{x}(0)}{(s+\lambda w)^2+w^2(1-\lambda^2)}\right\} + (\dot{x}(0) + \lambda wx(0))L^{-1}\left\{\frac{1}{(s+\lambda w)^2+w^2(1-\lambda^2)}\right\} \quad (33)$$

By definition and properties of inverse Laplace transform, yielding for an underdamped system:

$$L^{-1}\left\{\frac{(s+2\lambda w)x(0)+\dot{x}(0)}{s^2+2\lambda ws+w^2}\right\} = x(0)e^{-\lambda wt}\cos\left(w\sqrt{1-\lambda^2}\right)t + (\dot{x}(0) + \lambda wx(0))e^{-\lambda wt}\sin\left(w\sqrt{1-\lambda^2}\right)t \quad (34)$$

Suppose the free vibrations of MR damper system are critically damped, then the term $s^2 + 2\lambda ws + w^2$ of Equation (29) is a perfect square as $(s+w)^2$ and it gives,

$$L^{-1}\left\{\frac{(s+2\lambda w)x(0)+\dot{x}(0)}{s^2+2\lambda ws+w^2}\right\} = L^{-1}\left\{\frac{(s+2\lambda w)x(0)+\dot{x}(0)}{(s+w)^2}\right\} \quad (35)$$

Applying linearity property of the inverse Laplace transform to the R.H.S of Equation (35) we get;

$$L^{-1}\left\{\frac{(s+2\lambda w)x(0)+\dot{x}(0)}{s^2+2\lambda ws+w^2}\right\} = x(0)L^{-1}\left\{\frac{1}{s+w}\right\} + (\dot{x}(0) + wx(0))L^{-1}\left\{\frac{1}{(s+w)^2}\right\} \quad (36)$$

Inverting using Laplace transform, this leads to:

$$L^{-1}\left\{\frac{(s+2\lambda w)x(0)+\dot{X}(0)}{s^2+2\lambda ws+w^2}\right\} = x(0)e^{-wt} + \left(\dot{X}(0) + wx(0)\right)te^{-wt} \quad (37)$$

Suppose the free vibrations of MR damper system are overdamped, then the term $s^2 + 2\lambda ws + w^2$ of Equation (29) has two linear factors $(s - s_1)$ and $(s - s_2)$ where $s_1 = -w\left(\lambda + \sqrt{\lambda^2 - 1}\right)$ and $s_2 = -w\left(\lambda - \sqrt{\lambda^2 - 1}\right)$.

By partial fraction method and linearity of inverse Laplace transform we can write;

$$L^{-1}\left\{\frac{(s+2\lambda w)x(0)+\dot{x}(0)}{s^2+2\lambda ws+w^2}\right\} = \frac{[(s_1+2\lambda w)x(0)+\dot{x}(0)]}{s_1-s_2}L^{-1}\left\{\frac{1}{s-s_1}\right\} + \frac{[(s_2+2\lambda w)x(0)+\dot{x}(0)]}{s_2-s}L^{-1}\left\{\frac{1}{s-s_2}\right\}$$

$$L^{-1}\left\{\frac{(s+2\lambda w)x(0)+\dot{x}(0)}{s^2+2\lambda ws+w^2}\right\} = \frac{[(s_1+2\lambda w)x(0)+\dot{x}(0)]}{s_1-s_2}e^{-s_1t} + \frac{[(s_2+2\lambda w)x(0)+\dot{x}(0)]}{s_2-s_1}e^{-s_2t} \quad (38)$$

The inverse Laplace transform of the term $\frac{F_{re}(s)}{s^2+2\lambda ws+w^2}$ of Equation (29) is appeared by finding for the specific form of function $F(t)$, forming $\frac{F_{re}(s)}{s^2+2\lambda ws+w^2}$, and inverting by using definition and properties of inverse Laplace transform.

An interesting aspect of MR damping is that the damping force can be adjusted actively, simply by changing intensity of external magnetic force. This function of MR dampers leads to a large area of research for magnetorheological dampers as well as the control systems associated with implementing active control. In above mathematical solutions, the parameter λ is the damping coefficients associated with the Bingham plastic model which is concerned with damping performance of the magnetorheological damper, it is controlled by changing intensity of external magnetic force, and overall performance of the damper system is improved. These results also show that the proposed damper model can be used for control algorithm improvement and system performance evaluation.

For better performance of the MR damper framework, the damping ratio λ is evaluated from the decay rate of the curve. An alternative method for determining the damping ratio is finding the damping ratio between two consecutive peaks, and the computing the final values as the mean value from the individual peak-to- peak values in the decay.

7. SIMULATION

The equation of motion describing the dynamics of MR damper are written considering 1-DOF MR damper and used for simulation in MATLAB SIMULINK. Following the same procedure, a 1-DOF MR damper is modeled, which incorporates the Bingham model of MR damper and the performance of the two compared.

The first step in simulation of a magnetorheological damper is to draw the framework, demonstrating the required degrees of freedom, the mass matrix, stiffness matrix and damping control matrix, and showing applied external and internal forces. The 1-DOF MR damper framework to be followed throughout the simulation, appeared in Figure (3), consists of mass m, attached to spring with stiffness constant k and magnetorheological damper with damping coefficient c_{MR}.

The number of degrees of freedom (DOF) for a magnetorheological damper is the least number of geometrically independent variables required to specify the complete configuration of the proposed magnetorheological damper system. For our simplicity, the notation "X" will be utilized for degrees of freedom (DOF), by taking variable x_1 and x_2 for state space representation of magnetorheological damper. The proposed damper system shown in Figure (3) where mass m can move only along the vertical axis, hence a single degree of freedom (1-DOF) for mass m is sufficient.

The equation for motion of 1-DOF MR damper is,

$$m\ddot{x} + c_{MR}\dot{x} + kx = F(t) \quad (39)$$

where, x = Vertical displacement of mass m,
m = Mass of the body,
c_{MR} = Viscous damping coefficient,
k = Spring coefficient and
$F(t)$ = Total force on the System.

The differential equation (39) can be solved mathematically by using transfer function. The transfer function is the mathematical formulation giving the corresponding output value for each value possible value of the input to the system. The results are represented as characteristic transfer curve. In the given magnetorheological damper, force F(t) and magnetic force is the input to the magnetorheological damper and displacement $x(t)$ is the output of the damper.

By Laplace transform the transfer function for the above equation (39) is,

$$\frac{X(s)}{F(s)} = \frac{1}{ms^2 + c_{MR}s + k}$$

where $X(s) = L\{x(t)\}$
$F(s) = L\{F(t)\}$

This transfer function is solved in MATLAB.

The solution of the differential equation (39) is given by,

$$x(t) = \frac{1}{m}\left\{x(t_0) + \dot{x}(t_0)t + \int_{t_0}^{t}\int_{t_0}^{t} F(\alpha)d\alpha\right\} \quad (40)$$

Clearly, solution of equation (39) can only be obtained, given the force, if the initial position $x(t_0)$ and initial velocity $\dot{x}(t_0)$ are known. Hence, these constants define the state of the MR damper at the initial time t_0 and the MR damper is second order. The state variables for MR damper system are defined as,

$$x(t) = x_1(t) \quad (41)$$

$$\dot{x}(t) = x_2(t) \quad (42)$$

and the state variables vector is

$$X(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = [x_1(t) \quad x_2(t)]^T \quad (43)$$

Then the equation (39) reduces to,

$$\dot{x}_1(t) = x_2(t) \quad (44)$$

$$\dot{x}_2(t) = \left[\frac{F(t)}{m} - \frac{c_{MR}}{m}x_2(t) - \frac{k}{m}x_1(t)\right] \quad (45)$$

Thus, the state variables $x_1(t)$ and $x_1(t)$ governs two first order differential equations (44) and (45). Here we can see that the second order differential equation (39) has been reduced to two first order differential equations (44) and (45).

In MATLAB, Simulink is a graphical application for mathematical modeling and simulation of dynamic process. In this application, magnetorheological damper can be expressed as block diagrams. Each block diagram of this application will play certain role of operations on its inputs and generates an output response that can be sent to another block diagram through interconnections. Different types of block diagrams are accessible in simulink application.

Let us to simulate an ordinary differential equation (39) in simulink for the MR damper. The following parameters are used in the simulation. Body mass (m) = 10 kg, spring constant of spring (k) = 2000 N/m, damping coefficient of damper (c_{MR}) = $10\sqrt{2}$ to $50\sqrt{2}$ Ns/m , control force $F(t)$ on system= $100\ N$, initial position $x(0) = 5$, initial velocity $\dot{x}(0) = 0$.

The above equation (39) is a second order ordinary differential equation. By replacing the known values of m, k, c_{MR} and $F(t)$ in the equation (45) the equation (39) can be expressed as following IVP.

$$\dot{x}_1(t) = x_2(t) \tag{46}$$

$$\dot{x}_2(t) = \left[\frac{100}{m} - \frac{c_{MR}}{m}x_2(t) - \frac{2000}{m}x_1(t)\right] \tag{47}$$

Subject to initial conditions

$$x_1(t_0) = 5,\ x_2(t_0) = 0 \tag{48}$$

Thus, simulation in Figure 3 solves the equation (46) and (47) simultaneously, with appropriate initial conditions (48).

Now we can solve the above IVP numerically by the appropriate methods like Runge-Kutta method, Euler's method, Modified Euler's method or Picards method. Also in MATLAB, simulink application is used to solve such type of IVP by block diagrams available in simulink block library.

After making the essential connections, the Simulink model for the equation (40) is shown in the Figure 5.

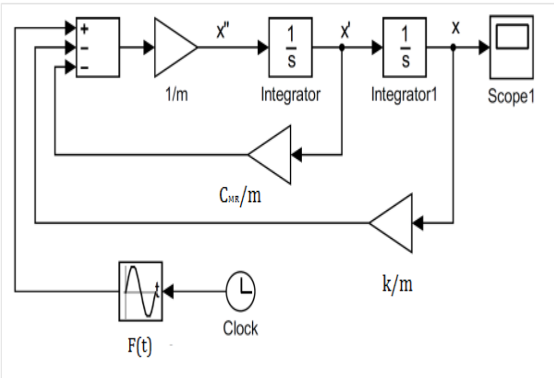

Figure 5. Simulink model

As we need to simulate the MR damper from $t = 0$ to $t = 10$, to fix the stop time of the simulation to 10 sec from the command window. The solver ODE4 (Runge-Kutta Method) is used to simulate the proposed MR damper system by setting a fixed step size as 0.001. We can also vary the start and end time from the same command window.

MATLAB code of the above equation is written and a graph of displacement versus time, velocity versus time and displacement versus velocity are drawn numerically as shown in Figure 6, Figure 7 and Figure 8 respectively.

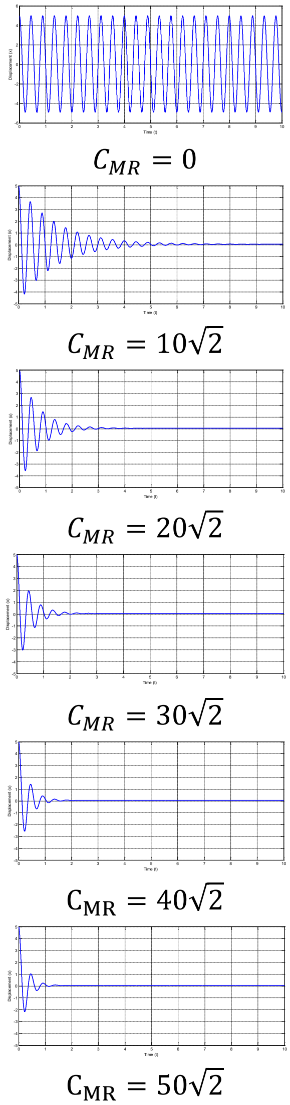

Figure 6. Velocity versus time

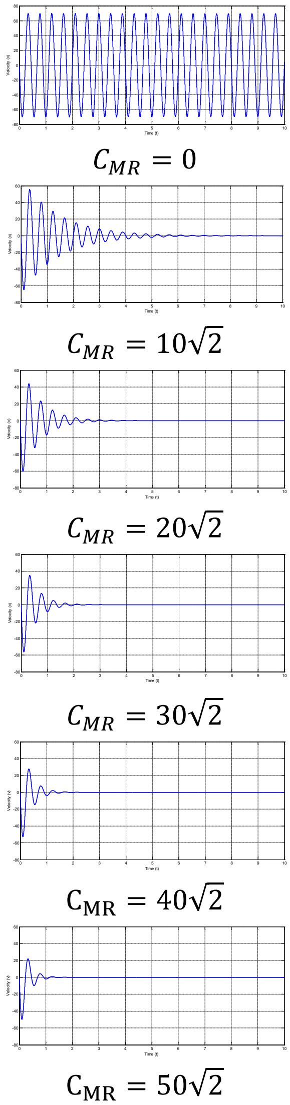

Figure 7. Displacement versus time

Simulation results demonstrate that both strategies equation modeling and state space modeling produce similar results. It shows that modeling is performed with great accuracy by understanding the dynamics of the MR damper system. Results are matching /validating with the historical data shown in renowned journals, books and literature [14–16].

8. CONCLUSIONS

This paper gives the background information on MR fluids, MR dampers, types of MR damper, MR dampers modeling and its applications. In the first part, the mechanical model of MR dampers has been evaluated. It is also shown that if velocity estimation is possible from displacement or acceleration, then velocity is only enough to use as model input while adding displacement and acceleration as model input will not improve the modeling error significantly.

This sub-optimal procedure is faster and simpler and also effective in closed-loop simulations.

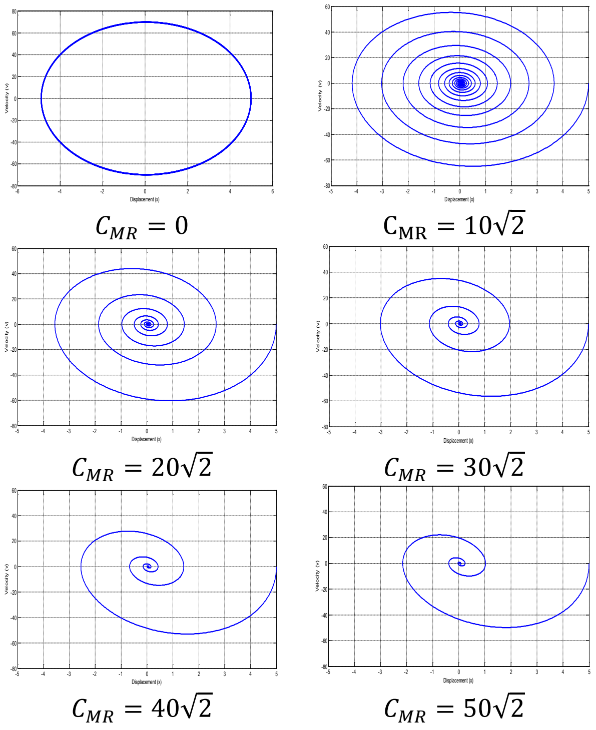

Figure 8. Displacement versus velocity

In the second part of this paper, a differential equation has been modeled by using Newton's laws of motion. The stiffness of the model has been determined from the mode shapes which are constructed from the frequency transfer functions obtained by the test data. The damping force has been determined from the estimated damping ratio through free decay tests, and from the corresponding frequency response, the resonance frequencies are determined. The validation of the model shows a quite satisfactory accuracy, and this model can be used for the calibration of the control strategies.

These types of MR dampers are useful to control vibrations in the field of automobiles, trains and civil structures. The future applications of MR damper systems are to achieve a more stability and comfortable journeys in vehicle.

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