# Dynamic Stability of Straight Cracked Pipes Resting on Winkler Elastic Foundation and Conveying Pulsatile Flow

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*Abstract:* - The present study concerns with the dynamic stability of a straight cracked pipe resting on a Winkler elastic foundation. The pipe conveys pulsatile flow. The fluid velocity is a trigonometric harmonic function of time. The objective is to examine the influence of foundation rigidity on the critical velocity of the pipe. A numerical solution of the associated boundary value problem is approached by using the Galerkin method. The harmonic function of the fluid velocity allows the Floquet theory to be applied in order to investigate the dynamic stability of the system.

The pipeline transport is used worldwide in many sectors of the economy. The safety problems of pipes attract much interest in science and industry. The paper rises attention on the stability of pipes with pulsatile flow as not much attention is paid on them.

*Keywords:* - dynamic stability, cracked pipe, elastic foundation, pulsatile flow.

## 1. INTRODUCTION

Fluid conveying pipes find applications in a number of areas of engineering.

For pipes conveying fluid with a constant velocity, it is known that the natural frequency of the pipe becomes lower when the velocity of the transported fluid increases. The velocity of the fluid corresponding to a natural frequency equal to zero is called critical velocity. At that point, the system is at the edge of loss of stability. When the pipe conveys pulsatile flow, the pipe loses stability even though the mean velocity of the fluid is smaller than the critical velocity [1].

Numerous articles nowadays analyze the linear and nonlinear dynamics of pipes conveying fluid, proving the actuality of the problem [2-6].

Lottati I. and Cornecki A, [7] investigated a fluid-conveying pipe, lying on Winkler elastic foundation. They found that it has a stabilizing effect on the pipe. The critical velocity of the fluid in a pipe resting on a Winkler elastic foundation is bigger compared to the critical velocity of the fluid in the same pipe, but without the foundation.

Ginsberg J. [8] considered a simply supported pipe with a pulsatile fluid. An analysis of the case of steady flow shows that the pipe exhibits the divergence type of instability. When the flow is pulsatile the pipe has regions of dynamic instability.

Cracks are the most encountered damage in the structures. They reduce the stiffness of the structural element which causes a decrease in its natural frequencies and a change in the mode shapes. In pipes conveying fluid, cracks lead to decrease in the critical velocity of the fluid.

In the present paper, a fluid-conducting tube resting on a Winkler elastic foundation is investigated. The results obtained reflect the dependence of the critical fluid velocity on the rigidity of the Winkler elastic foundation. The results also show the effect of an open crack on the critical velocity of the fluid.

A number of scientists are conducting research in the field of fluid-structure interaction. However, the tubes conveying pulsatile flow remain under-investigated area that lacks a researcher's attention.

## 2. PROBLEM FORMULATION

The present paper uses the Euler-Bernoulli beam theory to investigate the dynamic stability of a pipe of length $l$, conveying fluid and resting on a Winkler elastic foundation. The pipe, shown in Fig.1, is hinged at both ends. The pipe is supposed to have an open edge crack, which dimensions ($\theta_c$ and $b$) are shown in (Fig. 1). $b$ is the length of the crack. $\theta_c$ is the half central angle corresponding to the chord $b$. The crack severity is usually measured by the ratio

$\theta_c / \pi$ . The crack position along the length of the tube is fixed through the coordinate $x_c$. The crack is modeled as a rotational spring with a lumped stiffness $k_r$ [9] (Fig. 2).

The classical Winkler foundation is often used as a model in geotechnical analyses. According to the model, any point deflection at the surface of an elastic medium is proportional to the applied load in the point and is independent on the applied loads at other points of the surface. Thus, the mechanical model of the elastic medium consists of a series of closely spaced and mutually independent linear elastic springs with rigidity $k$ .

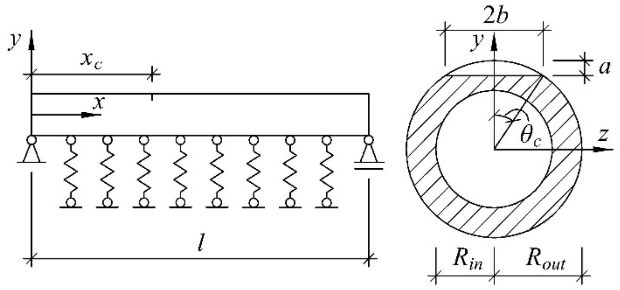


**Figure 1.** Static scheme of the investigated pipe

The pipe is divided into two segments. The first segment is the left-hand side of the crack, and the second – is the right-hand side of the crack.

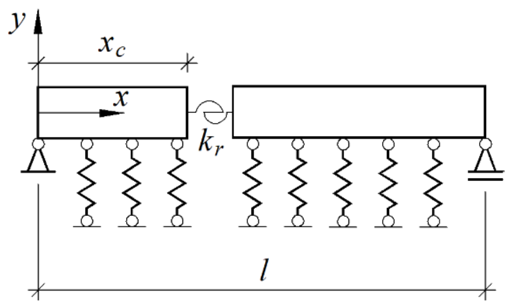


**Figure 2.** Mechanical model of the crack

The transverse vibration of a straight pipe conveying inviscid fluid and lying on a Winkler elastic foundation, with rigidity $k$ , is governed by the following differential equation

$$EI\frac{\partial^4 w}{\partial x^4}+m_f V^2\frac{\partial^2 w}{\partial x^2}+2m_f V\frac{\partial^2 w}{\partial x \partial t}+m_f\frac{dV}{dt}\frac{\partial w}{\partial x}+$$
$$+\left(m_f+m_p\right)\frac{\partial^2 w}{\partial t^2}+kw=0 \qquad (1)$$

where $t$ is the time, $w(x,t)$ is the lateral displacement of the pipe axis, $x$ is the coordinate along the axis, $EI$ is the rigidity of the pipe. The mass of the pipe per unit length is denoted by $m_p$ and the mass of the fluid per unit length of the pipe by $m_f$ .

The fluid velocity is the following trigonometric harmonic function of the time $t$ .

$$V=V_0\left(1+\delta\cos\left(\omega_f t\right)\right) \qquad (2)$$

where $V_0$ is the constant fluid rate, $\delta$ is the excitation coefficient and $\omega_f$ is the circular fluid pulsation frequency.

The spectral Galerkin method is applied to approximate the solution of the boundary value problem (1). According to this method, an approximate solution is sought in the form [10]:

$$w(x,t)=\sum_{i=1}^{n} y_i(x) z_i(t). \qquad (3)$$

In this expression, $z_i(t)$ are unknown functions. $y_i(x)$ are basic functions satisfying the boundary conditions of the tube. The eigenfunctions for the pipe with stationary fluid ( $V=0$ ) are used as basic functions in the present paper.

For a Bernoulli-Euler tubular beam filled with stationary fluid, one has

$$EI\frac{\partial^4 w}{\partial x^4}+\left(m_f+m_p\right)\frac{\partial^2 w}{\partial t^2}=0 \qquad (4)$$

Free vibration of the beam has the form

$$w(x,t)=y(x)e^{i\omega t} \qquad (5)$$

where $\omega$ is the natural frequency of the beam and $i=\sqrt{-1}$ .

The substitution of (5) in (4) yields

$$y_i^{IV}(x)=\gamma_i^4 y_i(x) \qquad (6)$$

where

$$\gamma_i=\sqrt[4]{\frac{\left(m_f+m_p\right)\omega_i^2}{EI}} . \qquad (7)$$

Substituting (3) in equation (1) one obtains the residual function, which does not vanish identically since $w(x,t)$ is not exact solution of equation (1). Here, and in the sequel, dots denote derivatives with respect to $t$ and primes denote derivatives with respect to $x$.

$$R(x,t)=\sum_{i=1}^{n}\Big[\big(m_f+m_p\big)y_i\ddot{z}_i+$$
$$+2m_fV_0\big(1+\delta\cos(\omega_f t)\big)y_i^I\dot{z}_i+EI\,y_i^{IV}z_i+$$
$$+m_fV_0^2\big(1+\delta\cos(\omega_f t)\big)^2y_i^{II}z_i-$$
$$-m_fV_0\delta\omega_f\sin(\omega_f t)y_i^I z_i+k\,y_i\,z_i\Big]. \qquad (8)$$

According to the standard Galerkin procedure, the residual function $R(x,t)$ should be orthogonal to the basic functions in the area $x\in[0;l]$:

$$\int_0^l R(x,t)y_k(x)\,dx=0\text{, for }k=1,...,n \qquad (9)$$

The result of the application of (9) is a system of $n$ differential equations about the unknown functions $z_i(t)$. This system for the differential equation (1) is:

$$\sum_{i=1}^{n}\int_0^l\Big\{\big(m_f+m_p\big)y_i\ddot{z}_i+2m_fV_0\big(1+\delta\cos(\omega_f t)\big)y_i^I\dot{z}_i+$$
$$+\Big[EI\gamma_i^4y_i+m_fV_0^2\big(1+\delta\cos(\omega_f t)\big)^2y_i^{II}-$$
$$-m_fV_0\delta\,\omega_f\sin(\omega_f t)y_i^I+k\,y_i\Big]z_i\Big\}y_k\,dx=0 \quad (10)$$

For the solution of system (10) is employed the described in [10] method.

The beam is divided to sections with length of $\Delta x$ .

The integrals in (10) are expressed in the following form

$$\int_0^l y_iy_k\,dx=\{y_i\}^T\{y_k\}\Delta x \qquad (11)$$

$$\int_0^l y_i^I y_k\,dx=\{y_i^I\}^T\{y_k\}\Delta x \qquad (12)$$

$$\int_0^l y_i^{II}y_k\,dx=\frac{1}{EI}\{M_i\}^T\{y_k\}\Delta x \qquad (13)$$

In equations (11), (12) and (13):

$\{y_i\}$ is a column vector of the lateral displacements of the nodes on the axis of the pipe, corresponding to the $i$-th eigenform of a pipe with stationary fluid;

$\{y_i^I\}$ is a column vector of the rotations of the nodes on the axis of the pipe, corresponding to the $i$-th eigenform of a pipe with stationary fluid;

$\{M_i\}$ is the vector of the bending moments associated with the $i$-th mode shape $\{y_i\}$.

The substitution of (11), (12) and (13) in (10) yields

$$\sum_{i=1}^{n}\Big\{\big(m_f+m_p\big)\{y_i\}^T\{y_k\}\ddot{z}_i+$$
$$+2m_fV_0\big(1+\delta\cos(\omega_f t)\big)\{y_i^I\}^T\{y_k\}\dot{z}_i+$$
$$+\Big[\big(EI\gamma_i^4+k\big)\{y_i\}^T\{y_k\}+$$
$$+\frac{m_fV_0^2}{EI}\big(1+\delta\cos(\omega_f t)\big)^2\{M_i\}^T\{y_k\}-$$
$$-m_fV_0\delta\,\omega_f\sin(\omega_f t)\{y_i^I\}^T\{y_k\}\Big]z_i\Big\}\Delta x=0 \quad (14)$$

Writing equation (14) in matrix form gives:

$$|M|\ddot{z}+|C(t)|\dot{z}+|K(t)|q=0 \qquad (15)$$

The equation (15) could be transformed in the following form

$$\begin{vmatrix}I & 0\\ 0 & M\end{vmatrix}\{\dot{q}\}+\begin{vmatrix}0 & -I\\ K(t) & C(t)\end{vmatrix}\{q\}=0\,, \qquad (16)$$

where

$$\{q\}^T=\{q_1=z_1;...;q_n=z_n;q_{n+1}=\dot{z}_1;...;q_{2n}=\dot{z}_n\} \qquad (17)$$

After some transformations for equation (16) one obtains

$$\{\dot{q}\}=|A(t)|\{q\}=0 \qquad (18)$$

where the coefficient matrix $|A(t)|$ is periodic with period $T$ , that is $|A(t+T)|=|A(t)|$ .

The Floquet theorem is applied to investigate the stability of the trivial solution $\{q\}\equiv 0$ [11]. According to the theorem the solution of the system (18) has the following form

$$\{q(t)\}=|\Phi(t)|\{q(0)\} \qquad (19)$$

where $|\Phi(t)|$ is the fundamental matrix, solution of the $T$ - periodic system (18). The fundamental matrix has the following form [12]

$$|\Phi(t)| = L(t)e^{Bt}. \qquad (20)$$

In (20) $L(t)$ is a periodic matrix that has an initial value $L(0) = I$. $B$ is a constant matrix.

The matrix $|\Phi(T)|$ is known as monodromy matrix. The eigenvalues of $|\Phi(T)|$ are known as characteristic multipliers. The stability of the system is determined by the modules of the characteristic multipliers of the periodic system (18) [12].

If all of the characteristic multipliers have modulus less than one, then the zero solution is asymptotically stable.

If all of the characteristic multipliers have a modulus less than one or equal to one, and if the algebraic multiplicity equals the geometric multiplicity of each characteristic multiplier with modulus one, then the zero solution is Lyapunov stable.

If one or more of the characteristic multipliers has a modulus greater than one, then the zero solution is unstable.

In principal the fundamental matrix is difficult to be determined in an analytic way, but there are methods to approximate it [13]. The period $T$ is divided into $k$ subintervals $\Delta t$. For each time interval is calculated the matrix $|A(t)|$.

$$A_i = A\left(\frac{T(2i-1)}{2k}\right) \qquad (21)$$

Then the monodromy matrix $|\Phi(T)|$ is calculated as in the following formula

$$\Phi(T) = \prod_{i=1}^{k} e^{A_i \Delta t} \qquad (22)$$

## 3. CRACK MODELING

It is considered that the bending vibrations of the Euler-Bernoulli beam is in the plane $x-y$ (Fig.1), which is also a plane of symmetry for the cross-section. The crack is assumed to be open. Castigliano's theorem is used to obtain the local flexibility in the presence of the crack [14]

$$c = \frac{\partial^2 U}{\partial M^2} = \frac{1-\nu^2}{E}\int_{-b}^{b}\int_{0}^{a}\frac{\partial^2\left(K_I^2\right)}{\partial M^2}dx\,dy, \qquad (23)$$

where $E$ and $\nu$ are respectively Young's module and Poisson's ratio. $K_I$ is the stress intensity factor of bending. $a$ and $b$ are the crack dimensions as shown in (Fig.1). $M$ is the bending moment.

$$K_I = \frac{M}{\pi R^2 t_p}\sqrt{\pi R \theta_c}\,F(\theta_c), \qquad (24)$$

where $R = 0{,}5(R_{in} + R_{out})$, $t_p$ and $\theta_c$ are respectively thickness of the pipe and the half central angle of the crack (Fig.1). $F(\theta_c)$ is calculated from the following formula [15]

$$F(\theta_c) = 1 + A_t\left[4{,}5967\left(\frac{\theta_c}{\pi}\right)^{1,5} + 2{,}6422\left(\frac{\theta_c}{\pi}\right)^{4,24}\right] \qquad (25)$$

$$A_t = \sqrt[4]{\frac{1}{8}\frac{R}{t_p} - \frac{1}{4}} \quad \text{for} \quad 5 \le \frac{R}{t_p} \le 10 \qquad (26)$$

$$A_t = \sqrt[4]{\frac{2}{5}\frac{R}{t_p} - 3} \quad \text{for} \quad 10 \le \frac{R}{t_p} \le 20 \qquad (27)$$

The equivalent rotational spring stiffness

$$k_r = \frac{1}{c}. \qquad (28)$$

## 4. NUMERICAL RESULTS

Numerical studies have been carried out for the system in Fig. 1.

The geometric and the material characteristics of the pipe are: the inner and the outer radii of the cross-section of the pipe are $R_{in} = 0.012\,m$ and $R_{out} = 0.014\,m$, Young's modulus $E = 210\,GPa$, the density of the material of the pipe $\rho = 7800\,kg/m^3$. The density of the flowing fluid is $\rho = 1000\,kg/m^3$. The excitation coefficient $\delta = 10$ and the circular fluid pulsation frequency $\omega_f = 5\,s^{-1}$.

The dimensions of the crack are $a = 0.001\,m$, $b = 0.005\,m$. The position of the crack is fixed with the coordinate $x_c = 1{,}2\,m$.

In the present paper 14 eigenfunctions $y_i(x)$ are used in the approximate solution (3).

For the pipe in Fig.1 is obtained the critical value of the constant fluid rate $V_{0,cr}$ for different rigidity of the Winkler elastic foundation. The results are shown in Fig.3.

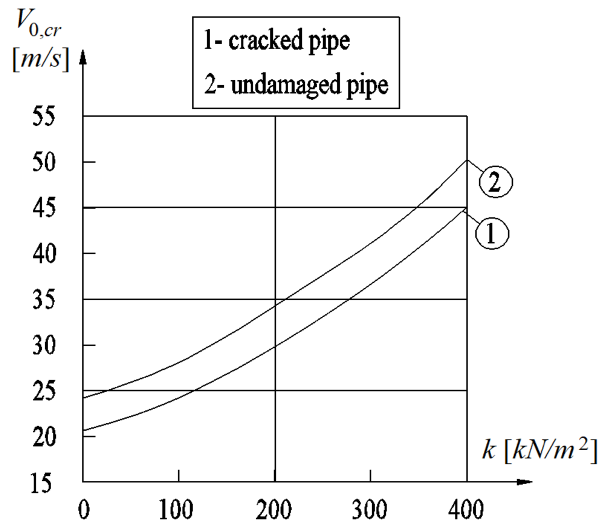


**Figure 3.** Critical value of the constant fluid rate versus the rigidity of the Winkler elastic foundation

## 5. CONCLUSIONS

The employed Floquet theory, in the case when the fluid velocity is a harmonic function of the time $t$, allows the investigation of the dynamic stability of the system. The applied in the paper method to approximate the monodromy matrix allows relatively easy determination of the critical value of the constant fluid rate.

The obtained results show that the Winkler elastic foundation has a stabilizing effect on the pipe - with increasing the rigidity of the foundation the critical value of the constant fluid rate increases. The crack has a destabilizing effect on the system, leading to decreasing of the critical value of the constant fluid rate.

The results obtained contribute to the safety of pipes conveying fluid. In order to avoid damages, the operator of the pipe shouldn't allow higher transportation velocities than the critical velocity of the system. As the critical velocity depends on many parameters of the system, among them the severity and position of the crack, the operator of the pipe, should strictly perform crack detection test and based on the results to correct the velocity of the fluid in order the damaged system not to lose stability.

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