
Enhancing Joint Parameter Identification with Sub Structure Synthesis Theory: A Non-linear Perspective

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Abstract: - Engineering structures that undergo fluctuating loads frequently exhibit structural nonlinearity. In a general perspective, linear concept can be utilized to partially analyze nonlinear systems, but in bolted joint structures non-linearity has to be considered for defining the joint behavior. The current research work provides a mathematical model for the parametric identification of non-linear structural elements. The frequency equation is derived using the idea of sub-structure synthesis with both non-linear and linear stiffness characteristics. The generated equation is further utilized in inverse analysis to estimate a non-linear parameter. The present work is numerically analyzed using a cantilever beam with a non-linear boundary condition. The method provides precise approximations for a broad range of non-linear stiffness values. The work concludes less than 10% of error between the developed mathematical model and experimentation.

Keywords: Vibration, Sub-Structure Synthesis Theory, Nonlinear Rotational Stiffness, Linear Translational Stiffness, Fast Fourier Transform, Frequency Domain Analysis.

List of Symbols and Abbreviations

K_1	Linear translational stiffness
K_2^*	Non-linear rotational stiffness
B	Subsystem (Bolted Joint)
C	Subsystem (Cantilever Beam)
L	Length of beam
FRF	Frequency Response Function
$[\beta]$	FRF matrix of subsystem B
$[\gamma]$	FRF matrix of subsystem C
β_{11}, β_{22}	Direct receptance for subsystem B
β_{12}, β_{21}	Cross receptance for subsystem B
γ_{11}, γ_{22}	Direct receptance for subsystem C
γ_{12}	Cross receptance for subsystem B
λ	Non-dimensional frequency
EI	Bending stiffness
ω	Flexural vibration's natural frequency
ρA	Linear mass density function
K_{4L}	Cubic type stiffness non linearity
K_2	Linear rotational stiffness
$\theta(t)$	Angular displacement or rotation of the system
F(t)	Forcing function, which is an external excitation or force acting on the system. (Sinusoidal force applied to the system)
Fo	Amplitude of the external force
$\theta^3(t)$	Cubic term, representing the cubic nonlinearity in the system
K_4	Cubic stiffness coefficient

θ_0	Non-linear system's harmonic response amplitude with applied harmonic force
Δ	Denominator of the receptance functions
X	Acceleration in the direction along X axis (side to side movement)
Y	Acceleration in the direction along Y axis (up and down movement)
Z	Acceleration in the direction along Z axis (depth or front-to-back movement)

1. INTRODUCTION

The force reconstruction approach was used which was purely based on the nonlinear system recognition technique where the base excitation was treated as an input [1]. A joint is a link between two or more parts of a structure, and it is an important part of any building. The classification of a joint is contingent upon its behavior, which can be either linear or nonlinear [2]. A joint having nonlinearity is characterized by a load-displacement relationship which deviates it from linearity. Nonlinear joints can be of types like geometrically nonlinear joints and material nonlinear joints [3]. Geometrically nonlinear joints are having the presence of significant rotations or displacements resulting from the joint's geometry, as opposed to other factors. When a material's nonlinear behavior such as creep, plasticity, or

viscoelasticity, contributes to joint deformation, at that case, joint is material nonlinear [4]. The complex structure of nonlinear joints renders their behavior challenging to anticipate. A thorough understanding of the behavior is essential for ensuring that structures are designed to withstand the loads they will experience throughout their lifespan [5]. The analysis of nonlinear joints necessitates the utilization of modern numerical techniques, such as finite element analysis, which can mimic the behavior of joints under a variety of different loading circumstances [6].

Nonlinear joints are utilized in engineering encompassing multiple disciplines including aerospace, civil, mechanical, and automotive engineering [7]. By improving the mathematical models with cubic stiffening nonlinearity, a better understanding of the nonlinear behavior in bolted joints can be achieved and a more realistic representation of the actual behavior of bolted joints can be studied [8]. In civil engineering, nonlinear joints are used to connect different structural elements in buildings, bridges, and other infrastructure [9]. In mechanical engineering, nonlinear joints are used in the design of machinery, such as gears and bearings. In automotive engineering, nonlinear joints are used to connect different parts of a car, such as the suspension and the chassis [10]. Every structural assembly must be attached in some way, whether it is by riveting, welding, or innovative, complicated fastenings like smart joints [11]. In spacecraft, connectors are used to assemble the substructures by joints and hinges. As the connector possesses nonlinear properties, it strongly affects the dynamic characteristics of the spacecraft [12]. Connectors serve an important function in spacecraft by allowing substructures to be attached and hinged. The nonlinear features of these connectors have a significant impact on the spacecraft's dynamic qualities [13]. Using force state mapping approaches, nonlinear joint model parametric identification can be identified by Iwan Modelling, Runge Kutta Method, Frequency Response Function (FRF) for computing FRFs of a substructure [14]. The coupling-identification method is also used to expressly improve the joint identification's precision [15]. To provide a generic equation for the proportional viscous damping ratio, which is utilised to determine response parameters, the concept of equivalent viscous damping as a nonlinear quantity was established [16]. Based on the idea of multi-harmonic balance, a generic technique for the identification of the lively aspects of the nonlinear joint has been created and tested, successfully identifying the nonlinear characteristics of joints [17]. Substructure synthesis theory generated a nonlinear element parametric identification frequency equation. The

nonlinear amplitude response was calculated using curve fitting. Experimental and analytical values matched well [18]. The Iwan model is also utilized to perfectly replace the experimental joint phenomenon and to describe the nonlinear property theory of elements that include pinning, macro and micro slip [19].

Rahmati [20] conducted an experiment using an Alumina Honeycomb Panel (AHP) that included multiple bolted joints using nonlinear dynamics modelling and parametric recognition. Dynamic FEM responses in the time domain were obtained using the nonlinear module, and they were later transformed into frequency domain analysis. Force state mapping approach was used to identify nonlinear joint parameters from time-domain acceleration data in response to single-frequency stimulation close to the initial natural frequency [21]. Jacobs et. al [22] combined frequency and time domain techniques, using time domain, restored force plots to define the amplitude and frequency characteristics of nonlinearities. These non-linear characteristics were then used in the output for the formulation of the nonlinear recognition, resulting in a model that can be used to simulate device responses with linear and non-linear frequency response functions of suspension systems of automobiles which was used in this method.

The task of designing nonlinear joints is a complex undertaking that necessitates a profound comprehension of the behavior of materials and structures. The type of loading the joint will encounter, the joint's geometry, and the qualities of the materials that will be utilized must all be taken into account when designing nonlinear joints. Additionally, there is a need for considering effects of corrosion, fatigue, and other parameters which can significantly affect the performance of a joint above timespan.

In the field of dynamic analysis of complicated structures, a set of techniques for substructure synthesis has been devised. The aforementioned techniques take into account the notion that a convoluted framework is comprised of interlinked sub-frameworks. The synthesis of the overall structure dynamics is achieved by suitably integrating the dynamics of the constituent substructures. The dynamics of the substructure can be analyzed independently and subsequently integrated to form the comprehensive overall structure. The aforementioned technique has been expanded for the purpose of dynamic system analysis and has been employed by numerous scholars in the recent era. Yang et al. [23] devised a combined parameter identification approach employing substructure synthesis and frequency response functions (FRFs). This article refined the approaches of Tsai and Chou

[24] and Wang and Liou [25] and derived joint identification equations. The joint model was matrixed. Rotational degrees of freedom were obtained from the correctly calibrated finite element model of the unconstrained (or free-free) structure. The joint's translational and rotational stiffness was then determined using substructure synthesis. The parameters substantially influence the results' correctness. Accurately identify the sensitive joint's rotational stiffness. In the insensitive zone, translational stiffness results are less accurate. Lee and Hwang [26] used frequency response function (FRF)-based substructuring and optimization to uncover joint structural parameters in complex systems. Gradient-based optimization was utilized to estimate joint parameters using FRF. Noise and stiffness parameters affected accuracy. Celic and Boltezar's [27] joint identification approach considered mass, stiffness, damping, and rotation. FRF data from a calibrated FE model estimated unmeasured FRFs and damping. Sjoval and Abrahamsson [28] used FRFs to create a non-parametric model for system identification in linear structural dynamics. Gillich and Nedelcu [29] performed experiments on cantilever beams with cracks and imperfect clamped ends using sophisticated models to ascertain the true modal characteristics and evaluate the location of the crack and fixture imperfection. Cristian and Gillich [30] outline a process for creating an artificially intelligent system that can determine whether a beam is impacted by cross-sectional cracks. It can also determine the natural frequency loss that transverse cracks cause, even when the beam is not securely fastened.

The task of identifying joint properties holds significant importance in the prediction of dynamic characteristics of mechanical and structural systems. The numerical methodologies employed in addressing structural dynamic issues, such as Finite Element Method (FEM), frequently yield outcomes that diverge from those obtained through experimental measurements. In earlier analysis, joints were simplified by assuming ideal boundary conditions such as a fixed joint or a simply supported joint etc. However, the analytical or FEM models with the simplified boundary conditions fail to predict the modal parameters accurately and often the deviation is significant enough asking for the need of proper joint modeling in the analysis.

The present research is focused upon the formulation of a mathematical model to effectively discern the parameters of non-linear structural components. To achieve this, the concept of substructure synthesis is used which combines both non-linear and linear stiffness characteristics for

formulation of the comprehensive equation that describes the dynamic behavior of the system. Further analysis was done by comparing the results from a mathematical model with experimental data. The derived frequency equation is then utilized in inverse analysis to estimate the non-linear parameter. The research work validates the efficacy of the mathematical model on a cantilever beam, shows a higher degree of accuracy while comparing with experimental results. With the increasing demand for complex and efficient engineering structures, the study of nonlinear joints will continue to be an important area of research and development.

2. DEVELOPMENT OF MATHEMATICAL MODELLING

For non-linear joint parameter estimation, a cantilever beam having non-linear rotational stiffness (K_2^*) and linear translational stiffness (K_1) is considered with as shown in fig.1 and fig.2.

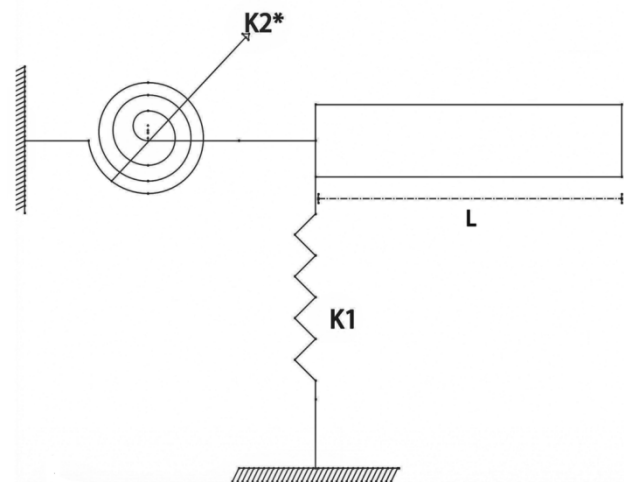


Figure 1. Cantilever beam model with nonlinear rotational spring stiffness, and linear translation spring stiffness K_1 and elastic support at fixed end

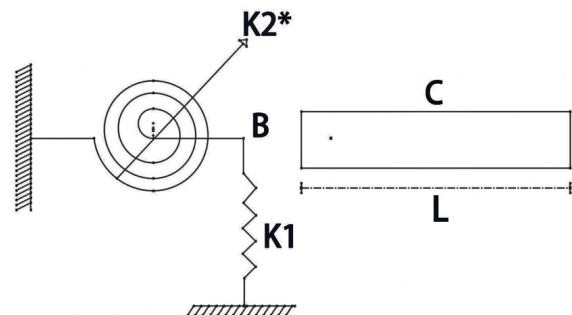


Figure 2. Cantilever Beam having sub-system C and B

Subsystem B's FRF Matrix is depicted as a diagonal matrix $[\beta]$ [18]

$$[\beta] = \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix} = \begin{bmatrix} \frac{1}{K_1} & 0 \\ 0 & \frac{1}{K_2^*} \end{bmatrix} \quad (1)$$

Where, β_{11} , β_{22} are direct receptance for subsystem B and β_{12} , β_{21} are cross receptance for subsystem B.

FRF Matrix for subsystem C $[\gamma]$

$$\gamma_{11} = \left(\frac{-L^3}{EI} \right) \left(\frac{1}{\lambda^3} \right) \left(\frac{F_5}{F_3} \right) \quad (2)$$

$$\gamma_{22} = \left(\frac{L}{EI} \right) \left(\frac{1}{\lambda} \right) \left(\frac{F_6}{F_3} \right) \quad (3)$$

$$\gamma_{12} = \left(\frac{L^2}{EI} \right) \left(\frac{1}{\lambda^2} \right) \left(\frac{F_1}{F_3} \right) = \gamma_{21} \quad (4)$$

The non-dimensional frequency (λ)

$$\lambda = \left[\frac{\omega^2 \rho A L^4}{EI} \right]^{1/4} \quad (5)$$

Where, L is beam length, EI is bending stiffness, ω is flexural vibration's natural frequency, ρA is the linear mass density function.

In this research, rotational stiffness is used as a cubic type stiffness non linearity for parametric identification with principle of sub structure synthesis.

Nonlinear K_2^* can be calculated by

$$K_2^* = K_2 + K_{4L} \quad (6)$$

Where K_2^* is the nonlinear stiffness, K_2 is the linear stiffness parameter, K_{4L} is the cubic type stiffness non linearity.

The derivation of the describing function for the nonlinearity of cubic type stiffness can be accomplished through the utilization of the harmonic balance method as below.

$$K_2 \theta(t) + K_4 \theta^3(t) = F(t) \quad (7)$$

$$\left[K_2 + \frac{3}{4} K_4 \theta_0^2 \right] \theta_0 = F_0 \quad (8)$$

From equation (6) and equation (8), we get,

$$K_{4L} \cong \frac{3}{4} K_4 \theta_0^2 \quad (9)$$

Where, K_4 is the cubic stiffness coefficient, θ_0 is a non-linear system's harmonic response amplitude with applied harmonic force.

Setting the denominator of the receptance functions (Δ) to zero, a frequency equation of composite system will be,

$$\Delta = (\beta_{11} + \gamma_{11})(\beta_{22} + \gamma_{22}) - (\beta_{12} + \gamma_{12})^2 = 0 \quad (10)$$

where, β_{11} , β_{22} are direct receptance of sub system B

β_{12} or β_{21} are cross receptance of sub system B

γ_{11} , γ_{22} are the direct receptance of sub system C

γ_{12} or γ_{21} are the cross receptance of sub system C

Now, rearranging the harmonic balance equation by using sub structure synthesis theory, we get,

$$K_1 K_2 + K_1 K_{4L} + [A_x]_{NL} K_1 - [B_x]_{NL} K_2 - [B_x]_{NL} K_{4L} = [C_x]_{NL} \quad (11)$$

$$K_1 K_2 + [A_y]_{LL} K_1 - [B_y]_{LL} K_2 = [C_y]_{LL} \quad (12)$$

Subtracting equation (12) from equation (11), we get,

$$\begin{aligned} \{[A_x]_{NL} - [A_y]_{LL}\} K_1 + \{[B_y]_{LL} - [B_x]_{NL}\} K_2 \\ + \{K_1 - [B_x]_{NL}\} K_{4L} \\ = \{[C_x]_{NL} - [C_y]_{LL}\} \end{aligned} \quad (13)$$

Eq.11 and Eq.12 signifies presence of the linear and non-linear joint parameter stiffness coefficients. Whereas, Eq.13 gives the correlation of linear translatory stiffness, non-linear rotational stiffness with cubic stiffness non-linearity.

3. EXPERIMENTAL SET UP AND PROCEDURE

Figure 3 and Figure 4 shows experimental arrangement of the cantilever beam, providing a visual representation of the setup. The specific one end was bolted while external excitation was provided on its free end.

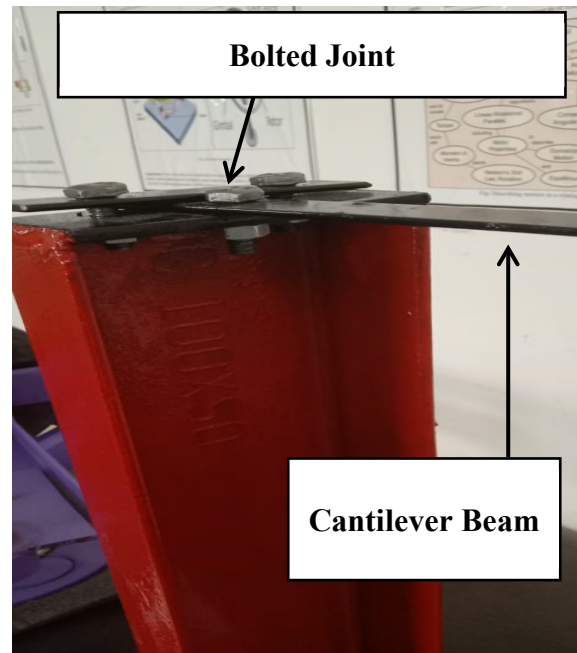


Figure 3. Actual Experimental set up of a cantilever beam bolted at its end

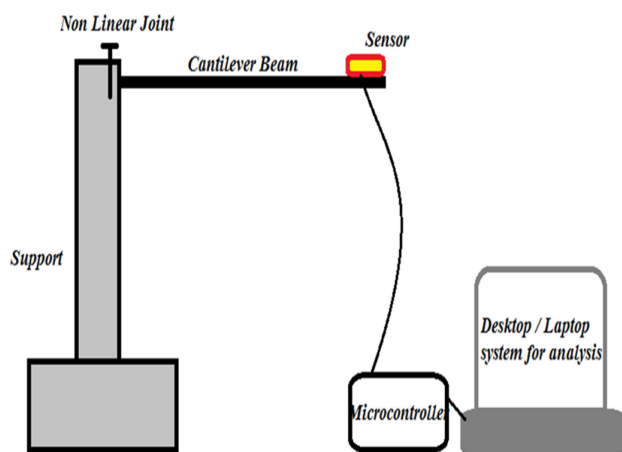


Figure 4. Cantilever beam bolted at its end

Accelerometer (sensor) Adxl335 was used to measure the acceleration of gravity resulting from motion vibration and Arduino (Microcontroller) ATmega328P was used for interfacing as shown in Figure 5 and Figure 6.

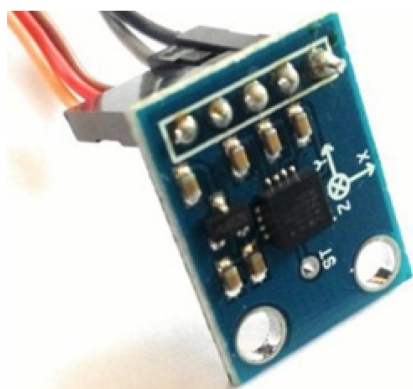


Figure 5. Accelerometer Adxl335



Figure 6. Arduino ATmega328P

The dimensions and material properties of beam are shown in the Table 1.

OROS OR34 (Power:15 VA max.10-28V) Compact Vibration Analyzer is used Gathering and documenting vibration data obtained from sensors positioned on or in close proximity to the equipment or structure being examined. DYTRAN General Purpose Accelerometer (3055B1, S/N 19535) was utilized to evaluate the natural frequencies. DYNAPULSE Impulse Hammer (MODEL 5850B)

was used to provide external excitation in order to capture the natural frequency.

Table 1. Geometrical and material properties of beam (Mild Steel Plate)

Dimensions	Material Properties of beam
Area of Cross section (A)	$(0.003 \times 0.025) \text{ m}^2$
Beam Length (L)	0.6 m
Beam Width (w)	0.025 m
Beam Thickness (t)	0.003 m
Elastic modulus (E)	$2.075 \times 10^{11} \text{ N/m}^2$
Density (ρ)	7800 Kg/m^3

A Fast Fourier Transform (FFT) analyzer was employed as a tool for measuring the natural frequencies. The accelerometer was placed on the bolted joint to capture frequency domain signals and an anvil was used for creating dynamic motion of cantilever beam. A signal is broken down into its distinct spectral components by an FFT analyzer, which then delivers frequency information about the signal.

Through the experimentation, around 1130 sample reading from accelerometer sensor ADXL335 were captured. Table 2 shows the sample readings along x, y and z axis.

Table 2. Sample reading from Accelerometer Sensor ADXL335

X	Y	Z	Acceleration
-0.01	0	-0.1	0.1004988
0.01	0	0.03	0.0316228
0.03	0.03	0.03	0.0519615
0.04	0	-0.04	0.0565685
0.06	0.01	-0.01	0.0616441
0.07	0.01	0.04	0.0812404
0.07	0.01	-0.01	0.0714143
0.07	0.03	-0.03	0.0818535
0.09	0.01	-0.03	0.0953939
0.1	0.01	0	0.1004988
0.09	0.04	0	0.0984886
0.09	0.01	-0.01	0.0911043
0.1	0.01	-0.03	0.1048809
0.1	0.01	0.01	0.100995
0.1	0.03	0	0.1044031
0.1	0.01	-0.03	0.1048809
0.1	0.01	-0.03	0.1048809
0.12	0.01	0	0.1204159
0.12	0.03	0	0.1236932
0.12	0.01	-0.03	0.1240967
0.12	0.03	-0.04	0.13

The frequency plot obtained from acceleration showing normalized frequency with respect to the magnitude is shown in Figure 7. The plots signifies the nature of frequency domain signals as the magnitude changes from maximum displacement to mean and then to minimum displacement.

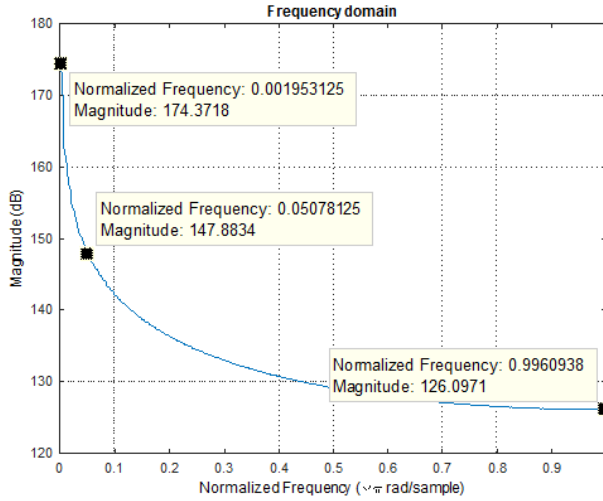


Figure 7. Normalized frequency Vs. Magnitude

4. RESULT AND DISCUSSION

Table 3 provides the results of identified non-dimensional natural frequencies.

Table 3: Non-Dimensional Natural Frequencies

Assumed value of stiffness parameters	Response Amplitude in radian	Non- dimensional natural frequency			
		λ_1	λ_2	λ_3	λ_4
	Ideal Value	1.8	4.537	7.249	9.5660
$K_1 = 1.79 \times 10^3$ N/m $K_2 = 5.98 \times 10^3$ N-m/rad $K_4 = 0.98 \times 10^6$ N-m/rad ³	0.01	1.6277	4.3253	7.019	9.4211
	0.02	1.6277	4.3254	7.022	9.4213
	0.04	1.6278	4.3261	7.027	9.4216
	0.06	1.6279	4.3264	7.053	9.4216
	0.08	1.628	4.3266	7.0685	9.4219
	1.00	1.6281	4.3271	7.0895	9.4232
	1.12	1.6283	4.3273	7.123	9.4237
	1.14	1.6286	4.32844	7.1831	9.4249

In a study, considering large response amplitudes, the non-linear zones within the system have been identified. For large response amplitude, the non-linear zone is identified, and for a given value of the stiffness parameters, the non-dimensional natural frequencies are calculated. Non-dimensional natural frequencies simplify system comparison and study by eliminating physical dimensions.

Significant variation at higher frequencies is seen, indicating the presence of nonlinear behavior. The presence of notable fluctuations at elevated frequencies suggests that the response of the system

is no longer exclusively reliant on the amplitude or frequency of the excitation.

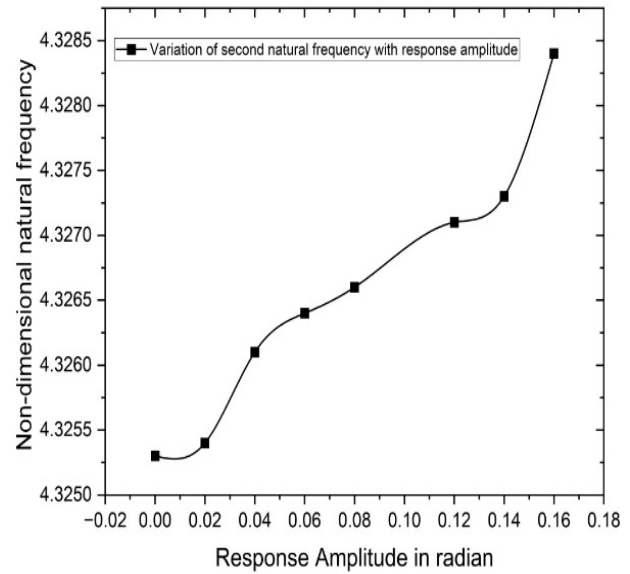


Figure 8. Second Natural Frequency Variation by Response Amplitude

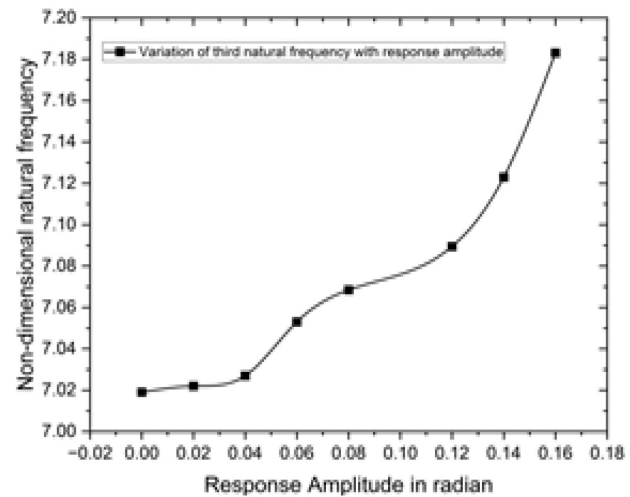


Figure 9. Third Natural Frequency Variation by Response Amplitude

Prior knowledge for linear stiffness parameters is necessary for the suggested method. The aforementioned parameters serve as an elementary structure for conducting comparisons and calibrations during the analysis. Fortunately, obtaining this data is comparatively uncomplicated, especially when accounting for diminished response rates within a particular interval. This is easily attainable at lower response rates considered as 0.01 radian to 1.14 radian. For joint parameter estimation, the deviation of equivalent linearized stiffness K_{3L} (N-m/rad) having amplitude response for mode 2 and mode 3 using curve fitting tool is shown in Figure 10 and Figure 11. The aforementioned figures were utilized as a visual depiction of the disparities between the

approximated linearized stiffness and the factual non-linear stiffness of the articulation.

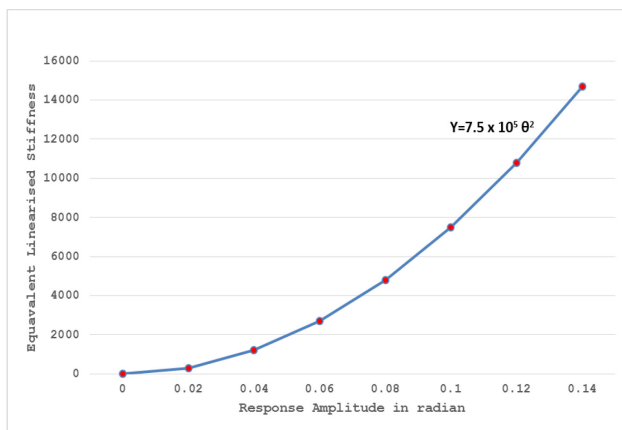


Figure 10. Variation of Equivalent Linearized Stiffness with Response Amplitude for Mode 2.

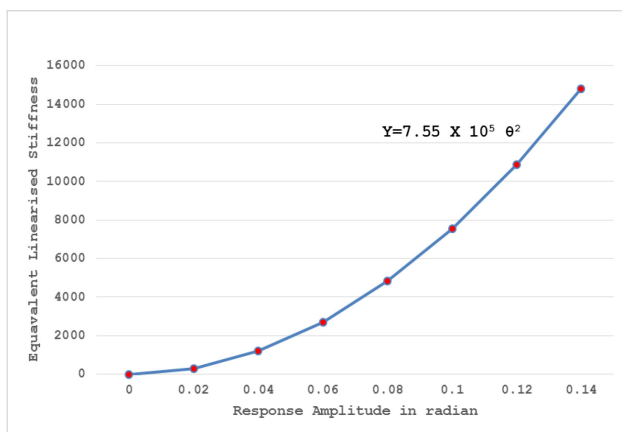


Figure 11. Variation of Equivalent Linearized Stiffness by Response Amplitude for Mode 3.

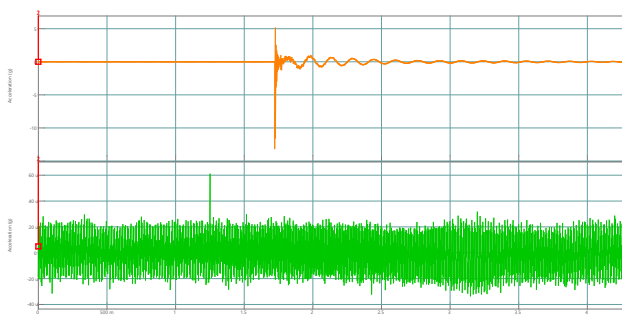


Figure 12. Time domain signal of beam under impact excitation

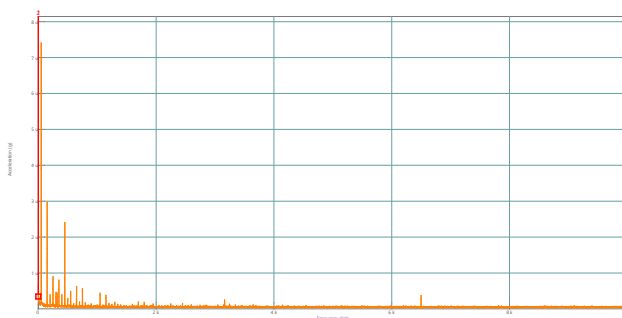


Figure 13. Vibration response of cantilever beam using real time FFT analyzer at bolted end

From Figure 12 and 13, it is observed that the frequency of a cantilever beam is often indicated by the first peak observed in the frequency domain. The presence of resonance, or a dominant vibration mode, is typically identified by the peak on the graph. The beam exhibits the highest level of vibration at this particular frequency when it is triggered. The presence of this peak suggests that the beam is resonating in close proximity to its natural frequency.

Table 4 provides the results of Estimated Cubic Stiffness Coefficient K_4 for modes 2, 3, 4 and 5. The aforementioned coefficient denotes the cubic stiffness component in the mathematical system employed for the analysis. It can be observed that an approximate K_4 value, cubic stiffness coefficient, is quite closer to the actual value. The K_4 values estimated exhibit a significant concurrence with the anticipated values derived from the system's performance. Similarly the percentage error is not more than 5% which shows the exactness of the model. This suggests that the mathematical model provides an outstanding approximation of the actual system behavior and indicates a high degree of precision.

Table 4. Cubic Stiffness Coefficient value of K_4

Mode	Estimated Stiffness Coefficient (K _c)	K ₄ =(4/3)K _c	Exact Value of K ₄	Percentage Error
Second	7.5x10 ⁵	1.000 x10 ⁶	0.98 x 10 ⁶	2.0 %
Third	7.55x10 ⁵	1.007x10 ⁶		2.681 %
Fourth	7.57x10 ⁵	1.009 x10 ⁶		2.874 %
Fifth	7.6x10 ⁵	1.013x10 ⁶		3.257 %

5. CONCLUSION

This research study focuses on the development of a technique for identifying the parameters of nonlinear structural elements. The frequency equations are obtained through the use of a sub-structure synthesis idea. The equation that was created was utilized in an inverse investigation, where theoretically obtained, data were employed to estimate a non-linear parameter. The response amplitude in the non-linear region was determined using a curve fitting technique, which revealed a cubic stiffness non-linearity in the rotational stiffness parameter. The method is validated by employing numerical case studies that imitate experimental measurements using theoretically produced data. The results demonstrate satisfactory agreement between the assumed and estimated values. Evidence shows that a technique is resilient to inaccuracies in measurements.

The research acknowledges that engineering structures subjected to varying loads often exhibit structural nonlinearity. While linear analysis can be somewhat applicable to nonlinear systems, it is crucial to consider nonlinearity while studying bolted joint constructions due to their unique behaviour. This work presents a mathematical methodology for identifying the parameters of non-linear structural elements. Non-linear parameters can be estimated by researchers using inverse analysis, which involves deriving the frequency equation from this approach. Due to the nonlinearity of the joint behaviour, this representation more accurately reflects the response of the system. However, current study exhibits restraint over a number of unknown non linear joint parameter which may results in complexities in mathematical modeling.

The further study also have a scope in applying damper system. The existing model can be enhanced by include damping characteristics of the joint. Subsequently, an appropriate approach can be proposed to solve the resulting complex frequency equation.

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