# A Method for Colored Noise Generation

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*Abstract:* - The present paper addresses the generation of power-law, colored digital noise signals (sequences) with arbitrary spectral slope. In the beginning, brief background information is given about some noise features. Further, a newly proposed method is described, based on generation of a white noise signal, its transformation into the frequency domain, spectral processing and inverse transform back into the time domain. Computer simulations are performed to confirm the consistency of the algorithm, including estimation of the power spectral density and the autocorrelation, along with example of its outperformance in comparison with the corresponding in-built Matlab® function.

*Keywords:* - colored noise, pink noise, red noise, blue noise, violet noise, generation

## 1. INTRODUCTION

The noise is a fundamental aspect of the reality. This is a ubiquitous phenomenon spanning innumerable fields of the nature. The unavoidable presence of the thermal noise in the universe is just enough to allude. This and other types of noises are presented in the structure of the nature itself: the arrangement of the cells in the human retina, the model of occurrence of natural disasters, the statistics of DNA sequences, etc. [1, 2].

The generation of noise signals with given parameters is a classic problem in the field of circuits and systems, signal processing, automation, mechanics, acoustics, vibration engineering, econometrics, electrical and electronics measurements and instrumentation, etc.

This paper discusses a method for generation of digital, pseudorandom colored (power-law) noise sequences, with arbitrary power spectral density (PSD) slope $\propto f^{\alpha}$ ($\alpha \in \mathbb{R}$), regardless of the used DSP device (trivial PC, PLC, FPGA, etc.). The basic idea of the method is firstly described in [3] and now, with modifications, independently proposed by the author.

The generated noise could be used as a test signal for software computer simulations or outputted via some kind of data acquisition system (i.e. a sound card or a professional DAQ device) for real-world experiments or measurement procedures.

There are two main approaches for obtaining power-law noises: physical and statistical. Physical approach refers to methods based on microscopic considerations of the underling physical process that produces the corresponding noise. Unfortunately each theory is specific to the device or system being considered, and therefore somehow limited. On the other hand, the statistical approach does not look at given device, system or process, but treat the phenomenon itself as a statistical subject and is more generalized and fundamental [4]. This statistical approach is adopted further in the paper.

In order to avoid possible misunderstanding of the noise essence, in the following a brief preliminary agreements are made.

The term "noise signal" is referred to a signal produced by a stochastic process. Further, one considered the noise signal itself and not the underling process. The term "colored noise" is used to refer to any non-white noise signal whose PSD is not a constant but is a function of the frequency.

The standard definition of the white noise signal implies an infinity bandwidth, hence an infinity energy (according to the Planck's law) and thus it is a purely theoretical construction and physically unrealizable. In the real-world applications the noise signal is considered white if it has flat PSD over the frequency range relevant to the context [4].

Often, it is incorrectly assumed the terms "Gaussian noise" and "white noise" to be used interchangeably. One must remind that Gaussianity refers to the amplitude probability distribution in the time domain, while the term "white" refers to the independently distributed power in the frequency domain and neither property implies the other.

The threshold value $\alpha = -1$ defines the pink noise and also marks the boundary between the predictable and unpredictable behavior of the generated noise signal [5]. For values $\alpha \le -1$ the noise signals are non-stationary and long-term correlated [6].

Note that the Gaussianity and stationarity of the generated signals are not a requirement, but they must satisfy the following conditions [7]:

1) to be non-stationary for $\alpha \le -1$;
2) to be scale-invariant (i.e. self-affine);
3) to have PSD with the desired slope over the entire frequency range of interest.

Scale invariance refers to the independence of the model from the scale of observation. The fact that the power-law noises are scale invariant is suggested by their PSD shape. If the frequency scale is changed, the original amplitude scaling can be obtained by simply multiplying with an appropriate constant [4].

Different types of colored noise signals are distinguished by the slope of their PSD (on a log-log scale). A few of them are listed in Table 1.

**Table 1.** Information about some types colored noise signals [1, 2, 8, 9, 10]

| Noise type | $\alpha$ | PSD slope dependence | Spectral slope rate | Examples |
|---|---|---|---|---|
| Violet noise | +2 | $\propto f^2$ | +6 dB/oct, +20 dB/dec | Signal from an acoustic thermal noise of water \| Differentiation of a white noise signal |
| Blue noise | +1 | $\propto f$ | +3 dB/oct, +10 dB/dec | Signal from a Cherenkov radiation process \| High-frequency filtering of a white noise signal |
| White noise | 0 | flat | 0 dB/oct, 0 dB/dec | Signal from a white noise process (e.g. Johnson–Nyquist thermal noise) |
| Pink noise | –1 | $\propto 1/f$ | -3 dB/oct, -10 dB/dec | Signal from statistical fluctuations of a number of natural processes \| Low-frequency filtering of a white noise signal |
| Red noise | –2 | $\propto 1/f^2$ | -6 dB/oct, -20 dB/dec | Signal from a Brownian motion (Winner process) \| Integration of a white noise signal |
| Black noise | $< -2$ | - | > -6 dB/oct, > -20 dB/dec | Model of the frequency of the natural disasters |

## 2. PROPOSED METHOD DESCRIPTION

An object of consideration is the generation of a real-world (i.e. mathematically real and time limited) discrete noise signal (sequence) with zero mean, unity standard deviation and arbitrary PSD slope $\propto f^{\alpha}$, i.e. amplitude spectrum density (ASD) slope $\propto f^{\alpha/2}$

$$cn[nT_s], \tag{1}$$

where: $n = \{1,...,N\} \in \mathbb{N}$ is the sample number;

$N \in \mathbb{N}$ – number of all samples;

$T_s$ – sampling time interval.

The duration of the sequence is

$$T = N \cdot T_s. \tag{2}$$

The generation of the signal is performed via processing of an i.i.d, zero mean, additive white Gaussian noise $wn[n]$. The last could be generated via one of the known methods [11, 12, 13].

One proposed a spectral processing technique, involving a manipulation of the amplitude spectrum of the white noise time sequence in order to obtain a new colored noise time sequence with an arbitrary spectral slope in the frequency domain. The phase spectrum is kept untouched since there is no need of manipulation.

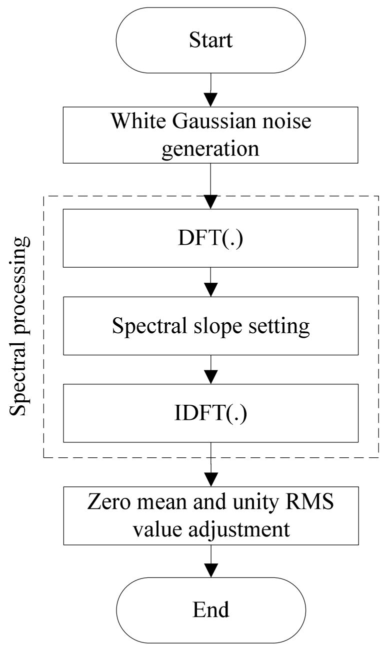



**Figure 1.** Algorithm of the proposed colored noise generation procedure. The generation of the desired colored noise signal is obtained via spectral processing of a white noise signal.

The generation algorithm has three basic steps (*cf.* Fig. 1): (i) generation of a white noise signal $wn[n]$ in the time domain; (ii) DFT on a $wn[n]$, manipulation of the complex spectral coefficients in order to obtain the new spectrum of the desired colored noise signal and IDFT on the last; (iii) signal conditioning of the colored noise signal $cn[n]$ in the time domain in order to ensure zero mean and unity standard deviation values.

The DFT/IDFT transforms are performed according to [14]:

$$\dot{WN}[k]=\frac{1}{N}\sum_{n=1}^{N}wn[n]\cdot e^{-j2\pi\frac{nk}{N}}\ , \tag{3}$$

$$cn[n]=\sum_{n=1}^{N}\dot{CN}[k]\cdot e^{j2\pi\frac{nk}{N}}\ . \tag{4}$$

where $k=\{1,...,N\}\in\mathbb{N}$ is the spectral component number.

The complex spectrum obtained using Eq. (3) has a structure as shown in Fig. 2. The spectrum has Hermitian symmetry where the right half is the unique one, including the DC and (eventually) the Nyquist component, and the right half is a conjugate flipped copy of the left one. Hence, it is more easily to manipulate only the right half of the spectrum and then to make a conjugate flipped copy of it.

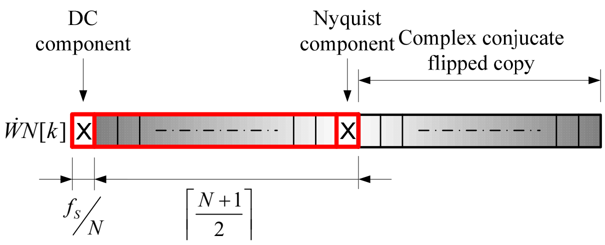



**Figure 2.** Structure of the discrete spectrum of the signals. The left half of the spectrum (located between the DC and Nyquist components) is the unique one. The right half is a conjugate flipped copy of the left one, excluding the DC and Nyquist components – they are unique.

The spectral slope setting procedure itself (*cf.* Fig. 1) is done as shown in Fig. 3. One must note that: (i) $\dot{WN}[k]$ is the complex amplitude spectrum and not a PSD of the white noise signal; (ii) the spectral coefficient $k$ and the linear frequency $f$ are uniquely linked; (iii) the division of the complex spectrum coefficients is equivalent to division of the amplitude spectrum while the phase spectrum remain untouched; (iv) any orthogonal transformation (e.g. DFT) of the Gaussian random signal will results a new Gaussian random signal with the same variance [15]. This ensures an independent distribution of both the amplitude and phase spectrums of the generated colored noise signal.

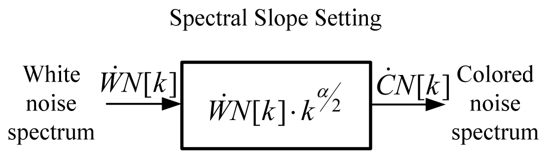



**Figure 3.** Block diagram of the spectral slope setting procedure. Note that the frequency index *k* is used instead of the actual frequency *f*.

Finally, the mean value of the colored noise signal is set to zero, and the RMS-value is set to unity:

$$\tilde{cn}[n]=cn[n]-\mathrm{E}\left[cn[n]\right], \tag{5}$$

$$\hat{cn}[n]=\frac{\tilde{cn}[n]}{\sqrt{\mathrm{E}\left[\tilde{cn}[n]^2\right]}}, \tag{6}$$

where $\mathrm{E}[\bullet]$ denotes the expected value of the signal.

These actions allow the user to set the desirable values of the mean and RMS values freely as

$$\mu+\sigma\cdot\hat{cn}[n], \tag{7}$$

where: $\mu$ is the desired DC value of the signal;
$\sigma$ – desired RMS value of the signal.

## 3. SIMULATION RESULTS

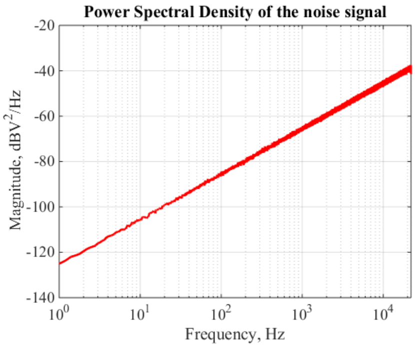
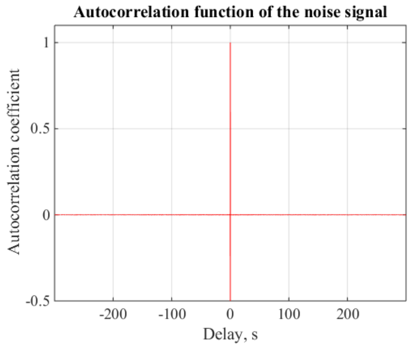
Computer simulations were performed with Matlab® to examine the performance of the proposed colored noise generation algorithm.

Noise sequences of different types were generated with duration $T$ = 300 s and length $N=13{,}23\cdot10^6$ samples at sampling frequency $f_s=44100$ Hz. The PSD and the auto-correlation function (ACF) of the generated signals are listed in Table 2.

The PSD is computed using the Welch's modified periodogram [16] with the Hamming window with length $win=132{,}3\cdot10^3$ and overlapping $ovrlp=\frac{3}{4}\cdot win$.

The visual examination of the PSD plots confirmed the desired slope rate of the generated signals and their scale-invariance. The expectation of the ACFs approves the thesis of long-term correlation of the signals for $\alpha\le-1$.

**Table 2.** PSDs and ACFs of some generated colored noise signals

<table>
<tr><th></th><th>Power Spectral Density</th><th>Autocorrelation Function</th></tr>
<tr><td>Violet noise</td><td>
</td><td>
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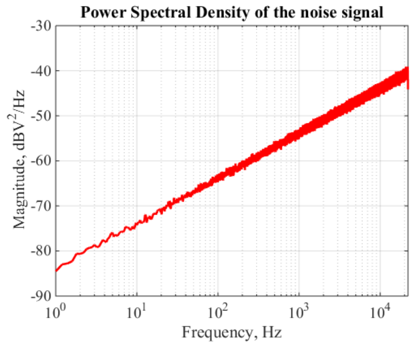
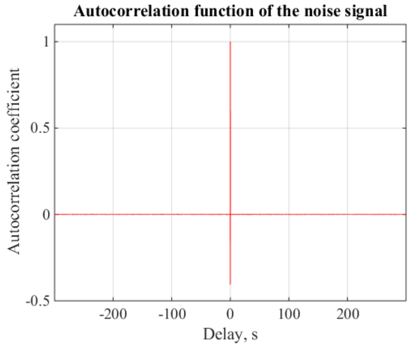
<tr><td>Blue noise</td><td>
</td><td>
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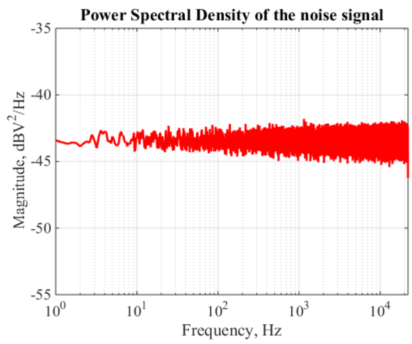
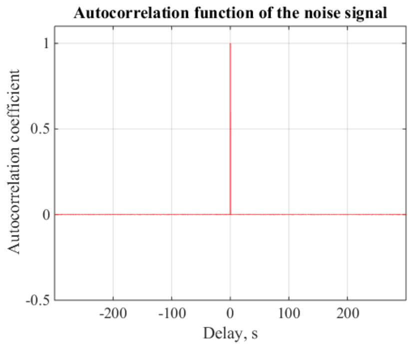
<tr><td>White noise</td><td>
</td><td>
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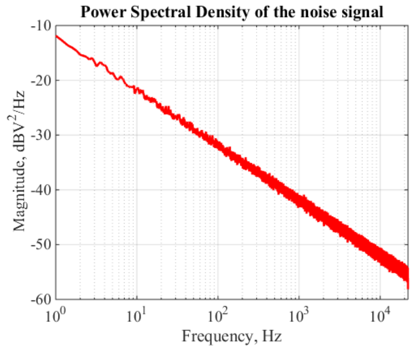
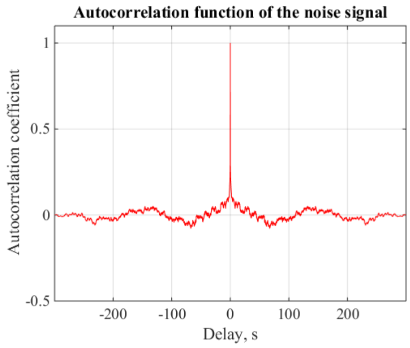
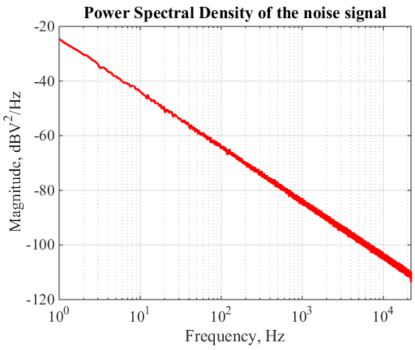
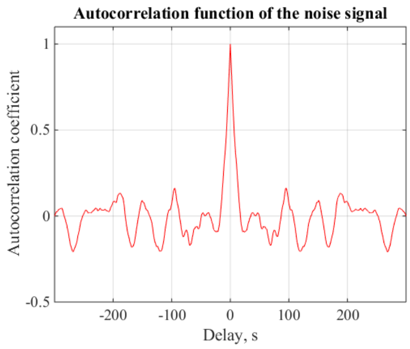
<tr><td>Pink noise</td><td>
</td><td>
</td></tr>
<tr><td>Red noise</td><td>
</td><td>
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Further, the generated signals are tested for stationarity using the Kwiatkowski–Phillips–Schmidt–Shin (KPSS) test [17] and the augmented Dickey–Fuller test (ADF) [18]. The goal is to show that the proposed algorithm generates non-stationary noise sequences for $\alpha \leq -1$ which is in good agreement with the theory [4, 7]. For all tests, the significance level is chosen to be 0,05. The results are shown in Table 3.

**Table 3.** Results from the noise signals' stationarity tests.

| Noise type | KPSS test | | ADF test | | Result comment |
|---|---|---|---|---|---|
| | *p*-value | Test result | *p*-value | Test result | |
| Violet Noise | 0,100 | The signal is trend-stationary | 0,001 | The signal has no unit root | The signal is certainly stationary |
| Blue Noise | 0,100 | The signal is trend-stationary | 0,001 | The signal has no unit root | The signal is certainly stationary |
| White Noise | 0,100 | The signal is trend-stationary | 0,001 | The signal has no unit root | The signal is certainly stationary |
| Pink Noise | 0,010 | The signal is not trend-stationary | 0,001 | The signal has no unit root | The signal is not stationary, but has no unit root. The signal is heteroscedastic |
| Red Noise | 0,010 | The signal is not trend-stationary | 0,109 | The signal has a unit root | The signal is certainly non-stationary |
| Black Noise | 0,010 | The signal is not trend-stationary | 0,651 | The signal has a unit root | The signal is certainly non-stationary |

**Table 4.** Speed comparison of the proposed and the corresponding in-built Matlab® functions

| Noise type | Short sequence, $T = 1$ s | | | Long sequence, $T = 300$ s (5 min) | | |
|---|---|---|---|---|---|---|
| | Authors' function time, s | Matlab® function time, s | Out-performing | Authors' function time, s | Matlab® function time, s | Out-performing |
| Violet Noise | 0,0032 | 0,0052 | 38,5 % | 1,0809 | 1,2854 | 15,9 % |
| Blue Noise | 0,0032 | 0,0051 | 37,3 % | 1,0293 | 1,2570 | 18,1 % |
| Pink Noise | 0,0040 | 0,0054 | 25,9 % | 1,2382 | 1,2904 | 4,0 % |
| Red Noise | 0,0031 | 0,0052 | 40,4 % | 0,9963 | 1,2324 | 19,2 % |

Finally, tests have been made to show the outperforming of the proposed algorithm in comparison with the in-build Matlab® function *dsp.ColoredNoise System object*™ [19], that implements an autoregressive method of order 63, based on [4]. The results are listed in Table 4, averaged over 100 runs of the two routines.

One can deduce that the author's function is faster than the Matlab® one, moreover the last allows values of $\alpha$ only in the interval $\alpha \in [-2,\ 2]$. In addition, the proposed method can be used in real-time applications, since the execution time is far less in comparison with the duration of a generated signal itself.

All simulations clarify that the proposed algorithm is consistent and produced noise sequences that meet the requirements listed above in Section 1.

## 4. CONCLUSIONS

In the paper a method for colored noise generation with arbitrary user-defined spectral slope is presented. The procedure is based on generation of a white noise time sequence, its spectral processing in the frequency domain and translation of the newly obtained spectrum back in the time domain. Every spectral line is weighted proportionally to its spectral number (i.e. frequency), so the overall ASD slope is proportional to the frequency by the law $f^{\alpha/2}$ and the PSD slope – by $f^{\alpha}$. Also, it is possible to control the average and RMS values of the generated colored noise sequence.

The method is tested in the Matlab® environment and the results clearly indicate its consistence. It is simple and quick and can be used to generate noise over frequency band of arbitrary size with arbitrary values of the PSD slope and ability of real-time operation.

The proposed method is a new impact in the noise generation practice. The possible applications including but are not limited to audio, acoustics, vibration and oceanographic engineering, microelectronics, neuroscience, econometrics, in measurements and simulation applications, including real-time ones. The algorithm is implemented in the Matlab® software environment as Matlab®-functions and accessible at [20, 21].

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