Estimation of The Frequency of Very Short Signals by Involving Artificial Neural Networks

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Abstract: - This paper proposes a method to accurately estimate the frequencies of signals that contain less than one and a half periods of the targeted frequency. A behavioral model is developed by finding the amplitudes of three points on the main lobe of the Discrete Fourier Transform (DFT) of a zero-padded generated harmonic signal. The amplitudes are normalized, and the distance between the generated and roughly estimated frequency is found. The signal is shortened, and the process is repeated. We train an Artificial Neural Network (ANN) with the obtained results, namely with the amplitudes as Input data and the distance as the Target. With this network, we succeeded in estimating the frequencies with high accuracy, the errors being of the order of millihertz.

Keywords: - structural health monitoring, frequency estimation, artificial neural network, short signals, modal analysis.

1. INTRODUCTION

Frequency estimation can be achieved using standard methods [1], such as Discrete Fourier Transform (DFT) or Fast Fourier Transform (FFT). These methods give excellent results for signals manifesting a more extended time but give erroneous results for short signals due to the significant distance between the spectral lines [2]. Different interpolation methods [3]-[7], including zero padding, can be applied to achieve better results. The latter method presumes to extend the signal length by adding zero-value points to the original signal [8] but when used on very short signals it provides additional information in the time domain, but it cannot enhance the true time-domain characteristics of the signal [9].

Other frequency estimation methods presume to calculate the spectra of a signal for different time durations and overlap the spectra [10] but this method is also limited when dealing with very short signals, because the time-domain information is limited, and the accuracy of characterizing transient behavior could be compromised [11]. Even if they have advantages, the methods listed are not accurate enough for many practical applications, need a long analysis time, use extensive computer resources, and cannot be automated.

To overcome the disadvantages, we proposed a learning-based method using an ANN to identify frequencies at a position between lines in the spectrum [12-13]. The use of ANN-based methods has been proven as reliable also for determining the natural frequencies of structures with known mechanical and physical parameters [14]. To estimate the frequencies, we first perform a zero-padding to double the length of the signal. Next, we calculate the signal's DFT, then shorten it and recalculate the DFTs. The frequencies and amplitudes for three points on the main lobe were determined for each signal length, representing the input data for training the ANN. The output consists of the distance calculated between the frequency of the first point and the generated frequency. The ANN is trained with normalized data; it accurately estimates the correction term regardless of the frequency or amplitude of the tested signal. Unfortunately, the method works well just when the signal has at least four cycles/periods of the targeted frequency [15].

The method proposed in this article refers to training an ANN with shorter signals, which contain up to one/one and a half cycle.

2. CREATION OF THE TRAINING DATABASE

2.1. Theoretical background

Starting from a discrete-time sinusoidal signal with zero initial phase angle, amplitude A, signal frequency f_R, sampling frequency f_S, number of samples N, sample index n, are given by the equation of the form:

$$a[n] = A\sin(2\pi f_R \frac{n}{f_S}) \quad n = 0...N \qquad (1)$$

The amplitudes in DFT are calculated for the given signal, and the frequencies are displayed on the spectral lines located at equal distances. For each spectral line, the real (R_e) and imaginary (I_m) parts are calculated using equations 2 and 3, and then the modulus is obtained, with the relation 4:

$$R_e\left(A_k\right) = \sum_{n=0}^{N-1} a[n]\cos\left(\frac{2\pi}{N}nk\right) \qquad (2)$$

$$I_m\left(A_k\right) = \sum_{n=0}^{N-1} a[n]\sin\left(\frac{2\pi}{N}nk\right) \qquad (3)$$

$$A_k = \sqrt{\left[R_E\left(A_k\right)\right]^2 + \left[I_m\left(A_k\right)\right]^2} \qquad (4)$$

The distance between two spectral lines, known as frequency resolution Δf, is calculated from the acquisition/generation time t and is:

$$\Delta f = \frac{1}{t} = \frac{fs}{N-1} \qquad (5)$$

In Eq. (5), we denoted with f_S the sampling rate and with N the total number of samples.

To fulfill the condition of at least three spectral lines belonging to the main lobe, the signal is extended by zero-padding and thus doubles the number of points of the signal, and implicitly the number of spectral lines without increasing the maximum value of the frequency that can be identified, i.e., f_{max}. According to Nyquist, f_{max} is determined with the relation:

$$f_{\max} = \frac{fs}{2} \qquad (6)$$

By doubling the number of spectral lines using a zero-padding method, the three neighboring terms A_{k-1}, A_k and A_{k+1} will be on the main lobe, as shown in Figures 1 and 2. If the signal is long enough, the main lobe will have the configuration shown in Figure 1, that is, it will be symmetric and maximizer A_k has a neighbor on the left.

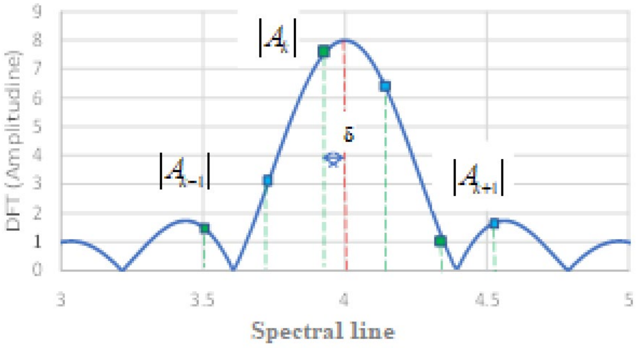

Figure 1. The DFT spectrum of a zero-padded sinusoid, for a short signal

For a very short acquisition/generation time, the spectrum will look like in Figure 2. It is observed that the main lobe is not symmetrical, and the highest value becomes A_{k-1} and will be displayed on the $k = 0$ spectral line.

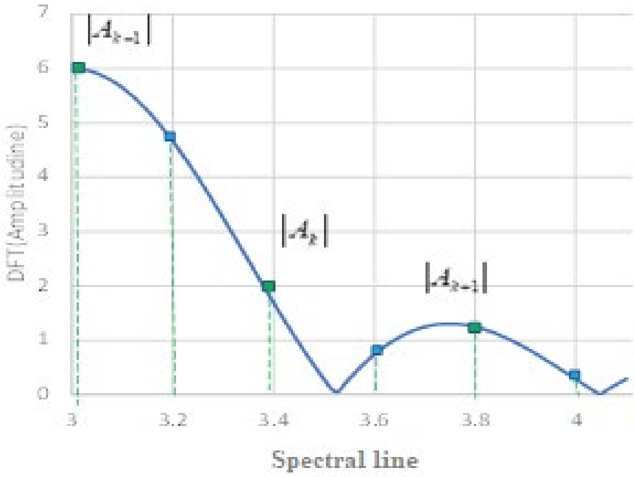

Figure 2. The DFT spectrum of a zero-padded sinusoid, for a very short signal

In both conditions, the correction coefficient δ can be calculated, as the distance between the spectral line k and the spectral interline where the real frequency f_R is located, with the relation:

$$\delta = \frac{f_R - f_{k-1}}{\Delta f} \qquad (7)$$

So, the correction coefficient is given by the ratio of the difference in frequencies to the frequency resolution for the given signal. Note that the value of δ can be positive or negative.

2.2. Calculating the database elements

To train the ANN we consider a sinusoidal signal of short length, for which we define: amplitude A=1, f_R=1 Hz, generated with f_S=1000 Hz. We start from N_i=650 samples and extend the signal by two samples until we reach N_f=1650 samples, representing 500 scenarios for input.

For the 500 signals, zero-padding is applied and amplitude values A_{k-1}, A_k and A_{k+1} are calculated, respectively the frequencies f_{k-1}, f_k and f_{k+1} are determined. The correction coefficient δ can then be calculated with relation (6). To automate the calculations, we developed the PyPad application. Figure 2 shows the sinusoidal signal obtained for N=1050 samples to which 1050 samples with zero amplitude are added.

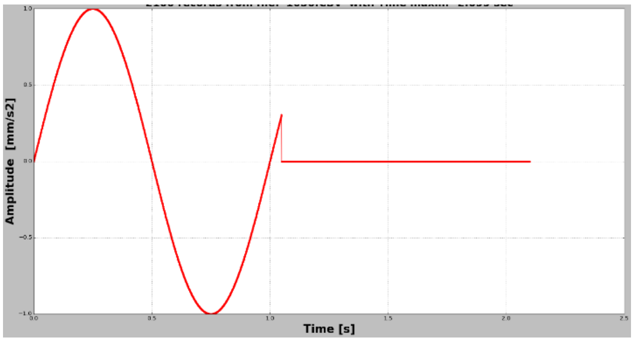

Figure 3. Sine wave after zero padding

By applying a standard DFT, the graph in Figure 4 is obtained, where one can observe the values of the amplitudes and frequencies that are of interest. The amplitude values are presented in Figure 4.

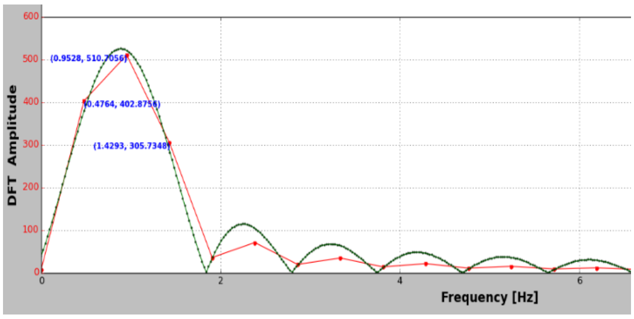

Figure 4. DFT amplitudes

After normalization, i.e. division by A_k, these become the Input data. Figure 5 graphically represents these values. The difference between the real frequency f_R and the one estimated on the spectral line k, i.e. f_k, constitutes the dataset for the target values. The values are calculated with equation (7).

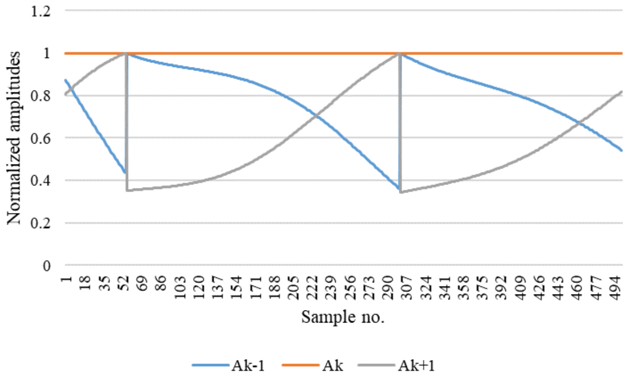

Figure 5. Normalized DFT amplitudes

3. THE TRAINING PROCESS

After calculating the training data using the described method, the normalized amplitudes for three points on the main lobe are used as input data for training a feedforward network to accurately depict the corresponding frequency, resulting in 501 training cases. The ANN is developed using the MATLAB software by considering a network architecture comprised of three input neurons, two hidden layers containing 30 neurons each and one output neuron, as illustrated in Figure 6.

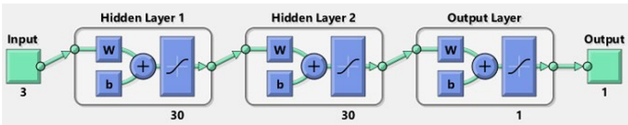

Figure 6. ANN architecture

The training parameters of the ANN are set to 0.01% learning rate at a maximum number of 1000 epochs. The data is split into input-output pairs by defining 70% of the data to be used for training, 15% for validation and 15% for testing (Figure 7).

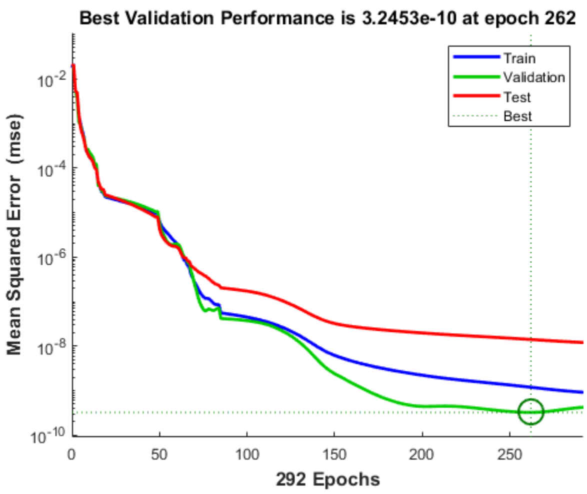

Figure 7. MSE plot for training, validation, and test

The validation performance is set to 30 validation steps, meaning that in the training phase, when the

ANN's performance reaches a considered optimum, the module will try to further improve the performance by executing the training for an extra 30 epochs. If the performance is not further improved, then the training will be considered optimum and stopped. The training evolution is asset using the Mean Squared Error shown in Figure 7, while the accuracy is illustrated by the coefficient of determination R^2.

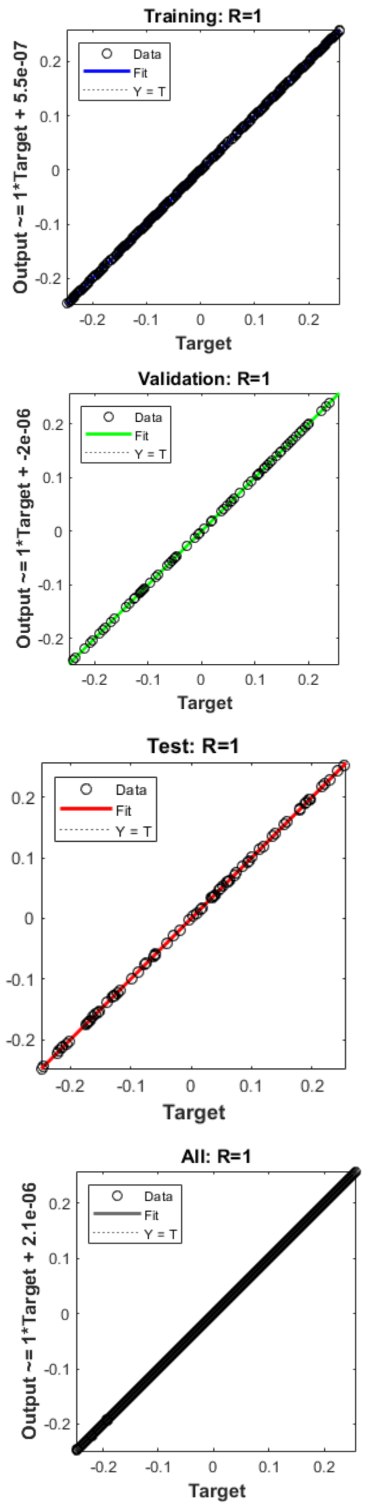

Figure 8. Regression plots for training, validation, and tests

4. TESTING THE NETWORK ON GENERATED SIGNALS

The developed method is tested using generated signal samples for a frequency of 1 Hz, by feeding them into the trained ANN to evaluate the ANN's performance. The results, referring to the predicted δ values, are compared with the input values, and the predicted Frequency, as presented in Table 1

Table 1. ANN results obtained for signal with $f = 1$ Hz

Sample N	Known δ	Predicted δ	Predicted Frequency [Hz]
654	0.153617	0.15362	1.000004
666	0.165624	0.16562	0.999994
676	0.17563	0.175619	0.999985
696	0.195641	0.195649	1.000012
710	0.209648	0.209637	0.999985
740	0.239662	0.239666	1.000006
780	-0.22064	-0.2207	0.99992
814	-0.18661	-0.1866	1.00002
862	-0.13858	-0.13859	0.999994
906	-0.09455	-0.09455	1.000003
972	-0.02851	-0.02852	0.999999
1034	0.033516	0.033516	1
1122	0.121554	0.121554	1
1218	0.217589	0.217587	0.999998
1258	-0.2426	-0.24276	0.999869
1288	-0.21258	-0.21256	1.000014
1318	-0.18257	-0.18257	0.999999
1362	-0.13855	-0.13855	1.000004
1624	0.123538	0.123538	1
1646	0.145544	0.145542	0.999999

The obtained results showcase a maximum frequency alteration of 0.000131 Hz between the real value and the predicted one which proves the high precision of the proposed approach. For instance, the algorithms presented in [3]-[5] are analyzed in [6] and much higher errors (0.26 Hz, 0.015 Hz, and 0.15 Hz) are obtained for significantly longer signals.

5. CONCLUSIONS

We propose an application to accurately estimate the frequencies of shorter signals, up to one cycle or one and a half cycles, using DFT and an ANN developed in the MATLAB software. The method uses input data to train the network, which can be determined by calculating the DFT amplitudes of a given signal, of known amplitude and frequency. The target data for the training process is represented by the correction coefficient, which is the distance between the generated frequency and the one obtained from the calculus.

With the help of these data, the amplitudes are normalized in the DFT and the correction coefficient δ is determined for the generated short signals. The method was tested on short signals of known amplitude and frequencies, being able to calculate the differences between the frequencies and calculate the correction coefficient. The results are extremely accurate, the errors being less than 0.000131 Hz for a short signal (time length t is shorter than a period T). Accurate results are obtained in both cases, when the analyzed signal has the length in the training length range or outside it.

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