Method To Determine the Elastic Constants of Polymeric Fibers Reinforced Composite Using Finite Element Vibration Analysis

Maria Luminița SCUTARU
Department of Mechanical Engineering, Transilvania University of Brașov, B-dul Eroilor,29, Romania, lscutaru@unitbv.ro

Ovidiu VASILE *
Department of Mechanics, Faculty of Biotechnical Systems Engineering, National University of Science and Technology Politehnica Bucharest, 313 Splaiul Independentei, 060042 Bucharest, Romania, ovidiu_vasile2002@yahoo.co.uk

* Author to whom correspondence should be addressed

Abstract: - In this paper, it is proposed a systematic presentation of the known methods for determining the elastic constants of a composite material using the vibration response. For a series of bodies with a simple shape, the value of the eigenfrequencies for different boundary conditions can be calculated with the standard formula (for example for beams, plates, or cylinders). These values depend on the elastic constants of the materials through simple formulas. So, if we know the eigenfrequencies for a certain particular shape of bodies made of a certain material, the elastic constants (or some of them) can be easily determined. The most accurate methods to determine these values of eigenfrequencies are, obviously, experimental methods. However, these methods are generally expensive in terms of time and resources. As a result, especially if we are in the design phase, it is very advantageous if we can determine these values from the calculations. For this, the Finite Element Method (FEM) is used in the paper. Cases are presented in which these values can be calculated with relative ease and therefore the values of the elastic constants can be obtained with less effort.

Keywords: - elastic constants, vibration, experimental measurement, FEM, virtual experiments.

1. INTRODUCTION

The current industry can no longer be conceived without the existence of composite materials. These materials have an unprecedented use at the moment, due to numerous advantages, both in terms of mechanical properties but also due to their ease of manufacture and low price. They have proven to be, many times, a better alternative than the use of classic materials. In almost all industrial branches, the use of these materials has been imposed, also in medicine, transport, in the military, aerospace, naval, construction, etc. industries. In addition to the countless advantages, they also have a high resistance compared to a low weight, so they become extremely suitable for structural applications. Obviously, the use of composites stimulated research on their properties and behavior in operation, which led to numerous articles in specialized journals. The literature that deals with the study of composites is enormous and concerns different aspects of their behavior or specific properties. Within the great diversity of composite materials, in this work we will deal with fiber-reinforced polymers whose properties are determined by the properties of the polymer matrix and by the reinforcing materials (fibers) in permanent connection [1]. The most important thing when using a composite in engineering applications is the knowledge of its properties and behavior in operation.

Obviously, the researchers' attention was focused on determining the physical (mechanical) properties of a composite if the properties of the component phases are taken into account. We are not aware of a unitary analysis and systematization of these methods. In the present paper we want to make a critical analysis for one direction from the multitude of methods for determining the mechanical properties of a composite, namely for determining the elastic constants of a composite using the analysis of its vibrations (which depend on the mechanical properties).

The development of methods for calculating elastic constants is a continuous desire of researchers in the field, since experimental measurements, which obviously represent the most reliable method, involve high costs of time and resources. Most studies focus on the study of non-linear materials, but methods have also been developed for materials with non-linear behavior [2]. Also, for certain materials, the behavior over time must be considered (polymers are generally viscoelastic materials) [3-5]. The influence of temperature represents another factor that can co-implicate the models [6].

The proposed methods for engineering constants require, in most cases, knowledge of the field of stresses and deformations for a certain state of loading. If particular loading cases are used, as in [7], then upper and lower limits are obtained for the homogenized constant values of the composite material, results that sometimes provide an estimate in a large range, which makes them useless for design [8-11].

Micromechanical analysis is often used to determine these values [12,13]. If we refer to the particular case of polymer composites reinforced with fibers, due to the frequent use of this type of composite, they have been intensively studied [14-25]. Analytical models are also used in these analyses [26]. The FEM proves to be, in all cases, an extremely powerful tool in studies aimed at determining the mechanical properties of polymer composites with fibers (and not only) [27-28]. Other studies that use different methods to determine the engineering constants are presented in [29-34].

Micromechanical models were among the first methods used to determine these quantities, in parallel with the use of FEM [35]. The analysis of polymer composites reinforced with carbon fibers, in which there are unwanted voids, is analyzed in [36] where the main analysis tool is the FEM. The increase in the content of voids found in the material of the composite matrix leads to a sharp decrease in the properties in the longitudinal direction. The obtained results are confirmed by the experimental measurements.

The research is also extended in other directions of use, such as in electro-technics. A micromechanical model for the study of smart composites reinforced with piezoelectric constituents is presented in [37] for a situation where electrical and magnetic effects appear. As a check, if these effects are missing, the results are similar to those obtained with classical methods. Other results obtained using the micromechanical model and FEM are obtained in [38-43].

In the frequently used methods (micromechanical models, homogenization theory, variational models) it is necessary to determine the stress and deformation field in order to obtain the elastic constants. This is a difficult operation and sometimes special load cases are used to avoid this challenge. However, the use of particular cases allows the determination of lower and upper limits for the elastic constants, something that can lead to obtaining useless results in practice (the range in which the values of the elastic constants can be found is too large). It follows that, in general, it is necessary to determine the field of stresses and deformations in more general cases in order to obtain more precise results.

To achieve this objective, the FEM represents the main analysis tool. Some results are reviewed to illustrate the field. A paper that analyzed a wider class of materials, including cement [44], uses FEM to obtain the stress and strain field using homogenization theory. The obtained values agree with those obtained with other calculation methods and are verified experimentally.

The determination of viscoelastic constants is done in [45]. The experimental measurements demonstrate the accuracy of the obtained results. An original method for determining the properties of a fiber-reinforced composite (carbon and glass) is developed in [46]. Other research in the field are presented in [47-48]. Numerous works propose the determination of the values of the elastic constants, using experimental methods [49-52].

The estimation methods presented in this short review are based on static models of the material. They establish relationships with the help of which it is possible to determine the mechanical properties of a material. For this, it is necessary to know the topology, geometry, and properties of the component phases. Most of the presented methods lead to the need to know the stress and deformation field considering certain loading states of the material. Obviously, the calculation of stresses and deformations is done, in most cases, with numerical methods, which are approximate methods. Errors are thus introduced, but at the moment these can be considered acceptable at the level of the existing calculation technique.

Elastic constants appear not only in the static description of a material subjected to certain forces and boundary conditions. If a dynamic analysis is done, of course, the results obtained will depend on the elastic constants of the studied material. Thus, a new method is envisioned to determine these quantities, using the classical model of simple bodies, for which there are formulas for determining the eigenfrequencies.

By then doing a FEM analysis of the respective body, introducing the properties of the component phases, the eigenfrequencies of the studied body can be determined, considering the exact distribution of the composite phases. By then comparing the two sets of data, theoretical and numerical, the engineering coefficients can be determined. This type of approach will be presented in the work. A theoretical basis for this type of estimation is also provided.

Of course, experimental methods remain the most reliable methods for determining elastic constants. Their disadvantage remains the high cost of the equipment and the time required to perform the experiments. For this reason, the method proposed in the work can be used in the design phase of a material, when it is necessary to obtain quick estimates of the properties of the resulting material by composing several phases. Then, after the design phase is finished, the prototype can be executed and the necessary mechanical tests can be performed. In this way, the availabilities offered by the numerical calculation methods can be used, but the results can be thoroughly verified using the experimental methods. The developed experimental methods are numerous and well documented [19,22,39,40,43].

2. THEORETICAL METHODS TO DETERMINE THE EIGENFREQUENCIES OF A SIMPLE BODY

In principle, any theoretical calculation method that offers us analytical expressions that contain the values of the elastic constants can be useful for the approach within the work. By determining the eigenfrequencies for different particular cases in which there are simple structural elements (beams, plane plates, cylinders, etc.) and comparing them with the analytical expressions containing the elastic constants, it is possible to determine their values. For composite materials, the problem is particularly interesting because it gives us the homogenized values of these elastic constants. Numerous studies determine the eigenfrequencies for different relatively simpler situations in the form of analytical expressions. These expressions have incorporated in them both the geometric characteristics of the studied element and the engineering constants that describe the mechanical properties. To begin with, a straight beam will be analyzed that is not loaded with any force along its length and for which the calculation relationships are well and definitively established. The transverse vibrations of such a bar, the torsional vibrations and the longitudinal vibrations will be presented. Then some cases of flat plates will be presented. Hollow cylinders will also be analyzed. The relations obtained for the calculation of the eigenfrequencies of these elements make it possible, by comparing these values with the eigenfrequencies obtained by other methods, to determine the elastic constants of the homogenized material. This method was often used together with the experimental methods for determining the eigenfrequencies. This method is the most indicated because the values obtained experimentally if the measurements are carried out carefully, will provide the most credible values. The disadvantage of this method is the fact that the experimental procedures involve high costs of time and resources. For this reason, the use of faster and less expensive methods is desirable. In this sense, the most indicated method is the FEM, which offers a virtual version of the experimental procedures. With this method, by inspecting the representation of the eigenmodes, it is possible to identify the classical and well-studied modes due to transverse, torsional, and axial vibrations. In this way, if the relations known from the classical analysis of straight beams or flat plates are used, some of the elastic constants of the homogenized material can be calculated relatively simply.

2.1.Beams

Using the classical model of the beam it is possible to obtain the eigenfrequencies for the beam, made by a homogenized material. We generally have three important modes of vibrations: bending, torsional and axial vibration.

a) Bending vibration. Considering the bending vibrations of the beam, they are described by the differential equation:

$$\frac{\partial^4 v}{\partial x^4}+\frac{\rho A}{E_z I_z}\frac{\partial^2 v}{\partial t^2}=0 \quad . \tag{1}$$

we denote:

$$k_s = D = \frac{E_z I_z}{A} \ ; \ p_o = \frac{1}{l^2}\sqrt{\frac{k_s}{\rho}} = \frac{1}{l^2}\sqrt{\frac{D}{\rho}} \quad . \tag{2}$$

For Eq. (2) is considered a solution of the form:

$$v(x,t) = Y(x)\sin(pt+\theta) \quad . \tag{3}$$

The differential equation (1) becomes:

$$\frac{d^4 Y}{dx^4} - p^2\frac{\rho A}{E_z I_z}Y = 0 \tag{4}$$

or:

$$\frac{d^4 Y}{dx^4} - \frac{p^2}{l^4 p_0^2}Y = 0 \tag{5}$$

or else:

$$\frac{d^4 Y}{dx^4} - \alpha^4 Y = 0 \quad ; \quad \alpha^4 = \frac{p^2}{l^4 p_0^2} \quad . \tag{6}$$

The circular eigenfrequency is:

$$p_i = \alpha_i^2\sqrt{\frac{E_z I_z}{\rho A}} \quad ; \quad i = 1,2,3,..... \quad , \tag{7}$$

and the solution:

$$Y_i = C_{1i}\sin\alpha_i x + C_{2i}\cos\alpha_i x + C_{3i} sh\alpha_i x + C_{4i} ch\alpha_i x \tag{8}$$

The boundary conditions express the liaisons of the beam with the background. In engineering practice, there are five cases more often met. For every of these cases, it is possible to obtain the constants appearing in Eq.(8), writing the boundary conditions.

Simply Supported Beam

The end conditions become:

- at $x=0$; $Y_i\big|_{x=0}=0$; $\left.\dfrac{d^2Y_i}{dx^2}\right|_{x=0}=0$

- at $x=l$; $Y_i\big|_{x=l}=0$; $\left.\dfrac{d^2Y_i}{dx^2}\right|_{x=l}=0$

It results:

$$C_{1i}\sin\alpha_i l=0;\quad C_{2i}=0;\quad C_{3i}=0;\quad C_{4i}=0 \quad , \tag{9}$$

and:

$$\alpha_i l=i\pi \quad , \tag{10}$$

so:

$$\alpha_i=\frac{i\pi}{l} \quad . \tag{11}$$

Considering (7) it results:

$$p_i=\frac{i^2\pi^2}{l^2}\sqrt{\frac{E_zI_z}{\rho A}} \quad ; \quad i=1,2,3,..... \tag{12}$$

and the ratio:

$$\frac{p_i}{i^2}=\frac{\pi^2}{l^2}\sqrt{\frac{E_zI_z}{\rho A}}=ct \quad ; \quad i=1,2,3,..... \tag{13}$$

or:

$$p_1=\frac{p_2}{4}=\frac{p_3}{9}=\frac{p_3}{16}.... \tag{14}$$

The transversal Young's modulus can be computed with the relation:

$$E_z=\frac{p^2l^4\rho A}{\pi^4 i^4 I_z} \quad ; \quad i=1,2,3,..... \tag{15}$$

Clamped beam at both ends

The end conditions become:

$$\text{-at } x=0 \quad ; \quad Y_i\big|_{x=0}=0 \quad ; \quad \left.\frac{dY_i}{dx}\right|_{x=0}=0 \tag{16}$$

$$\text{-at } x=l \quad ; \quad Y_i\big|_{x=l}=0 \quad ; \quad \left.\frac{dY_i}{dx}\right|_{x=l}=0 \tag{17}$$

It results the system:

$$\begin{aligned} &0=C_{2i}+C_{4i}\\ &0=C_{1i}\sin\alpha_i l+C_{2i}\cos\alpha_i l+C_{3i}sh\alpha_i l+C_{4i}ch\alpha_i l\\ &0=C_{1i}+C_{3i}\\ &0=C_{1i}\cos\alpha_i l-C_{2i}\sin\alpha_i l+C_{3i}ch\alpha_i l+_i C_{4i}sh\alpha_i l \end{aligned} \tag{18}$$

from where:

$$C_{3i}=-C_{1i}=0 \quad ; \quad C_{4i}=-C_{2i} \quad . \tag{19}$$

The system becomes:

$$\begin{aligned} &0=C_{1i}(\sin\alpha_i l-sh\alpha_i l)+C_{2i}(\cos\alpha_i l-ch\alpha_i l)\\ &0=C_{1i}(\cos\alpha_i l-ch\alpha_i l)-C_{2i}(\sin\alpha_i l+sh\alpha_i l) \quad . \end{aligned} \tag{20}$$

The condition that the determinant of this system is zero, offers finally:

$$\cosh\alpha_i l\cos\alpha_i l=1 \tag{21}$$

and α_i are the solutions of the transcendental equation:

$$\cosh\alpha l\cos\alpha l=1 \quad . \tag{22}$$

The first ten solutions $\beta=\alpha l$ of the transcendent equation $\cosh\beta\cos\beta=1$ are: 4.730040; 7.853204; 10.995607; 14.137165; 17.278759; 20.420352; 23.561944; 26.703537; 29.845130; 32.986722. The constant α_i will be:

$$\alpha_i=\frac{\beta_i}{l} \quad . \tag{23}$$

Knowing α_i, it is possible to determine the transversal Young's modulus with the relation:

$$E_z=\frac{p_i^2\,\rho A}{\alpha_i^4 I_z}=\frac{4\pi^2\nu_i^2\,l^4\rho A}{\beta_i^4 I_z} \quad ; \quad i=1,2,3,..... \quad . \tag{24}$$

It results:

$$\frac{p_i}{\beta_i^2}=\sqrt{\frac{E_zI_z}{\rho A l^4}}=ct \quad ; \quad i=1,2,3,..... \quad . \tag{25}$$

The beam clamped at one end and simply supported at the other

The end conditions become:

$$\text{-at } x=0 \quad ; \quad Y_i\big|_{x=0}=0 \quad ; \quad \left.\frac{dY_i}{dx}\right|_{x=0}=0 \tag{26}$$

$$\text{-at } x=l \quad ; \quad Y_i\big|_{x=l}=0 \quad ; \quad \left.\frac{d^2Y_i}{dx^2}\right|_{x=l}=0 \tag{27}$$

To obtain a non-null solution for the coefficients it must have:

$$\tan\alpha_i l=\tanh\alpha_i l \quad . \tag{28}$$

and using the notation: $\beta_i=\alpha_i l$ remains to solve the equation:

$$\tan\beta_i=\tanh\beta_i \quad . \tag{29}$$

The first eight solutions of this equation are: 3.927; 7.069; 10.210; 13.352; 16.493; 19.635; 22.776; 25.918. Eq. (24) remains valid for all the cases.

Beam clamped at one end and free at the other

The end conditions are:

$$\text{-at } x=0 \quad ; \quad Y_i\big|_{x=0}=0 \quad ; \quad \left.\frac{dY_i}{dx}\right|_{x=0}=0 \tag{30}$$

-at $x = l$; $\left.\frac{d^2Y_i}{dx^2}\right|_{x=l} = 0$; $\left.\frac{d^3Y_i}{dx^3}\right|_{x=l} = 0$ (31)

To obtain a non-null solution for the coefficients it must have:

$$\cosh\alpha_i l \cos\alpha_i l = -1 \quad (32)$$

$\beta_i = \alpha_i l$ the first eight values of this parameter are: 1.875; 4.694; 7.855; 10.996; 14.137; 17.279; 20.420; 23.562 and using Eq. (24) is possible to obtain the Young's modulus.

Beam with two free ends

Now, the end conditions become:

-at $x = 0$; $\left.\frac{d^2Y_i}{dx^2}\right|_{x=0} = 0$; $\left.\frac{d^3Y_i}{dx^3}\right|_{x=0} = 0$ (33)

-at $x = l$; $\left.\frac{d^2Y_i}{dx^2}\right|_{x=l} = 0$; $\left.\frac{d^3Y_i}{dx^3}\right|_{x=l} = 0$ (34)

and as in the previous case when the beam is clamped at both ends it results the conditions (21).

b) Torsional vibration. If torsional vibrations are studied, then the shear modulus can be easily determined. At the distance x from the left end of the beam the dynamic equilibrium equation offers [53-56]:

$$GI_p \frac{\partial^2\phi}{\partial x^2} = J\frac{\partial^2\phi}{\partial t^2} \quad . \quad (35)$$

where we have:
G - shear modulus;
I_p - the inertia moment of the area;
J - the unitary mass moment of inertia;
φ - the rotation angle of the current area.

For a homogeneous, continuous beam there exists the relation $J = \rho I_p$, so the Eq. (1) becomes:

$$GI_p \frac{\partial^2\phi}{\partial x^2} = J\frac{\partial^2\phi}{\partial t^2} \quad . \quad (36)$$

The initial conditions at moment $t = 0$ for the differential equation are, if the beam is clamped at both ends:

$$\varphi(x,0) = f(x) \quad ; \quad \left.\frac{\partial\varphi}{\partial t}\right|_{t=0} = g(x) \quad ; \quad (37)$$

$$\varphi(0,t) = 0 \quad ; \quad \varphi(l,t) = 0 \quad ; \quad (38)$$

Choosing for Eq. (2) a solution of the form:

$$\varphi(x,t) = \Phi(x)\sin(pt+\theta) \quad . \quad (39)$$

Eq. (36) becomes:

$$\frac{d^2\Phi}{dx^2} + p^2\frac{\rho}{G}\Phi = 0 \quad . \quad (40)$$

The function $\Phi(x)$ represents the function amplitude *t*. The solution is:

$$\Phi(x) = C_1\sin\alpha x + C_2\cos\alpha x \quad . \quad (41)$$

where:

$$\alpha^2 = p^2\frac{\rho}{G} \quad . \quad (42)$$

The boundary conditions (4) are:

$$\Phi(0) = 0 \quad ; \quad \Phi(l) = 0 \quad , \quad (43)$$

and offer:

$$C_2 = 0 \quad ; \quad C_1\sin\alpha l = 0 \quad , \quad (44)$$

The solutions are:

$$\alpha_i = \frac{i\pi}{l} \quad ; \quad i = 1,2,3,..... \quad , \quad (45)$$

The circular eigenfrequencies of the vibration are given by the relation:

$$p_i = \frac{i\pi}{l}\sqrt{\frac{G}{\rho}} \quad ; \quad i = 1,2,3,..... \quad . \quad (46)$$

The shear modulus can be computed with the relation:

$$G = \frac{p_n^2\, l^2\rho}{n^2\pi^2} \quad ; \quad n = 1,2,3,..... \quad . \quad (47)$$

From Eq.(46), it results:

$$p_1 = \frac{p_2}{2} = \frac{p_3}{3} = ... = \frac{p_n}{n} \quad , \quad (48)$$

Eq. (48) is useful to verify how good are the results. If we obtain more results for G, an average can be made.

c) Longitudinal vibration. The analysis of longitudinal vibrations of a beam allows the Young's modulus to be easily determined. For this, only longitudinal vibrations will be analyzed. They are described by the differential equation is:

$$\frac{\partial^2 u}{\partial x^2} = \frac{\rho}{E}\frac{\partial^2 u}{\partial t^2} \quad . \quad (49)$$

In this equation u represents the axial displacement of the bar at distance x, ρ is the density and E is the axial Young's modulus. In the case of longitudinal vibrations of a bar, the form of the obtained equation is like the one encountered in torsional vibrations. Consequently, the same method of solving this equation can be applied. It obtains:

$$\frac{d^2\Phi}{dx^2}+p^2\frac{\rho}{E}\Phi=0 \quad . \tag{50}$$

The initial conditions can be written as:

$$u(x,0)=f(x) \quad ; \qquad \left.\frac{\partial u}{\partial t}\right|_{t=0}=g(x) \tag{51}$$

If the beam is clamped at both ends the boundary conditions are:

$$u(0,t)=0 \quad ; \qquad u(l,t)=0 \tag{52}$$

and (as in the case of torsional vibrations) the circular eigenfrequencies are given by the relation:

$$p_i=\frac{i\pi}{l}\sqrt{\frac{E}{\rho}}=\alpha_i\sqrt{\frac{E}{\rho}} \quad ; \quad i=1,2,3,..... \quad . \tag{53}$$

The relation that gives us the Young's modulus of the homogenized material is:

$$E=\frac{p_n^2\, l^2\rho}{i^2\pi^2} \quad ; \quad i=1,2,3,..... \quad . \tag{54}$$

The relation (48) remains valid.

2.2. Rectangular orthotropic plates

The displacement on a direction perpendicular on the plate is:

$$v(x,y)=C_{mn}\sin m\pi\frac{x}{a}\sin n\pi\frac{y}{b} \tag{55}$$

where a and b are the dimensions of the rectangular plate. The circular eigenfrequencies corresponding to this form of vibration are, for an isotropic plate:

$$\omega_{mn}=\frac{\pi^2}{a^2}\sqrt{\frac{D}{\rho}}\left[m^4+2m^2n^2R^2+n^4R^4\right] \tag{56}$$

where R=a/b. In this way, the eigenfrequencies can be related to the elastic properties of the plate material [57,58]. In the literature, there are numerous works that link the eigenfrequencies of bodies of a particular shape (beams, plates, cylinders) to the elastic properties of the material from which these bodies are made, thus allowing the described method to be applied. Other results in the domain can be found in [57-65].

3. FINITE ELEMENT METHOD USED TO DETERMINE THE EIGENFREQUENCIES

It can be considered that the FEM has reached a high degree of maturity and provides very good results in many areas such as strength of materials, vibration, material analysis, calculus of structure, etc. In the paper, this method will be used to determine the stress and strain field in a material specimen composed of several phases if the mechanical properties of these phases are known, as well as how they are arranged within the composite. The eigenfrequencies and eigenmodes of vibration of the analyzed material specimen will also be determined. Knowing what the eigenfrequencies of the specimen considered to be made of a homogenized material, it will be possible to determine the elastic constants of the material by a simple comparison. Only an example application of this method will be presented to not load unnecessary the presentation [56].

So, if we have determined the eigenvalue with FEM, it is possible to obtain now the shear modulus with Eq. (47):

$$G=\frac{4\nu_i^2\, l^2\rho}{i^2} \quad ; \quad i=1,2,3,..... \quad . \tag{57}$$

With FEM, more eigenfrequency values can be obtained. It results in more shear modulus values will be obtained. To obtain a more precise value, it is necessary to average these values.

In the calculation performed with FEM, the pure torsion modes can be easily identified. These modes will be analyzed to determine the shear modulus. Thus Fig. 1 represented the FEM model of the considered beam. Take a sample of the analyzed material, small in size, to describe as well as possible the composite material used. It is made up of four cylindrical carbon fibers, parallel, which are incorporated into a polymeric epoxy matrix. Young's modulus for the carbon fiber used is 86.960 GPa. For the matrix incorporating carbon fibers, Young's modulus is 4.140 GPa. The density of the materials is 1850 kg/m^3 for matrix and 2000 kg/m^3 for carbon fiber respectively. Poisson's ratio is 0.22 and 0.34 for matrix and carbon, respectively. In Fig. 2 shows the torsional vibration modes of the bar.

So, knowing the eigenfrequency, the eigenmode corresponding to this frequency, the length and the density of the homogenized material, it is possible to obtain the shear modulus.

If the density of the fiber is ρ_f, the density of the matrix is ρ_m, the ratio of the fiber is ν_f and the ratio of the matrix is $v_m = 1 - v_f$, the density of the homogenized material is:

$$\rho = v_f \rho_f + v_m \rho_m \tag{58}$$

By calculation, the eigenfrequencies of the beam fixed at the two ends were determined and the first three values were obtained: 82,288 Hz;164,757 Hz; 247,584 Hz. Using these values, it is possible to obtain the shear modulus: 5.212 GPa; 5.224 GPa; 5.243 GPa having an average value G=5.227 GPa.

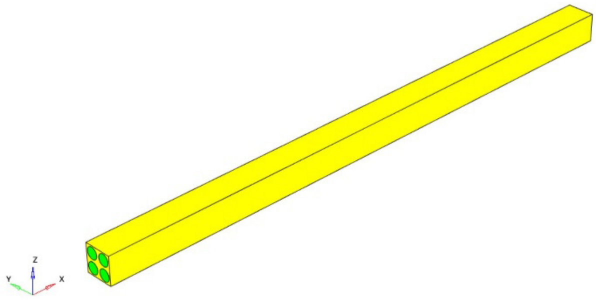

Figure 1. The model of the studied beam

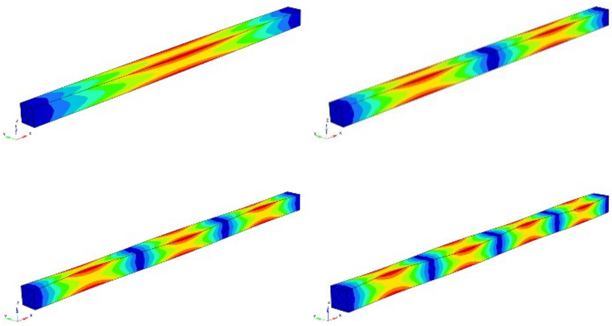

Figure 2. The first four eigenmodes of torsional vibration

4. CONCLUSIONS

The homogenized elastic constants that define the behavior of a polymeric composite material, reinforced with cylindrical fibers can be calculated using different analytical formulas, obtained by various methods. These methods are very laborious and assume, in most situations, the field of stresses and deformations for different loading situations of the composite. Experimental methods obviously represent the most suitable solution for this type of problem, but they require the preparation of the experiment and therefore significant time and resources from the project phase.

In the present paper we make a review of the methods that allow us to obtain the elastic constants of a composite material using the classical theories of homogeneous body to determine the eigenfrequencies. Then FEM is used to determine these values for a specimen of the material, considering the exact composition, phases, and dimension of the constituent phases. Comparing the obtained FEM values with the values used in classical theory, it is possible to obtain some of the homogenized material constants. This method presents the major advantage of simplicity and the possibility of quickly obtaining sufficiently accurate estimates in a process of designing a system that has composite materials in its composition. If several FEM determinations are considered for the same engineering constant, by averaging the results the obtained precision can be improved. Obviously, the presented method is accurate if we are within the assumptions considered for the calculations made. In this case the errors will be due to the inherent errors that appear in the use of FEM, but this method is currently accurate enough to satisfy engineering needs. We can talk about a virtual experiment, using FEM, to determine the eigenfrequencies.

The need to have sufficiently precise estimates of the mechanical properties of the composite materials used in a project has been determined, since the beginning of the use of composites, numerous studies and research to solve this problem.

Polymeric composites reinforced with cylindrical fibers are a type of composites often used in engineering practice and researchers have been focusing on this type of materials for a long time. Multiple methods have been developed and analyzed for this. The main models developed since the beginning of the use of composites, namely the micromechanical models and the theory of homogenization, imply in each case, the determination of the field of stresses and strains for a certain load of the analyzed body. The scientists tried to avoid this difficult calculation by considering some particular loading cases, but in this case, upper and lower limits are obtained for some elastic constants, which can be, in certain circumstances, a rather imprecise result. The paper aims to analyze the problem considering the dynamic behavior of the materials, linking the eigenfrequencies obtained with FEM to the elastic constants of the materials, which appear in formulas obtained on classic beam or plate models.

So, it was possible to determine some important elastic constants used in engineering as axial tensile modulus, transverse tensile modulus, axial shear modulus, transverse shear modulus, axial Poisson's ratio, transverse Poisson's ratio, or bulk modulus (the method applied depend on the obtained composite material: homogeneous and isotropic, transversely isotropic, orthotropic).

In engineering practice, other situations of increased complexity can be encountered, involving thermal effects, humidity or other factors that cannot be neglected. For these cases, it is necessary to look for existing results or to develop classic models that can also take these effects into account when determining the eigenfrequencies of the studied bodies. The additional parameters that describe the newly appeared situations must be identified and used in the modeling. Each time this happens, the appropriate model must be considered, and the necessary calculations must be performed.

In current practice to determine Young's modulus, experimental measurements are used. Then comparing these values with calculated values considering different theories of the beam, it is possible to determine the elastic constants of the material. These could be Young's modulus, shear modulus or Poisson’s ratio. Within the paper this basic idea of the method stated above was taken over but instead of making the experimental determination of the eigenfrequencies of a material specimen a quick calculation is made with FEM. In this way it is possible to quickly determine the elastic constants of the materials. This method presents the advantage of being able to be applied quickly and involving low costs compared to the experimental methods. Being a numerical solution method, the use of FEM depends on a number of factors such as the number of elements, the fineness of the discretization, the appropriate geometric description, the type of finite element chosen, etc. In this work, the authors did not investigate the influence of these factors, but wanted the possibility of applying this method successfully in the case of this type of problem. The influence of each of the previously mentioned factors can be studied in further research.

In the paper, the authors tried to present different methods to determine the eigenfrequencies of a sample of material manufactured in a simple form, depending on the elastic constants of the materials considered as homogenized materials and depending on their geometry. This presentation has sought to illustrate the most common and simple methods to achieve this. With the help of the provided relationships, it is possible to determine the values of the elastic constants of some fiber-reinforced polymer composite materials in the design phase. Obviously, there can be more complicated relationships and more complex theories for solving problems with more parameters, and literature offers us numerous such examples. The method presented in the paper can be used successfully in these situations as well. The proposed method allows a quick determination, with relatively low costs and in a short time, of the elastic constants of a composite material, considered a homogenized material made up of a single material, homogeneous on a macroscopic scale.

REFERENCES

[1] Katouzian M., On the Effect of Tempeature on Creep Behavior of Neat and Carbon Fiber Reinforced PEEK and Epoxy- A micromechanical Approach. Ph.D. Thesis, University of Muenchen, Germany, 1995.

[2] Findley W.N., Khosla G., Application of the Superposition Principle and Theories of Mechanical Equation of State, Strain, and Time Hardening to Creep of Plastics under Changing Loads, *J. Appl. Phys.*, 1955, 26, 821. https://doi.org/10.1063/ 1.1722102

[3] Hashin Z., On Elastic Behavior of Fibre Reinforced Materials of Arbitrary Transverse Phase Geometry, *J. Mech. Phys. Solids*, 1965, 13, pp. 119–134.

[4] Hashin Z., Shtrikman S., A Variational Approach to the Theory of the Elastic Behavior of Multiphase Materials, *J. Mech. Phyds. Solids*, 1963, 11, pp. 127–140.

[5] Hashin Z., Rosen B.W., The Elastic Moduli of Fiber-Reinforced Materials, *J. Appl. Mech.*, 1964, 31, pp. 223–232, https://doi.org/10.1115/1.3629590.

[6] Bowles D.E., Griffin O.H., Jr. Micromechanics Analysis of Space Simulated Thermal Stresses in Composites. Part I: Theory and Unidirectional Laminates, *J. Reinf. Plast. Compos.*, 1991, 10, pp. 504–521

[7] Zhao Y.H., Weng G.J., Effective Elastic Moduli of Ribbon-Reinforced Composites, *J. Appl. Mech.*, 1990, *57*, pp. 158–167. https://doi.org/10.1115/1.2888297.

[8] Hill R., Theory of Mechanical Properties of Fiber-strengthened Materials: I Elastic Behavior, *J. Mech. Phys. Solids*, 1964, *12*, pp. 199–212.

[9] Hill R., Theory of Mechanical Properties of Fiber-strengthened Materials: II Inelastic Behavior, *J. Mech. Phys. Solids*, 1964, *12*, pp. 213–218.

[10] Hill R., Theory of Mechanical Properties of Fiber-strengthened Materials: III Self-Consistent Model, *J. Mech. Phys. Solids*, 1965, *13*, pp. 189–198.

[11] Hill R., Continuum Micro-Mechanics of Elastoplastic Polycrystals, *J. Mech. Phys. Solids*, 1965, *13*, pp. 89–101.

[12] Aboudi, J. Micromechanical characterization of the non-linear viscoelastic behavior of resin matrix composites. *Compos. Sci. Technol.* 1990, *38*, 371–386. https://doi.org/10.1016/0266-3538(90)90022-w.

[13] Aboudi J., *Mechanics of Composite Materials—A Unified Micromechanical Approach*; Elsevier: Amsterdam, The Netherlands, 1991.

[14] Fattahi A.M., Mondali M., Theoretical study of stress transfer in platelet reinforced composites, *J. Theor. Appl. Mech.*, 2014, *52*, 3–14.

[15] Fliegener S., Hohe, J. An anisotropic creep model for continuously and discontinuously fiber reinforced thermoplastics. *Compos. Sci. Technol.*, 2020, *194*, 108168. https://doi.org/10.1016/j.compscitech.2020.108168.

[16] Vlase S., Negrean I., Marin. M., Nastac S., Kane's Method-Based Simulation and Modeling Robots with Elastic Elements, Using Finite Element Method, *MATHEMATICS*, 2020, 8 (5).

[17] Xu B., Xu W., Guo F., Creep behavior due to interface diffusion in unidirectional fiber-reinforced metal matrix composites under general loading conditions: A micromechanics analysis, *Acta Mech.*, 2020, *231*, pp. 1321–1335. https://doi.org/10.1007/s00707-019-02592-8.

[18] Vlase S., Teodorescu P.P., Itu C., Scutaru M.L., Elasto-Dynamics of a Solid with a General "Rigid" Motion using FEM Model, *Romanian Journal of Physics*, 2013, 58 (7-8), pp. 882-892.

[19] Lal H.M.M., Xian G.-J., Thomas S., Zhang L., Zhang Z., Wang H., Experimental Study on the Flexural Creep Behaviors of Pultruded Unidirectional Carbon/Glass Fiber-Reinforced Hybrid Bars, *Materials*, 2020, *13*, 976. https://doi.org/10.3390/ma13040976.
[20] Wang Z., Smith D.E., Numerical analysis on viscoelastic creep responses of aligned short fiber reinforced composites, *Compos. Struct.*, 2019, *229*, 111394. https://doi.org/10.1016/j.compstruct.2019.111394
[21] Gheorghe V., Scutaru M.L., Chircan E., Ungureanu B., Ulea M., New Design of Composite Structures Used in Automotive Engineering, *Symmetry-Basel*, 2021, 13 (3).
[22] Tebeta R.T., Fattahi A.M., Ahmed N.A., Experimental and numerical study on HDPE/SWCNT nanocomposite elastic properties considering the processing techniques effect, *Microsyst. Technol.*, 2021, *26*, pp. 2423–2441.
[23] Vlase S., Negrean I., Marin M., Scutaru M.L., Energy of Accelerations Used to Obtain the Motion Equations of a Three-Dimensional Finite Element, *Symmetry-Basel*, 2020, 12 (2).
[24] Selmi A., Friebel C., Doghri I., Hassis H., Prediction of the elastic properties of single walled carbon nanotube reinforced polymers: A comparative study of several micromechanical models, *Compos. Sci. Technol.*, 2007, *67*, pp. 2071–2084.
[25] Stanciu A., Teodorescu-Drăghicescu H., Vlase S. et al, Mechanical behavior of CSM450 and RT800 laminates subjected to four-point bend tests, *Optoelectron. Adv. Mater. Rapid Commun*., 2012, 6, pp. 495–497.
[26] Tran A.B., Yvonnet J., He Q.C., Toulemonde, C., Sanahuja J., A simple computational homogenization method for structures made of linear heterogeneous viscoelastic materials, *Computer Methods in Applied Mechanics and Engineering*, 2011, Vol. 200, Issue 45-46, pp. 2956-2970.
[27] Katouzian M., Vlase S., Scutaru M.L., Finite Element Method-Based Simulation Creep Behavior of Viscoelastic Carbon-Fiber Composite, *Polymers*, 2021, *13*, 1017. https://doi.org/10.3390/polym13071017
[28] Niculiță C., Vlase S., Bencze A. et al, Optimum stacking in a multi-ply laminate used for the skin of adaptive wings, *Optoelectron. Adv. Mater. Rapid Commun*., 2011, 5, pp. 1233–1236.
[29] Fung Y.C., *Fundamentals of Solid Mechanics*, Prentice-Hall, 1965
[30] Schapery R.A., Stress Analysis of Viscoelastic Composite Materials, *Journal of Composite Materials*, 1967, Vol.1, p.228.
[31] Morris D.H., Yeow Y.T., Brinson H.F., The Viscoelastic Behavior of the Principal Compliance Matrix of Unidirectional Graphite Epoxy Composite, VPI&SU, Blacksburg, VA, VPI-E-79-9, February, 1979.
[32] Huang Z.-M., Constitutive relation, deformation, failure and strength of composites reinforced with continuous/short fibers or particles, *Composite Structures*, 2021, Vol. 262, 113279.
[33] Huang Z.M., A micromechanics approach to stiffness and strength of unidirectional composites, *J. Reinf. Plastics Comp.*, 2019, 38, pp.167-196.
[34] Hinton M.J., Kaddour A.S., Soden P.D., The world-wide failure exercise: Its origin, concept and content, *The World-Wide Failure Exercise*, 2004, pp. 2-28.
[35] Ahmadi M., Ansari R., Hassanzadeh-Aghdam M.K., Micromechanical finite element analysis of Young's modulus, yield strength and thermal expansion coefficient of nano-sized ceramic particle/metal matrix nanocomposites, *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, 2023, 45 (9).
[36] Oz F.E., Computational examination of the effect of voids on the mechanical response of composites with emphasize on the cure hardening behavior, *Mechanics of Advanced Materials and Structures*, 2023, Early acces.
[37] Christofi I., Hadjiloizi D.A., Georgiades A.V., Dynamic micromechanical model for smart composite and reinforced shells, *ZAMM-ZEITSCHRIFT FUR ANGEWANDTE MATHEMATIK UND MECHANIK*, 2022, 102 (5).
[38] Gupta M., Ray M.C., Kundalwal S.I., Dynamic modelling and analysis of smart carbon nanotube-based hybrid composite beams Analytical and finite element study, *Proceedings of the Institution of Mechanical Engineers. Part L-Journal of Materials-Design and Applications*, 2021, 235 (10), pp. 2185-2206
[39] Qin F.P., Lu F.C., Huang L., Numerical simulation and experimental validation of ratchetting deformation of short fiber-reinforced polymer composites, *Composites, Part B-Engineering*, 2023, 266.
[40] Manchiraju V.N.M., Bhagat A.R., Dwivedi V.K., Estimation of Elastic Constants Using Numerical Methods and Their Validation Through Experimental Results for Unidirectional Carbon/Carbon Composite Materials, *Japan-Journal of Metrology Society of India*, 2023, 38 (4), pp. 923-937.
[41] de Morais A.B., Transverse moduli of continuous-fibre-reinforced polymers, *Composite Science and Technology*, 2000, 60 (7), pp. 997-1002.
[42] Fuchs M.B., Paley M., Miroshny E., Evaluation of failure criteria for fiber composites using finite element micromechanics, *Journal of Composite Materials*, 1998, 32 (8), pp.766-782.
[43] Astaraki S., Zamani E., Mohamadipoor R., Determination of mechanical properties of nanocomposites reinforced with spherical silica nanoparticles using experiments, micromechanical model and finite elements method, *Journal of Composite Materials*, 2023, 57 (17), pp. 2689-2702.
[44] Li Y., Li Y.Q., Evaluation of elastic properties of fiber reinforced concrete with homogenization theory and finite element simulation, *Construction and Building Materials*, 2019, *200*, pp.301-309
[45] Nguyen A.V., Nguyen T.C., Homogenization of Viscoelastic Composite Reinforced Woven Flax Fibers, 11th Joint Canada-Japan Workshop on Composites / 1st Joint Canada-Japan-Vietnam Workshop on Composites, *DESIGN, MANUFACTURING AND APPLICATIONS OF COMPOSITES*, 2016, pp.187-192
[46] Matsuda T., Ohno N., Predicting the elastic-viscoplastic and creep behaviour of polymer matrix composites using the homogenization theory, *Creep and Fatigue in Polymer Matrix Composites*, 2011, pp. 113-148
[47] Tian W.L., Qi L.H., Jing Z., Numerical simulation on elastic properties of short-fiber-reinforced metal matrix composites: Effect of fiber orientation, *Composite Structures*, 2016, 152, pp. 408-417
[48] Zhu T.L., Wang Z., Research and application prospect of short carbon fiber reinforced ceramic composites, *Journal of the European Ceramic Society*, 2023, 43 (15), pp. 6699-6717.
[49] Aravindh S., Venkatachalam G., Investigation on Elastic Constants of Microfibril Reinforced Poly Vinyl Chloride Composites Using Impulsive Excitation of Vibration, *Polymers*, 2022, 14, 5083. https://doi.org/10.3390/polym14235083
[50] Salvetat J.P., Bonard J.M., Thomson N.H., Kulik A.J., Forró L., Benoit W., Zuppiroli L., Mechanical properties of carbon nanotubes, *Applied Physics A- Materials Science & Processing*, 1999, VL 69, IS 3, pp. 255-260, DOI 10.1007/s003390050999.

[51] Lourie O., Wagner H.D., Evaluation of Young's modulus of carbon nanotubes by micro-Raman spectroscopy, *Journal of Materials Research*, 1998, Vol. 13, Issue 9, pp. 2418-2422, DOI: 10.1557/JMR.1998.0336

[52] Rajak D.K., Pagar D.D., Menezes P.L., Linul E., Fiber-Reinforced Polymer Composites: Manufacturing, Properties, and Applications, *POLYMERS*, 2019, vol. 11, iss. 10, 1667, DOI: 10.3390/polym11101667

[53] Katouzian M. et al, Modeling Study of the Creep Behavior of Carbon-Fiber-Reinforced Composites: A Review, *Polymers*, 2023, *15*, 194. https://doi.org/10.3390/polym15010194.

[54] Rades M., *Mechanical Vibration, II, Structural Dynamics Modeling*, Printech Publishing House: Bangalore, India, 2010.

[55] Vlase S., Marin M., Ochsner, A., Considerations of the transverse vibration of a mechanical system with two identical bars, *Proceedings of the Institution of Mechanical Engineers. Partl L-Journal of Materials, Design and Applications,* 2019, 233 (7), pp.1318-1323.

[56] Itu C., Scutaru M.L., Vlase S., Elastic Constants of Polymeric Fiber Composite Estimation Using Finite Element Method, *Polymers,* 2024, *16*, 354. https://doi.org/10.3390/polym16030354.

[57] Berthelot J.M., *Dynamics of Composite Materials and Structures*, Institute of Advanced Mechanics, Le Mans, France, 2010.

[58] Berthelot J.M., *Composites Materials, Mechanical Behavior and Structural Analysis*, Springer, New-York, 1999.

[59] Bratu P., Dobrescu C., Nitu, M.C., Dynamic Response Control of Linear Viscoelastic Materials as Resonant Composite Rheological Models, *Romanian Journal of Acoustic and Vibration*, 2023, vol. 20, issue 1, pp. 73-77

[60] Scutaru M.L., Vlase S., Marin M., Symmetrical Mechanical System Properties-Based Forced Vibration Analysis, *Journal of Computational Applied Mechanics,* 2023, vol. 54, Issue 4, pp. 501- 514, DOI: 10.22059/JCAMECH.2023.366509.890

[61] Scutaru M.L., Vlase S., Marin M., Analytical mechanics methods in finite element analysis of multibody elastic system, *BOUNDARY VALUE PROBLEMS*, 2023, Vol. 2023, Issue 1, 97, DOI: 10.1186/s13661-023-01784-5

[62] Scutaru M.L., Marin M., Vlase S., Dynamic Absorption of Vibration in a Multi Degree of Freedom ElasticSystem, *Mathematics*, 2022, Vol. 10, Issue 21, 4045, DOI: 10.3390/math102140.

[63] Tufisi, C., Minda,A.A., Burtea,D.G., Gillich, G.R. Frequency Estimation using Spectral Techniques with the Support of a Deep Learning Method. Romanian Journal of Acoustics and Vibration, 19 (1), 49-55, 2022.

[64] Barboni,L., Gillich,G.R., Chioncel,C.P.,Hamat, C.O. Mituletu,I.C. A method to precise determine the Young's modulus from dynamic measurements. 7th International Conference on Advanced Materials and Structures - AMS 2018 28–31 March 2018, Romania, Vol.416, 012063, 2018, DOI 10.1088/1757-899X/416/1/012063.

[65] Gillich,G.R., Samoilescu, G., Berinde, F., Chioncel, C.P. Experimental determination of the rubber dynamic rigidity and elasticity module by time-frequency measurements. Materiale Plastice, 44, 1, 18-21, 2007