Investigation of the Propagation of Uncertainty in the Measurement of the Scattering Matrix of a Duct Element Containing a Porous Material

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Abstract: - The need for a theoretical simulation of the experiment is intended to determine which parameters of the experiment are most sensitive to error, and to take these into account when measuring and reducing errors. This paper presents a theoretical simulation of the experimental procedure of measuring the scattering matrix and the acoustic power attenuation of a duct element containing a porous material, in order to identify the parameters affecting the measurements. The Monte Carlo technique is used to evaluate the influence of errors in measurement parameters (the separation distance, the length of porous material, and the incident pressure modulus) on the measured quantities (scattering matrix coefficients and the acoustic power attenuation). This method is considered a reference for examining the propagation of uncertainties in theoretical methods. The results of the uncertainty simulation provided 95% confidence intervals and maximum and minimum errors for each parameter.

Keywords: - Porous materials, The scattering matrix, Acoustic power attenuation, Monte Carlo method.

1. INTRODUCTION

Porous materials occupy an important place in acoustics because they control and attenuate sound waves. They are used to reduce noise and to improve sound quality in the sound insulation of buildings, as well as to control vibrations of structures and machines. One of the industrial applications of porous materials is its use in duct elements to reduce the noise inside and through it. In this specific application, the duct element containing the porous element can be presented by its scattering matrix. The scattering matrix is a good tool to define the acoustic behavior of duct systems because it predicts how acoustic waves are propagated and reflected. It presents an intrinsic characteristic of the duct element. For this, many researchers focused their attention on the use of the scattering matrix in their studies of acoustic propagation in ducts. such as Kani et al. [1,2] employed the scattering matrix to characterize the acoustic behaviour of duct elements containing porous material. Subsequently, they proposed two inverse techniques: the first consists of a multi-level identification method based on the simplex optimization algorithm and the other is based on the genetic algorithm. Both methods are applied to evaluate the physical parameters of the porous material placed in a duct by calculating its scattering matrix. Masmoudi et al. [3] carried out a numerical calculation of the effect of increasing and decreasing duct diameter on the acoustic performance of a lined wall or duct, using the simple Delany-Bazely model which depends solely on the flow resistivity. Besides, Dhief et al. [4] used the same matrix to examine duct discontinuities using more complex configurations and two types of liner: a perforated plate supported by an air cavity or by a porous material. They demonstrated that the second type of liner absorbed acoustic waves better than the first. In addition, Douha et al. [5] evaluated the influence of variation of duct parameters on the acoustic power attenuation of duct discontinuities through the numerically calculated multimodal scattering matrix. This analysis was performed on the basis of six configurations in order to show the effect of the geometries of the discontinuities and the position of the liner in the duct systems. Then, Hannachi et al. [6] established a numerical simulation of an experimental procedure for computing the diffusion matrix. The simulation was compared with the transfer matrix method in order to assess the degree of agreement between experiment and theory.

Recently, Calmettes et al. [7] presented a scattering matrix formalism for acoustic power in duct networks and Gorazd et al. [8] analyzed an acoustic reflective muffler using the same matrix. This diffusion matrix is the result of previous studies based on experimental methods [9,10]. Other works show that the diffusion matrix was determined using analytical and numerical methods [9,11 - 16]. In addition, some researchers developed tools to evaluate the uncertainties and their influence in the scattering matrix measurement procedure based on works like [17,18]. In fact, Sitel et al. [9] developed an analytical simulation of the measuring experience of the scattering matrix of a duct element combined with the two-microphone method to study the sensitivity of the scattering matrix coefficients as well as the acoustic power attenuation to certain experimental parameters such as temperature, microphone location, and modal coefficients. This simulation showed that the influence of temperature is negligible except near the cut-off frequencies of the high modes. A large effect of the modal coefficients especially on the reflection coefficients was also observed. Then, Taktak et al. [19] developed a simulation of the scattering matrix measurement using the Monte Carlo technique to provide an evaluation of the uncertainties of three predominant experimental parameters, namely temperature, modulus, and phase of the total modal pressures on the measurement of this matrix.

The objective of this paper is to develop an analytical simulation of the scattering matrix measurement experience to estimate the influence of errors on some parameters (the separation distance, length of the porous material, and incident pressure modulus) on the scattering matrix coefficients of a duct element containing a porous material. The Monte Carlo technique is then integrated to this simulation to analyze, in a plane wave situation, the propagation of errors on measured parameters (scattering matrix coefficients and the acoustic power attenuation).

The outline of this paper is as follows: in section 2, a short review of the general definition of the scattering matrix of a duct element containing a porous material and the physical interpretation of its coefficients is presented. The simulation of the experiment is presented in Section 3 with the use of the Monte Carlo method to calculate the 95% confidence intervals on the scattering matrix coefficients and the attenuation of the acoustic power attenuation. Finally, the simulation results are presented and discussed in section 4.

2. DEFINITION OF THE SCATTERING MATRIX [S]

The scattering matrix, denoted [*S*], relates the incoming and outgoing pressures within and across the section to be characterized (*S*) as shown in Figure 1.

This matrix depends only on the acoustic characteristics of the duct (its geometry, the impedance of its walls...). When only the plane wave is propagating in the studied duct, the scattering matrix [*S*] is defined as follows:

$$\begin{Bmatrix} P^{I-}(z_1) \\ P^{II+}(z_2) \end{Bmatrix} = [S] \begin{Bmatrix} P^{I+}(z_1) \\ P^{II-}(z_2) \end{Bmatrix} \tag{1}$$

$$[S] = \begin{bmatrix} S^{11} & S^{12} \\ S^{21} & S^{22} \end{bmatrix} \tag{2}$$

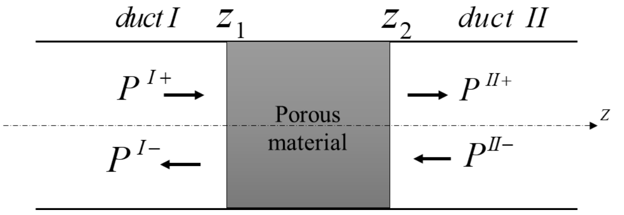


Figure 1. Incoming and outgoing waves of the studied duct element containing a porous material from sides I and II.

Seen the symmetry of the studied duct, the scattering matrix contains the transmission and the reflection coefficients on each side of the studied duct as follows:

$$\begin{cases} S^{11} = S^{22} = R \\ S^{12} = S^{21} = T \end{cases} \tag{3}$$

R : The reflection coefficient of the plane wave.

T : The transmission coefficient of the plane wave.

3. UNCERTAINTY PROPAGATION IN THE EXPERIMENTAL PROCESS OF MEASURING THE SCATTERING MATRIX

The objective of this part is the evaluation of error effects on the measurement of the scattering matrix as well as the acoustic power attenuation. For this, the experimental procedure is theoretically simulated and then the Monte Carlo method was introduced to this simulation.

3.1. Simulation of the Experiment

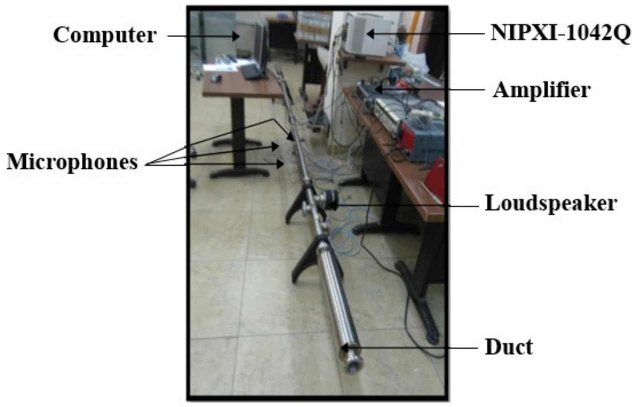


Figure 2. Experimental setup for measuring the scattering matrix of a duct element containing a porous material.

The Figure 2 and the Figure 3 present a schematic of the experiment set up to be numerically simulated as used in the work of Kani et al. [1] to measure the scattering matrix coefficients: The porous material is placed inside a duct element located between z_A and z_B. This duct element is connected to two anechoic terminations placed on each side of the duct to reduce the corresponding reflection. Four microphones (2 on each side) are placed in different locations to measure the acoustic pressures in different locations. The experimental setup also contains two acoustic sources at each side of the duct element to generate the acoustic wave inside the duct.

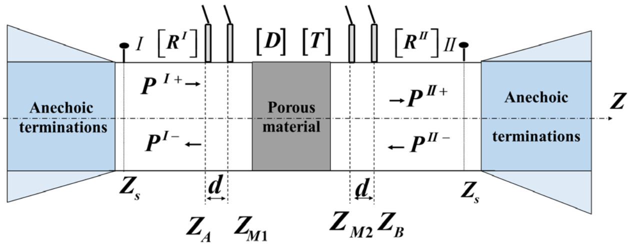


Figure 3. Representation of the simulated experimental setup for measuring the scattering matrix of a duct element.

As presented in Sitel et al. [9] and Taktak et al. [19], the experiment's simulation principle is based on the analytical calculation of the modal coefficients of the acoustic pressure in the four measurement sections z_A, $z_A + d$, z_B and $z_B + d$ for an incident pressure generated by a point source.

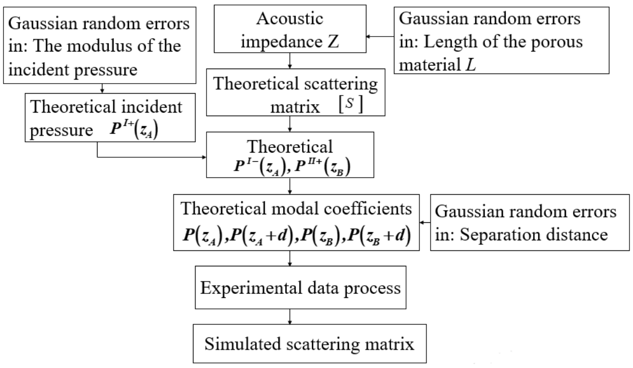


Figure 4. Diagram of Monte Carlo uncertainty estimation by simulation of experiment.

The following steps, depicted in Figure 4 , are used to carry out this simulation:

- Theoretical calculation of the scattering matrix using the Transfer Matrix Method [20].
- Computation of incident pressure wave generated by a point source.
- Calculation of the reflected and transmitted pressures on each side of the test element.

a. Calculation of the incident pressure

The incident pressure in the semi-infinite duct obtained from a point source is defined by its amplitude $A_s = 1$ and its phase $\varphi_s = 0$, and located at

($z_s$et θ_s). Its expression at z_A associated to the mode (0, 0) is given by Taktak et al. [19]:

$$P^{I+}(z_L) = \alpha\, A_S\, e^{jk(z_L - z_s)} \tag{4}$$

where $\alpha = \dfrac{1}{2jk}$.

b. Calculation of the Reflection and Transmission

$$[T] = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = \begin{bmatrix} \cos\left(k(\omega)L\right) & j\,Z(w)\sin\left(k(\omega)L\right) \\ \dfrac{j}{Z(w)}\sin\left(k(\omega)L\right) & \cos\left(k(\omega)L\right) \end{bmatrix} \tag{5}$$

$Z(\omega)$ is the characteristic impedance and $k(\omega)$ is the acoustic wave number of the porous material:

$$Z(\omega) = \sqrt{\rho(\omega)K(\omega)} \tag{6}$$

$$k(\omega) = w\sqrt{\frac{\rho(\omega)}{K(\omega)}} \tag{7}$$

The expressions of $\rho(\omega)$ and $K(\omega)$ respectively the dynamic mass density and the bulk modulus are expressed according to the Johnson-Champoux-Allard-Lafarge (JCAL) model [21]:

$$\rho(w) = \frac{\alpha_\infty \rho_0}{\phi}\left(1 - j\frac{\sigma\phi}{w\rho_0\alpha_\infty}\sqrt{1 + j\frac{4\alpha_\infty^2\eta\rho_0 w}{\sigma^2\Lambda^2\phi^2}}\right) \tag{8}$$

$$K(w) = \frac{\gamma P_0}{\phi}\left(\gamma - \frac{\gamma - 1}{1 - j\dfrac{8\eta}{\Lambda'^2 N_{Pr}\rho_0 w}\sqrt{1 + j\dfrac{\Lambda'^2 N_{Pr}\rho_0 w}{16\eta}}}\right)^{-1} \tag{9}$$

ϕ is the porosity, σ is the flow resistivity, α_∞ is the tortuosity, Λ and Λ' are respectively the characteristic viscous and thermal lengths, ρ_0 is the air density, η is its dynamic viscosity, N_{Pr} is its Prandtl number, γ is its specific heats ratio and P_0 is the atmospheric pressure.

The scattering matrix coefficients are deduced from the transfer matrix coefficients as follows Hu et al. [22]:

$$S^{11} = \frac{X^+ - W^+}{X^+ + W^+} \tag{10}$$

$$S^{22} = -\frac{X^- + W^-}{X^+ + W^+} \tag{11}$$

$$S^{12} = \frac{X^+ W^- - W^+ X^-}{X^+ + W^+} \tag{12}$$

$$S^{21} = \frac{2}{X^+ + W^+} \tag{13}$$

$$X^{\pm} = T_{11} \pm \frac{T_{12}}{Z_0} \tag{14}$$

$$W^{\pm} = Z_0 T_{21} \pm T_{22} \tag{15}$$

The acoustic power attenuation W_{att} of the studied duct element is defined in decibel $[dB]$ as follows [2,23]:

$$W_{att} = 10\log_{10}\left(\frac{W^{in}}{W^{out}}\right) = 10\log_{10}\left(\frac{|d_1|^2 + |d_2|^2}{\lambda_1|d_1|^2 + \lambda_2|d_2|^2}\right) \tag{16}$$

W^{in} is the incoming acoustic power and W^{out} is the outgoing acoustic power.

λ_1 and λ_2 are the eigenvalues of the matrix $[H] = [S]^H \cdot [S]$ where the subscript "H" denotes the conjugate transpose. $[V]$ is the matrix of the eigenvectors of the matrix $[H]$.

- d_1 and d_2 are the component of the vector $\{d\}$ which is calculated by the following equation:

$$\{d\} = \sqrt{\frac{1}{2Z_0}} \cdot [V]^H \cdot \begin{Bmatrix} P^{I+} \\ P^{II-} \end{Bmatrix} \tag{17}$$

After calculating the scattering matrix using the transfer matrix method, the transmission and reflection coefficients can be deduced.

The reflection at z_A and the transmission between z_A and z_B whose expressions are given respectively by:

$$R^I = S^{11} \tag{18}$$

$$T = S^{21} \tag{19}$$

For the studied case:

$$R^{II} = 0 \qquad (20)$$

The transmitted pressure P^{II+} and the reflected pressure P^{I-} are given by:

$$P^{I-} = R^{I}.\ P^{I+} \qquad (21)$$

$$P^{II+} = T.\ P^{I+} \qquad (22)$$

c. Calculation of Transmitted, Reflected and Retrograde Pressures

From the simulated transmitted and reflected pressures, the total pressures can be determined for the four measuring sections located at z_A , $z_A + d$, z_B and $z_B + d$ corresponding to microphones positions:

$$P(z_A) = P^{I+} + P^{I-} \qquad (23)$$

$$P(z_A + d) = e^{ikd} \cdot P^{I+} + e^{-ikd} \cdot P^{I-} \qquad (24)$$

$$P(z_B) = P^{II+} \qquad (25)$$

$$P(z_B + d) = e^{ikd} \cdot P^{II+} \qquad (26)$$

These pressures are used as input to the simulated experimental procedure to recalculate the scattering matrix and the acoustic power attenuation affected by errors.

3.2. MONTE CARLO METHOD

The Monte Carlo method is widely used in various fields to estimate the uncertainties propagation and to evaluate the reliability of results. Recent studies, such as Hannachi et al. [24,25], Hadj Kacem et al. [26] and Ghorbel et al. [27] applied the Monte Carlo method to quantify the impact of input parameter uncertainties on their model results and to obtain confidence intervals for their predictions.

This method has four general steps that must be followed. First, it is necessary to define the type of random distribution that will be used for each input parameter. In the present case, a normal distribution has been chosen. Next, random errors are introduced into the input parameter values to create random samples according to the chosen distribution. This creates a set of input data that covers a range of possible values for each input parameter. Once the random samples have been generated, the model is run for each set of input data. By doing this for a large number of random input sets, an output distribution can be generated for each output parameter. Finally, the information obtained from the model runs is synthesized to obtain descriptive statistics such as the 95% confidence interval and the maximum and minimum errors for each input parameter. This allows quantifying the uncertainty associated with the solution and estimating its reliability.

4. THEORETICAL SIMULATION RESULTS

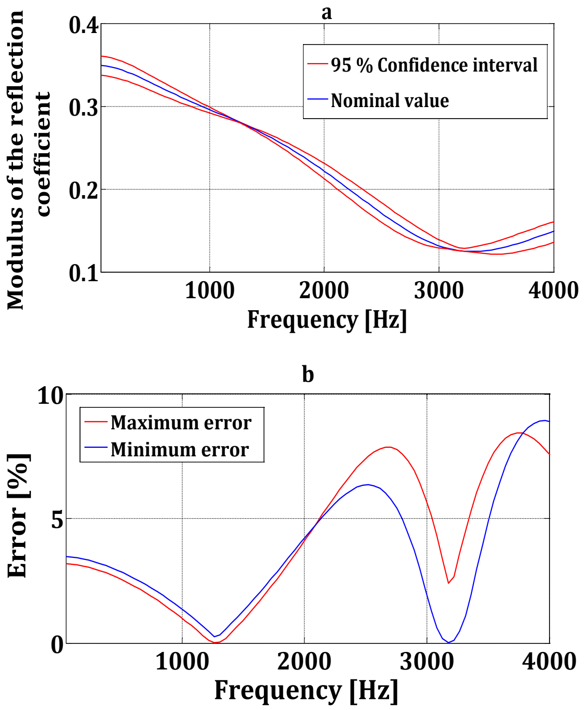
The simulation is performed with a porous material located in a duct element between z_L and z_R in the studied frequency band (0-4000 Hz). The characteristics of the used porous material are presented in Table 1. The used separation distance between the two microphones on each side of the studied duct is 40 mm.

Table 1. Studied porous material characteristics.

Flow resistivity ($Nm^{-4}s$)	10000
Porosity	0.88
Tortuosity	1
Thermal Permeability (m^2)	$0.831\ 10^{-8}$
Viscous characteristic length (µm)	92.07
Thermal characteristic length (µm)	141.3
Length (mm)	50

The 95% confidence intervals for the scattering matrix coefficients and the acoustic power attenuation are calculated, as shown in the following figures, using random errors following a normal distribution generated in the experimental simulation code, with a variation of ± 5% applied to each input parameter's mean value. To ensure the convergence of results, a large number of simulations were performed. In our study, 10,000 iterations are chosen. The effect of three parameters is studied: the porous material length (*L*), the separation distance between two microphones (*d*) and the incident pressure modulus (P). The figures 5, 6 and 7 showed respectively these effects for each studied parameter on the scattering matrix coefficients as well as the acoustic power attenuation by presenting the 95% confidence band and the maximum and the minimum errors due to the introduced errors.

Reflection coefficient (I)



Transmission coefficient (II)

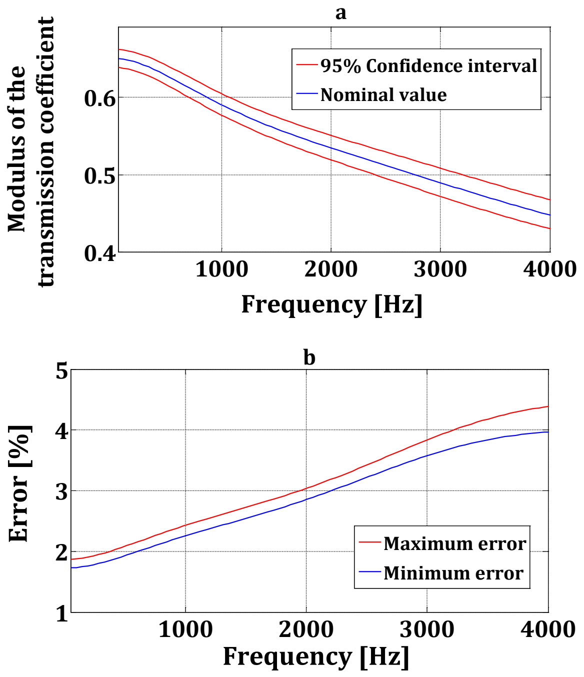


Figure 5 (A). Influence of porous material length uncertainties on acoustic performance: 95% confidence interval (a) and corresponding error (b) - *Reflection coefficient (I) / Transmission coefficient (II)*

Figure 5 (A, B) presents the effect of porous material length errors on the scattering matrix coefficients and the acoustic power attenuation by presenting the 95% confidence for the reflection coefficients (I-a), transmission coefficient (II-a), and the acoustic power attenuation (III-a) and the corresponding errors (I-b, II-b, and III-b).

Acoustic Power Attenuation(III)

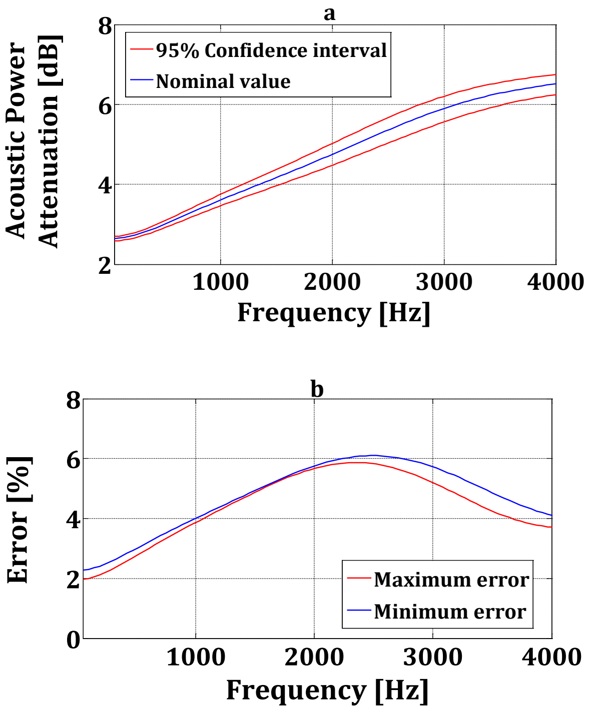


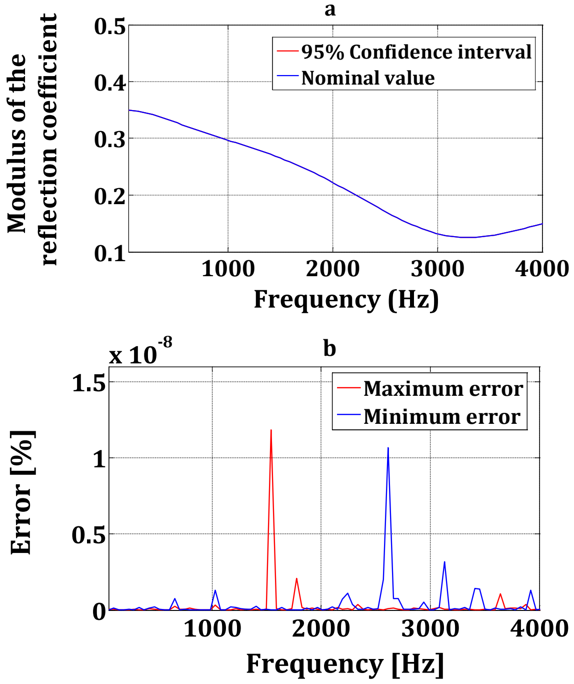
Figure 5 (B). Influence of porous material length uncertainties on acoustic performance: 95% confidence interval (a) and corresponding error (b) - *Acoustic Power Attenuation(III)*

These curves showed that scattering coefficients are sensitive to errors related to the length of this material. The effect of uncertainties of this parameter on the reflection and transmission coefficients is significant, as shown by the thickness of the confidence interval, which is large. The maximum and minimum errors range from 0% to 9% for the reflection coefficient, while the transmission coefficient has a maximum and minimum error of 1.8, with a constant increase of 4 for the maximum error and 4.5 for the minimum error. Similarly, the sensitivity of acoustic attenuation to errors associated to the porous material length is significant, as shown by the wide confidence interval, with an error ranging from 2% to 6%.

Figure 6 (A, B) presents the effect of errors related to the distance between the microphones on the scattering matrix coefficients and the acoustic power attenuation by presenting the 95% confidence for the reflection coefficients (I-a), transmission coefficient (II-a), and the acoustic power attenuation (III-a) and the corresponding errors (I-b, II-b, and III-b.

It can be observed that the errors related to this parameter have a negligible influence. This error has a negligible effect on the reflection and transmission coefficients, with very narrow confidence intervals.

Reflection coefficient (I)



Transmission coefficient (II)

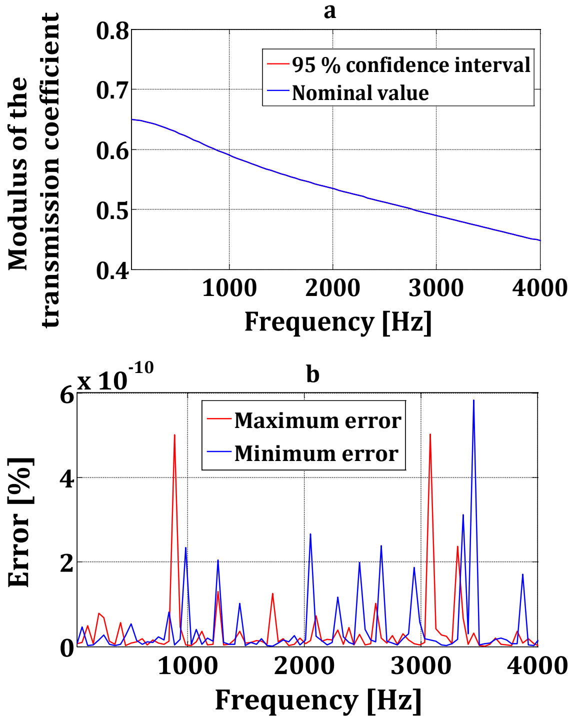


Figure 6 (A). Influence of the separation distance between two microphones uncertainties on acoustic performance: 95% confidence interval (a) and corresponding error (b) - *Reflection coefficient (I) / Transmission coefficient (II)*

These results are confirmed by the data presented in Figure 6 (I-b), (6-II-b) and (6III-b)) which displays the variation of the maximum and minimum errors. In the frequency range between 50 and 4000 Hz, both the minimum and maximum errors are null when the separation distance varies by ± 5% from its nominal value.

Acoustic Power Attenuation(III)

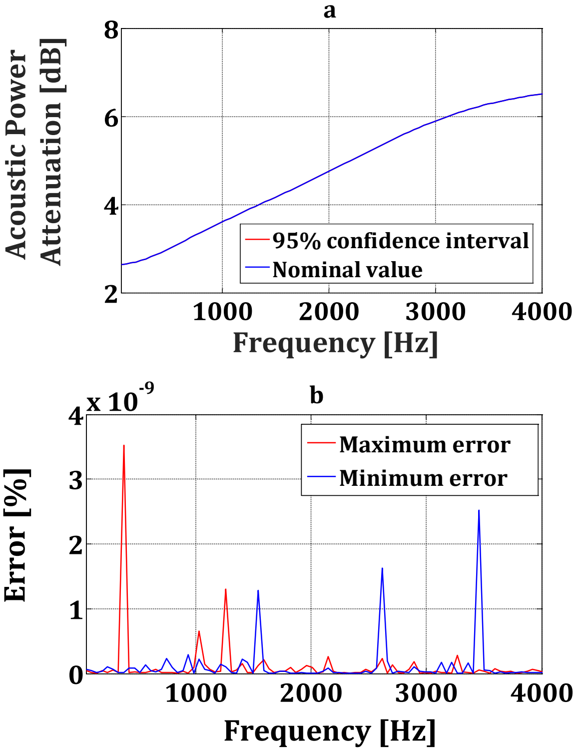


Figure 6 (B). Influence of the separation distance between two microphones uncertainties on acoustic performance: 95% confidence interval (a) and corresponding error (b) - *Acoustic Power Attenuation(III)*

Reflection coefficient (I)

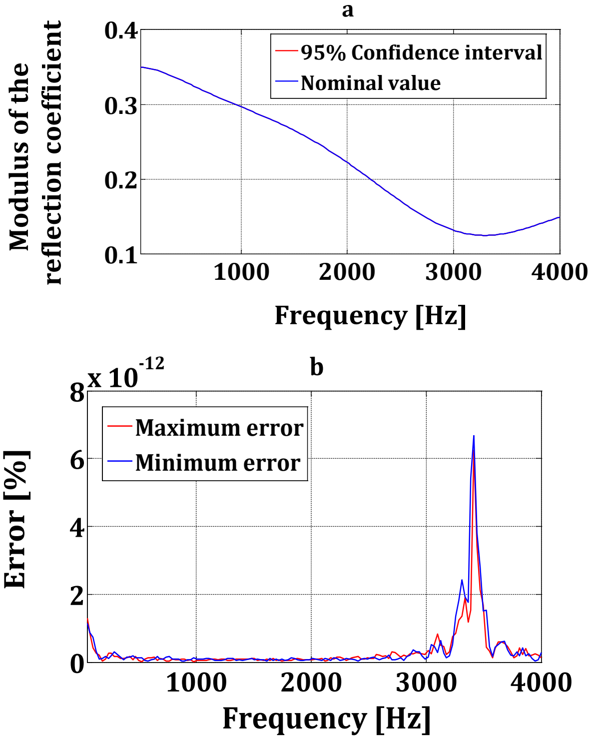
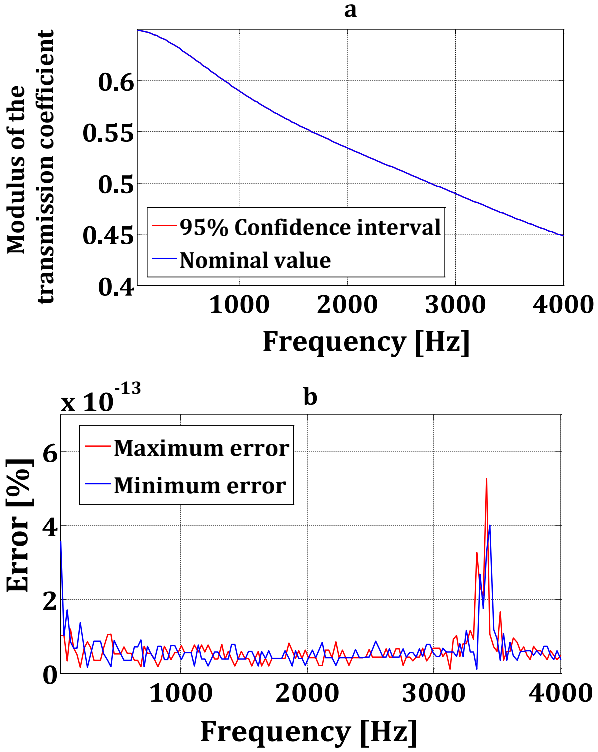


Figure 7 (A). Influence of the incident pressure modulus uncertainties on acoustic performance: 95% confidence interval (a) and corresponding error (b) - *Reflection coefficient (I)*

Transmission coefficient (II)



Acoustic Power Attenuation(III)

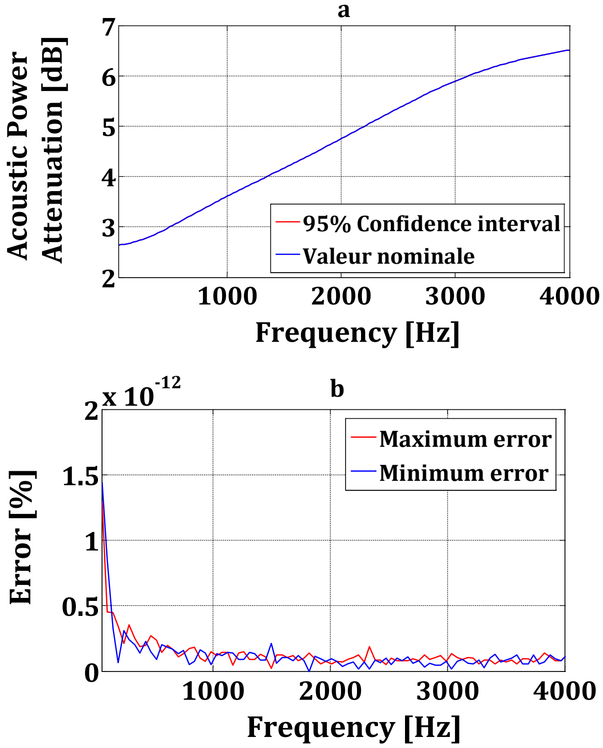


Figure 7 (B). Influence of the incident pressure modulus uncertainties on acoustic performance: 95% confidence interval (a) and corresponding error (b).

As presented in Figure 7 (A, B), a variation of ± 5% of the nominal value of the incident pressure modulus does not affect the acoustic performance (reflection coefficient, transmission coefficient, and acoustic attenuation). The confidence intervals in Figure 7 (I-a), (II-a) (III-a) are narrow, and the maximum and minimum errors in Figure (I-b), (II-b) (III-b) are close to zero. So, it can be concluded that this parameter has no effect.

5. CONCLUSIONS

A theoretical simulation of the experiment of measuring the scattering matrix and the acoustic power attenuation of a duct element containing a porous material was performed to identify the parameters affecting this measurement using the Monte Carlo technique by determining the 95% confidence interval of the measured scattering matrix coefficients and sound power attenuation.

This study allowed us to conclude that the length of the porous material has the most significant influence on the acoustic parameter's measurements, unlike other parameters such as separation distance and incident pressure modulus, which have no effect.

The obtained results show that acoustic power attenuation is the acoustic parameter that is least affected by input errors. This study justifies the choice of this parameter in a previous study by Kani et al. [15] in the cost function of an inverse technique for determining the porous parameters of a porous material in a pipe element. The use of this parameter ensures that the results are less sensitive to parameter uncertainties.

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