Free Vibration Analysis of Viscoelastic and Magnetorheological Elastomer Composite Sandwich Beam Under Various Boundary Conditions

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Abstract: - Vibration analysis of three-layer symmetrical sandwich beams with a viscoelastic and magnetorheological elastomer (MRE) core and conductive aluminum skins subjected to magnetic or non-magnetic field intensity was carried out under various boundary conditions. A numerical simulation of the bending vibration behavior of these beams using the Abaqus calculation code was carried out. The simulation results show that the absorption capacity of the developed magnetorheological elastomer is better than that of the viscoelastic material. The loss factor and shear modulus were found to be strongly influenced by the application of a magnetic field. The results demonstrated that the natural frequencies of the beam having a magnetorheological elastomer core can be tuned and adjusted intelligently.

Keywords: - Free vibration, Numerical simulation, Sandwich beam, Viscoelastic materials, Magnetorheological elastomer.

1. INTRODUCTION

From the point of view of their importance in the development of sciences such as sciences involving aeronautics and fields of civil and mechanical engineering, materials and structures with high capacity of deformations, isolating and reducing the undesirable sounds and vibrations are the subject of important discussions nowadays. Developing new complex structures, sometimes called intelligence that help absorb the vibrations; thus reducing the appearance of resonance phenomena represents the focus of some recent research in the field of engineering materials. For this reason, a large interest is brought to studying simple structures, such as sandwich composite beams or plates with incorporated viscoelastic cores, sometimes involving the use of magnetorheological elastomers. The latter loaded with iron particles, improves the rigidity of the structure when subjected to deformation and allows its stability after the application of the magnetic field over it, which creates an attractive force between the iron particles, thus stiffening the structures, and in case of vibration, helps reducing the oscillations of the systems.

Some previous recent research had already focused on the use of viscoelastic MRE and non-MRE core in composite sandwich structures, such as Banerjee [1] who studied analytically the free vibration of three layered symmetric sandwich beams by developing its dynamic stiffness matrix. Natural frequencies and associated mode shapes for the proposed sandwich beam were calculated using the Wittrick-William algorithm. The exact dynamic stiffness method was also used in the work of Howson and Zare [2] to define the flexural motion of a three-layered sandwich beam but with unequal faceplates. Eigenvalues were also calculated by the use of the Wittrick-William algorithm. In a second work by Banerjee et al. [3] free vibration analysis, using an experiment and analytical approach based on the dynamic stiffness matrix method for a three-layered beam was conducted. This time the layers have unequal thicknesses and each layer is idealized by the Timoshenko beam theory. Zhou and Wang [4] Studied the field-dependent stiffness of a smart sandwich beam composed of a soft-based core with MRE and non-MRE parts and assumed nonconductive upper and lower skins, in order to neglect the induced effects by the magnetoelastic loads appearing in the vibrating body. Results showed that the MRE field-dependent modulus causes the changes in the anti-resonance frequencies. However, a small change in the resonant frequencies was noticed. Two later works were reported by Zhou and Wang [5-6] on composite MRE core sandwich beam, this time with conductive skins, and a solution for the applied magnetoelastic load on the elastic conductive layers under a uniform magnetic field was analytically formulated. Deng and Gong [7-8] investigated the vibration reduction and frequency shift capability by using a new adaptive tuned vibration absorber (ATVA) based on MRE properties, which are controllable by an applied magnetic field. Results show that such an ATVA model can cause the variation of natural frequency from 55 to 82 Hz and in the second work from 25.5HZ to 40HZ. Another adaptive tuned vibration absorber (ATVA) was designed by Sun et al. [9] this time with multilayer MRE sheets. The newly designed model was compared with two kinds of the previous model with one MRE layer in terms of performance. Choi et al. [10] conducted an experimental and numerical study on the vibration characteristics of sandwich beams with steel skins and magnetorheological elastomer cores. The core damping properties against different combinations of applied magnetic field intensity were investigated in this work and frequency response results for both approaches were compared. Arvin et al. [11] studied numerically using finite element formulation, the free and forced vibration of a composite sandwich beam with a viscoelastic core layer. in their work, higher order theory was achieved based on independent transverse displacements through the thickness for the two upper and lower layers and taking also into consideration the effects of the young modulus which means the effect of axial displacements as well as the rotational inertia for the core layer. The analysis is more complex and realistic compared to the previous standard theory of Mead and Markus's work [12] generally used for modeling sandwich three-layered beams based on several simplification assumptions such as all the three beam layers have the same transverse displacement and the core layer deforms only to shear associated with transverse movement. Results allowed us to show the difference between the presented method and the Mead and Markus method in terms of accuracy. The study also shows the effect of other parameters on natural frequencies and loss factor variation, such as the thickness of layers and the fiber orientation angle of the upper and lower composite skins. Nayak et al. [13] performed an analytical study based on the Galerkin method. First, the study deals with the free vibration of a symmetric sandwich beam having MRE incorporated as a core layer and subjected to different boundary conditions, and second, investigates the parametric instability regions of the same structure, subjected to periodic axial load. Two later works were conducted by the same author [14-15]. The first one aimed to analyze the free and forced vibration of the same composite structure as in the previous work [13], this time using the finite element method (FEM). The results validate the analytical method used in the previous work [13]. The second conducted work was to determine the parametric instability regions of a sandwiched beam with a fully MRE core layer, using the finite element method (FEM). The instability regions firstly were investigated without considering the damping effects of the MRE, then another study was performed while taking into consideration the damping of the core layer. In this study, sandwich panels with carbon/epoxy composite skins and a magnetorheological elastomer (MRE) honeycomb core in different proportions of magneto/elastomer (w/w%) are manufactured and studied numerically and experimentally by F. S. Eloya et al. [16]. Yildrim et al. [17] made an experimental investigation of the nonlinear dynamics of a geometrically imperfect magneto-rheological elastomer sandwich beam. The structure was composed of only two layers, a magnetorheological elastomer, and another aluminum layer. An extensive study was by H. Barman et al. [18] to find out the different factors that affected the performance of a sandwich beam with a magnetorheological elastomer embedded in its core. In this work, Aguib et al. [19] studied the buckling instability phenomenon of magnetorheological elastomer plates subjected to compressive load. Navazi, et al. [20] conducted a recent study investigating in term of dynamic properties such as natural frequency and loss factor, the free vibration of a rotating magnetorheological tapered sandwich beam. To develop an appropriate governing equation for the rotating structure, the Ritz method via the Lagrange equation was used in this work. More recently in an experimental study conducted by De Souza Eloy et al. [21] the first composite sandwich structure with a honeycomb core filled with MRE was manufactured and tested. The dynamic properties of this new structure were analyzed and evaluated by conducting a free and forced vibration test. Results showed a reduction in vibration amplitudes and frequency shift when increasing the magnetic field intensity. Aguib et al. [22] studied the effect of the magnetic field on the mechanical properties of magnetorheological elastomers. Tourab et al. [23] studied the effect of the magnetic field and temperature on the mechanical properties of magnetorheological elastomers. In this article, the problem of determining the natural vibration frequencies of fixed-fixed sandwich beams is studied by M. E. Raville et al.[24]. one analyzed by an energy approach in which the Lagrangian multiplier method is used to satisfy the boundary conditions of the problem. In this work, Omkar JARALI et al. [25] used a machine-learning and regression model to determine the locations and severity of the delamination in the Fiber Metal laminate cantilever beams. The data set related to delamination location, severity, and bending natural frequencies was obtained using the Finite Element Analysis. Settet et al. [26] studied the three-point bending of a magnetorheological elastomer beam. In this research paper, analytical analysis, experimental work, and finite element simulations are combined to analyze the vibration behavior at different delamination sizes and different stacking sequences [27]. In this study, Suxia Hou et al. [28] carried out modeling and analysis of damping and vibrations in the innovative sector of composites and annular plates composed of a magnetorheological elastomer (MRE) reinforced with glass fibers. An algorithm to assess transversal cracks in composite structures based on natural frequency changes due to damage was proposed by Gillich et al [29]. Gillich et al. [30] showed in their paper the mathematical relations between the deflection of a cantilever beam at the free end and its capacity to store energy. It is shown by means of the finite element method FEM that by the occurrence of a crack, the stored energy decreases proportionally with the deflection increase. In their work, Gillich et al. [31] introduced a damage severity estimator to express the crack evolution as a function of stored energy by analytical, numerical, and experimental analyses. In their work, Sorin Vlase et al. [32] made an analysis of the influence of anthropogenic and natural vibrations on the position of the target, located at the end of a guide beam. In their paper, M.L. Scutaru et al. [33] used the properties involved by the symmetries that exist in mechanical systems for the analysis of the forced response to vibrations.

In this paper, we present a numerical simulation of the vibration behavior of sandwich beams having once a viscoelastic core (passive control), and another time a magnetorheological elastomer core (adaptive or active control). Passive control in the dynamics of a structure is quite common, and this solution is not satisfactory, it presents a major weakness in several areas. For this reason, and because of the lack of adequate efficient solutions, there have been only a few attempts to use magnetorheological elastomers that have variable and adaptive dynamic properties, controlled in real-time. However, in many cases and in the field of structural dynamics, the control of the system parameters in real-time is confused with a punctual and adequate selection.

2. PROBLEM FORMULATION

For viscoelastic material exposed to shear dynamic loading, Shear stress and strain relation based on the linear viscoelastic theory in the pre-yield regime can be written as [20]:

$$\tau = G^{*}\gamma \tag{1}$$

where represents the shear stress. Is the shear deformation and is the shear complex modulus of the viscoelastic material, given by [16, 19]:

$$G^{*} = G^{'} + iG^{"} = G^{'}(1 + i\eta) \tag{2}$$

The stiffness of the viscoelastic material is characterized by the storage also called conservative modulus $G^{'}$, the real part of the above complex number. However, the viscous behavior is referred to by the dissipative or loss modulus $G^{"}$, the imaginary part of the shear complex modulus and $i = \sqrt{-1}$.

The mechanical properties (storage modulus, loss modulus, and loss factor) of a viscoelastic material are determined by the following relationships:

$$G^{'} = G_{\infty} + G_0 \sum_{i=1}^{N} \frac{g_i \tau_i^2 \omega^2}{1 + \tau_i^2 \omega^2} \tag{3}$$

$$G'' = G_0 \sum_{i=1}^{N} \frac{g_i \tau_i \omega}{1+\tau_i^2 \omega^2} \tag{4}$$

$$\eta = \tan(\delta) = \frac{G''}{G'} \tag{5}$$

where G_∞ is the equilibrium modulus, τ_i is the relaxation time, g_i is the weight coefficient, G_0 is the instantaneous modulus, ω is the angular frequency.

The dynamic model represented in this section is a MRE sandwich beam as depicted in Fig.1, The beam is made of three layers; the upper and lower skins are the elastic, undammed, and conductive parts. The medium layer is the core layer and represents the viscoelastic MRE charged with ferromagnetic particles dispersed within the elastomeric matrix. When the magnetic field is applied to the beam, the core rheological properties will widely change.

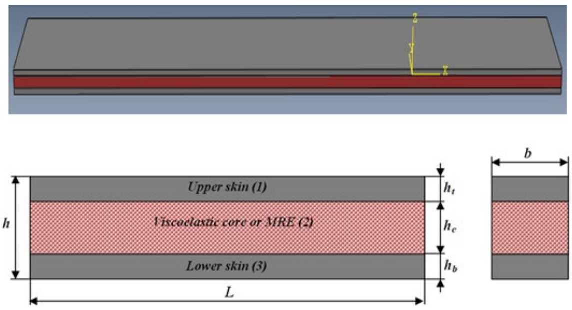

Figure1. Viscoelastic and MRE beam model constitution

To describe the vibration of three layers viscoelastic sandwich beam, previous theory based on some assumptions has been adopted by many previous works [14, 20] dealing with multiple problems. The assumptions are as follows:

- The top and bottom elastic layers behave as Euler –Bernoulli beams when subjected to deformation.
- Since the MRE is considered incompressible, the same transverse displacement is taken for all three layers.
- The MRE core layer deforms only to shear associated with axial displacements and its normal stress and strain are ignored due to the small value of the elastic modulus.
- Perfect bonding between layers, which means no slippage and delamination between the layers during deformations, is considered.

3. NUMERICAL SIMULATION

In this section, the free vibration simulation analysis of variant composite beams involving the MRE and having different boundary conditions has been carried out.

The results are given in terms of natural frequency comparisons with the present simulations and previously published experimental and analytical results.

In our simulation of the vibratory behavior using the computation code Abaqus, the elastomer in the last case is considered as a viscoelastic solid where its behavior is described following an appropriate viscoelastic Maxwell model (Figure 2).

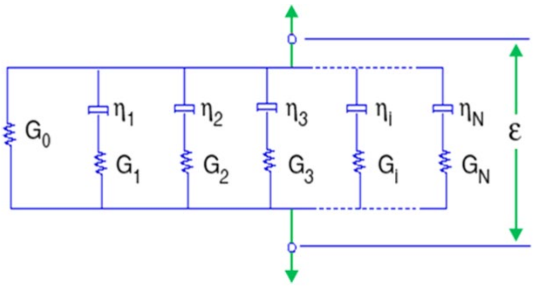

Figure 2. Rheological generalized Maxwell model

The magnetorheological elastomer is identified using Abaqus by the generalized Maxwell model (Figure 2). In this study, we used a five arms Prony series. The MRE properties determined experimentally by the DMA, are given in the Table 1.

Table 1. Mechanical properties of MRE

B=1T		
Storage modulus G' (Pa)	Loss modulus G'' (Pa)	Loss factor
4221381.54	832541.324	0.1972201
3753492.19	736542.684	0.1962286
3056915.52	63541.354	0.0207861
2832541.21	53246.843	0.0187982
2732514.62	49652.351	0.0181709

The parameters of the Prony series are given as follows:

$$g_i = \frac{G_i}{G_0 + \sum_{i=1}^{n} G_i} \tag{6}$$

$$\tau_i = \frac{G_i}{\eta_i} \tag{7}$$

To model the bending behavior of the different beams studied, a reduced-size C3D20R solid element, of the quadratic brick type with 20 nodes of dimensions ($0.2*0.2*0.2mm^3$) is used to discretize the skin and the core in MRE.

3.1. Study Case 1: Clamped-Clamped

The first example represents the case of a two fixed ends sandwich beam as that reported in the work of Raville et al. [24], Howson and Zare [2] and

Banerjee et al. [3]. The model has constitutively the following system parameters and materials properties: length $L = 1218.7$ mm; width, $b = 25.4$ mm; core thickness, $h_c = 6.3475$ mm; skin thickness, $h_t = h_b = 0.40624$ mm; $E_t = E_b = 68.9$ GPa; $E_c = 179.14$ MPa; $G_c = 68.9$ MPa ; $\rho_t = \rho_b = 2687.3$ Kg/m^3; $\rho_c = 119.69$ Kg/m^3.

The mesh of this beam is given in Figure 3. The number of elements assigned by different parts of the beam are given as follows: 1547876 elements generated in part 1 and also on part 3, 24766016 elements generated in part 2.

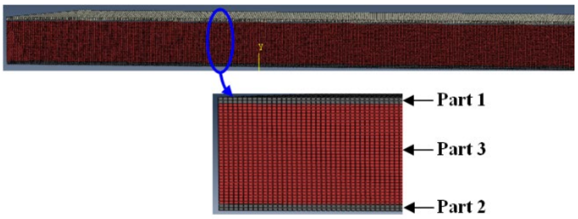

Figure 3. Beam mesh

Table 2, shows the results of the natural frequencies, where very good agreement is observed between the present work (Abaqus simulation) results and the experimental and theoretical results found by the previous works [2, 3, 24]. One must note that the two first natural frequencies that are less than 100Hz were note reported by Raville et al. [24] in their experiment due to the lack of equipment at the time [3]. For a beam clamped at both ends, the boundary conditions imposed at *x = 0* and *x = L* are *w(0)=w'(0)=w(L)=w'(L)=0.*

Table 2. Eigenfrequencies for the first seven eigenmodes

Mode	Present (Hz)	Raville et al. [22] (Hz)	Howson et al. [2] (Hz)	Banerjee et al. [3] (Hz)
1	34.753	-	34.597	34.346
2	93.491	-	93.100	91.386
3	177.83	185.5	177.16	171.69
4	283.75	280.3	282.78	270.36
5	407.55	399.4	406.33	383.27
6	545.76	535.2	544.33	506.88

3.2. Study Case 2: Clamped-Free

Hear the natural frequencies of a sandwich beam with a viscoelastic core layer and two upper and lower aluminum skins as that in the work of Arvin et al. [11] are studied. The beam is fixed on one side and free to vibrate on the other side and has the following physical parameters and material properties. length $L = 177.8$ mm ; width $b = 12.7$ mm; top and bottom skin thicknesses $h_t = h_b = 1.52$ mm; the viscoelastic core thickness $h_c = 0.127$ mm; $E_t = E_b = 70.370$ GPa; $E_c = 2.097$ MPa ; $\rho_t = \rho_b = 2770$ Kg/m^3 ; $\rho_c = 970$ Kg/m^3 ; $\nu_t = \nu_b = 0.3; \nu_c = 0.49$; $G^* = 0.7037(1 + 0.3i)$.

The mesh of this beam is given in Figure 4. The number of elements assigned by different parts of the beam is given as follows:

448056 elements generated on part 1 and also on part 3, 56007 elements generated on part 2.

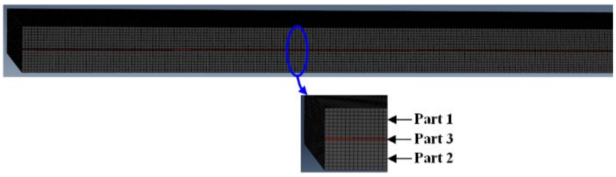

Figure 4. Beam mesh

The results are reported in Table 3 for only the first three natural frequencies. In their theoretical work based on the finite element method Arvin et al. [11] extracted natural frequencies considering four different studies, hear the comparison is made with the fourth case of their study where Shear and Young moduli, rotary inertia, and longitudinal kinetic energy of the core were considered in the finite element formulation. Results show good agreement with a difference of 1.515 % in the third mode. For a beam clamped-free, the boundary conditions imposed at *x = 0* and *x = L* are *w(0)=w'(0)=0, M(L)=M'(L)=0.*

Table 3. Eigenfrequencies for the first three eigenmodes

Boundary conditions	Mode	Present (Hz)	Arvin et al. [11] (Hz)
C-F	1	65.56	65.009
	2	303.44	299.57
	3	761.46	750.09

3.3. Study Case 3: Cantilever sandwich beam (Clamped-pinned-free)

Table 4 shows the natural frequencies of a cantilevered sandwich beam example as in the work of Banerjee et al. [3] and Nayak et al. [13]. In this case, the beam has the following material properties and physical parameters : $E_t = E_b = 72$ GPa; $E_c = 1.5$ MPa; $G_c = 0.5$ MPa; $\rho_t = \rho_b = 2770$ Kg/m^3; $\rho_c = 950$ Kg/m^3; length, $L = 500$ mm; width, $b = 25$ mm; core thickness, $h_c = 18$ mm; skins thicknesses, $h_t = h_b = 2$ mm.

The mesh of this beam is given in Figure 5. The number of elements assigned by different parts of the beam are given as follows: 3125000 elements generated in part 1 and also in part 3, 28125000 elements generated in part 2.

Results show very good agreement with the experimental work of Banerjee et al. [3] and the theoretical work of Nayak et al. [13] for the first and last natural frequencies. However, a remarkable difference is observed when comparing the results of the present work and the Nayak et al. [13] work with the results of the theoretical work of Banerjee et al. [13].

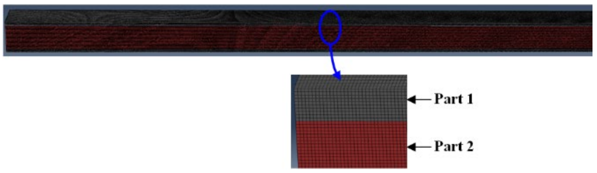

Figure 5. Beam mesh

Taking into consideration that a tenth-order differential equation of motion was used in Banerjee et al. [3] work, on the other hand, a fourth-order differential equation of motion was used in Nayak et al. [13] study. Let us consider the case of a cantilever beam subject to the following boundary conditions: $w(0)=0,\ M(L)=M'(L)=0$.

Table4. Eigenfrequencies for the first three eigenmodes

Boundary condition	Mode	Present (Hz)	Banerjee et al. [3] (Hz)	Nayak et al. [13] (Hz)
Cantiliver	1	12.324	11.25	12.308
	2	43.189	33.75	43.204
	3	91.66	93.75	94.427

3.4. Study Case 4: Simple-Simple sandwich beam

Considering the case of a simply supported beam with an MRE embedded core layer as that treated in the work of Zhou and Wang [6]. The beam is subjected to a magnetic field intensity of 1T. Taking the physical parameters as the following: length $L=150$ mm; width $b=15$ mm; core thickness $h_c=2$ mm; skins thicknesses $h_t=h_b=0.1$ mm.

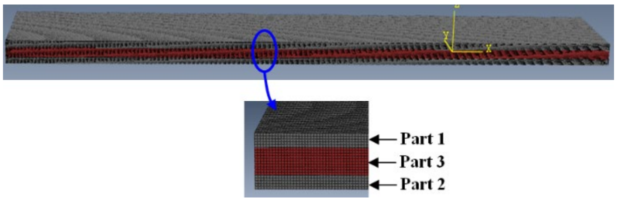

Figure 6. Beam mesh

The material's properties are as follows: $E_t=E_b=72$ GPa; $E_c=1.7$ MPa; $G_c=0.6208$ MPa; $\rho_t=\rho_b=2770$ Kg/m^3; $\rho_c=1100$ Kg/m^3 as reported in the work of Nayak et al. [13].

The mesh of this beam is given in Figure 6. The number of elements assigned by different parts of the beam are given as follows: 56250 elements generated in part 1 and also in part 3, 562500 elements generated in part 2.

Natural frequencies of the MRE beam are compared with those obtained by Zhou and Wang [6] and Nayak et al. [13] and reported in Table 5. A good agreement between the present simulation and the two previous theoretical works is observed. It may be noted that Nayak et al. [13] used a fourth-order differential equation of motion in their analysis, however, higher order theory is used in the work of Zhou and Wang [6]. For a beam simply supported at both ends, the boundary conditions imposed at $x=0$ and $x=L$ are $w(0)=w(L)=0,\ M(0)=M(L)=0$.

Table 5. Eigenfrequencies for the first three eigenmodes

Boundary condition		Mode	Present (Hz)	Zhou [6] (Hz)	Nayak [13] (Hz)
S-S	B=1T	1	68.323	68.816	67.338
		2	145.93	152.44	144.92
		3	223.80	225.08	222.14

The simulation results are plotted and compared with those found in the literature for the different cases. It is observed that the boundary conditions have a considerable influence on the natural frequencies of the beam. We also note that the viscoelastic layer has an important influence on the vibratory amplitude of the beam, especially in the case where the layer is made of viscoelastic magnetorheological elastomer which has high damping capacity.

Figure 7 shows the variation of the natural frequency as a function of the vibration modes of a Clamped-Clamped sandwich beam.

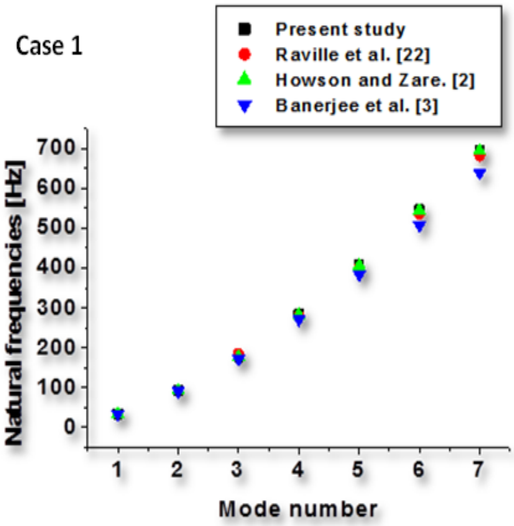

Figure 7. Variation of the natural frequency as a function of the vibration modes

The error between the present work and the works done by Raville et al. [24], Howson and Zare [2], and Banerjee [3] is shown in Figure 8. We note a good correlation between the results found where the maximum error does not exceed 7.67%.

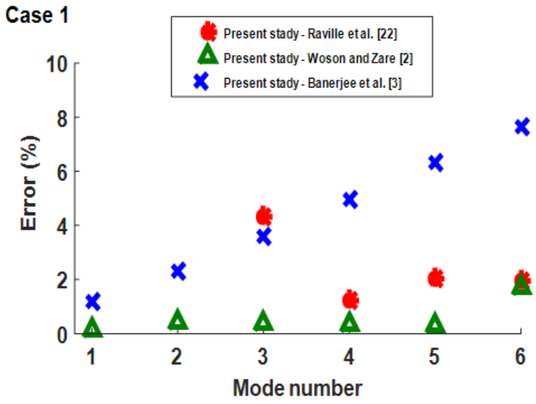

Figure 8. Error between different studies

The different vibration modes are given in figure 9.

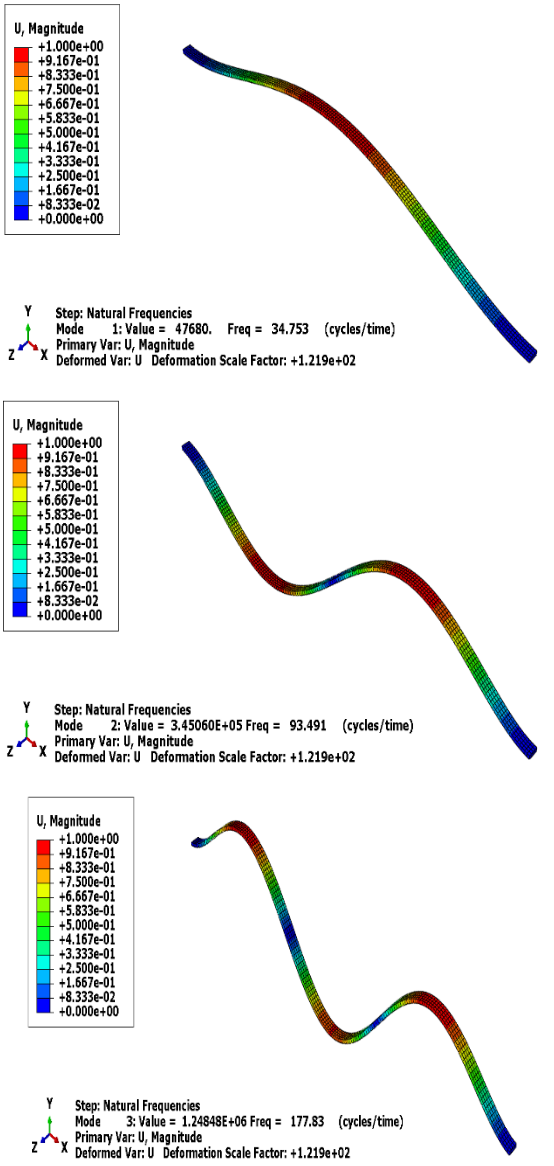

Figure 9. Eigenfrequencies and modes simulations for the first case study

Figure 10 shows the variation of the natural frequency as a function of the vibration modes of a Clamped-Free sandwich beam.

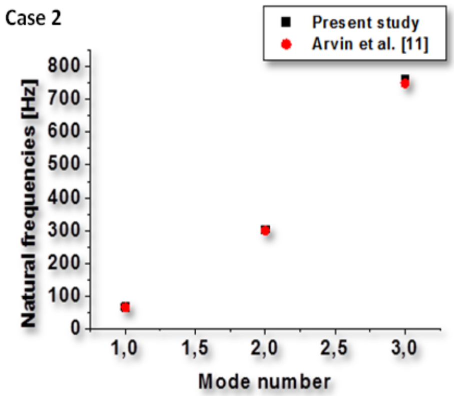

Figure 10. Variation of the natural frequency as a function of the vibration modes

The error between the present work and the work done by Arvin et al. [11] is shown in Figure 11. We note a good agreement between the results found where the maximum error does not exceed 1.51%.

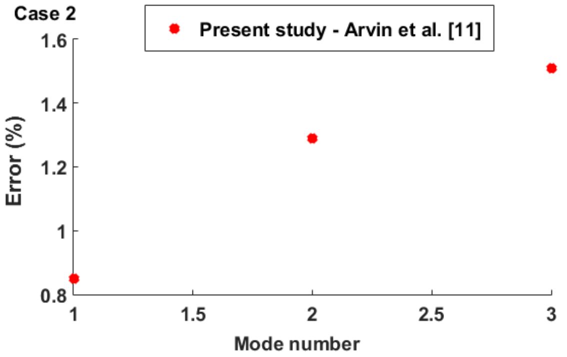

Figure 11. Error between different studies

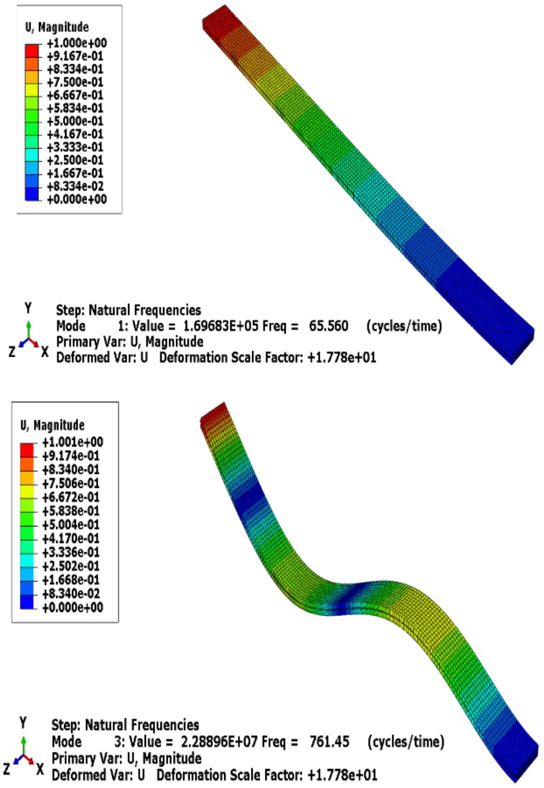

Figure 12. Eigenfrequencies and modes simulations for the second case study

The different vibration modes are given in figure 12.

Figure 13 shows the variation of the natural frequency as a function of the vibration modes of a Clamped-Free sandwich beam.

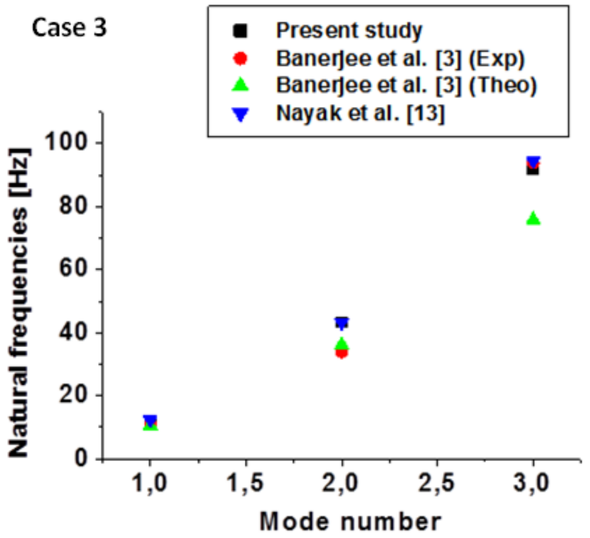

Figure 13. Variation of the natural frequency as a function of the vibration modes

The error between the present work and the works done by Banerjee et al. [3] and Nayak et al. [13] is shown in Figure 14. We note a good agreement between the present work and work carried out by Nayak et al. [13] where the maximum error does not exceed 3.02%. On the other hand, we note a divergence between our work and the work done by Banerjee et al. [3] where the error reaches a value of 27.97%.

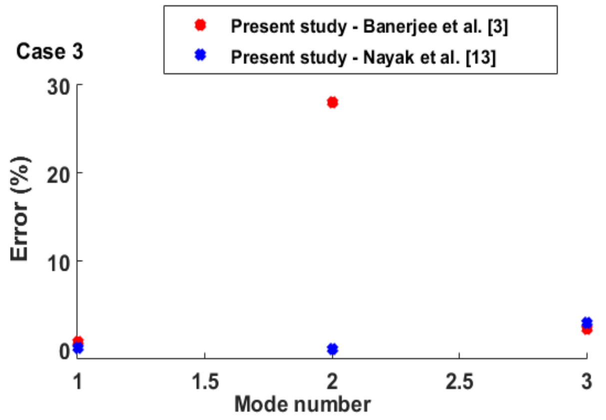

Figure 14. Error between different studies

The different vibration modes are given in figure 15.

Figure 16 shows the variation of the natural frequency as a function of the vibration modes of a Clamped-Free sandwich beam.

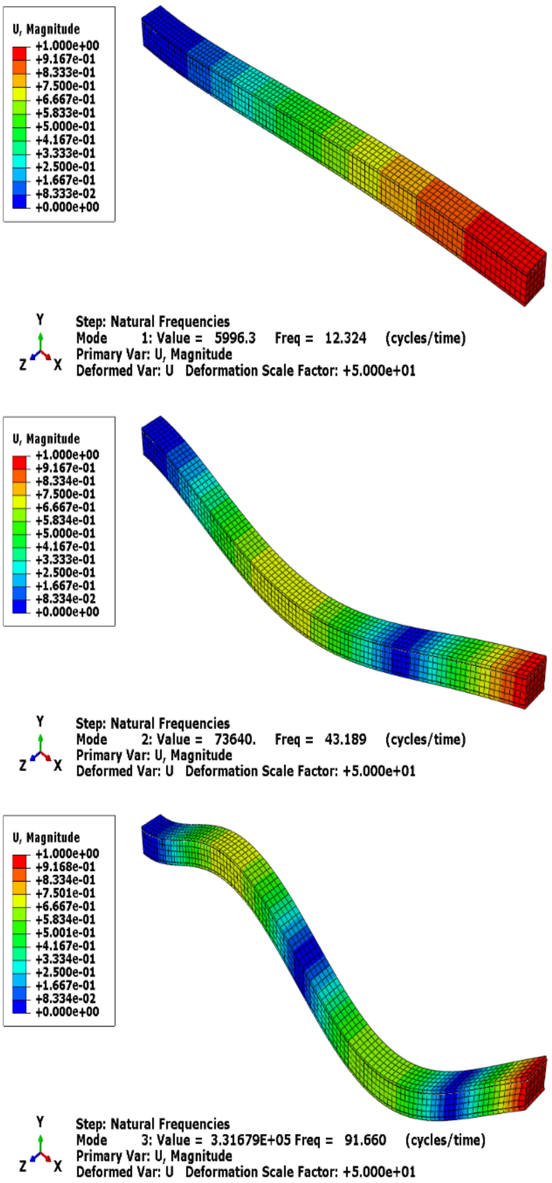

Figure 15. Eigenfrequencies and modes simulations for the third case study

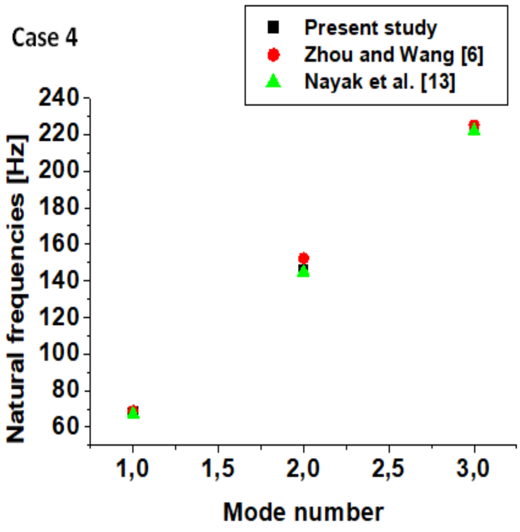

Figure 16. Variation of the natural frequency as a function of the vibration modes

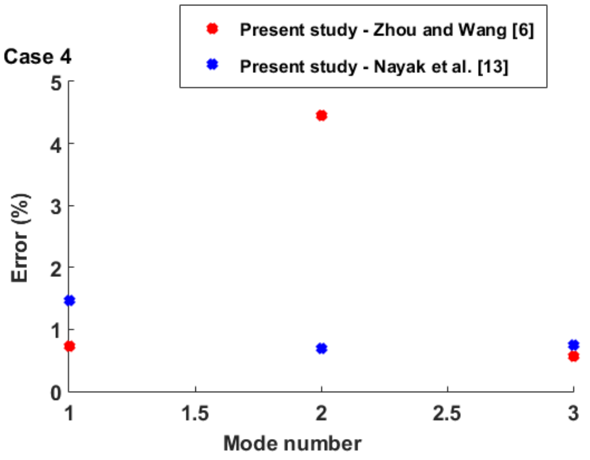

Figure 17. Error between different studies

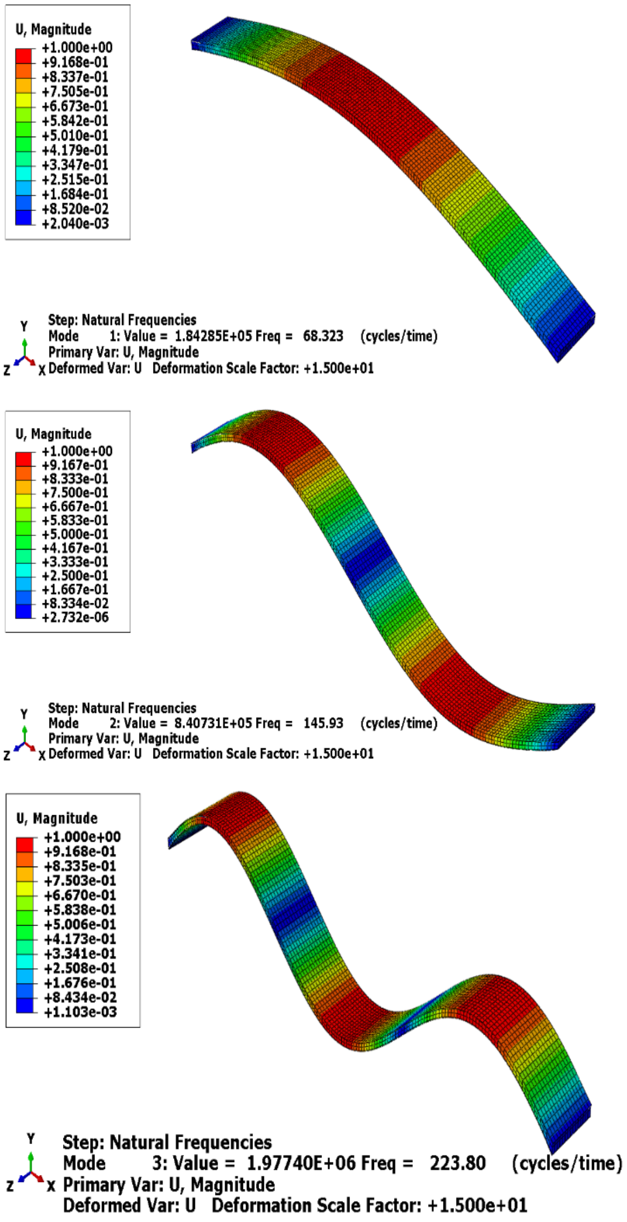

Figure 18. Eigenfrequencies and modes simulations for the fourth case study

The error between the present work and the works done by Zhou and Wang [6] and Nayak et al. [13] is shown in Figure 17. We note a good correlation between the results found where the maximum error does not exceed 4.46%.

The different vibration modes are given in Figure 18.

Figures 7, 10, 13, and 16 represent the vibration modes for the different cases. These figures show the influence of the geometry and properties of the viscoelastic material and of the MRE on the values of the natural frequencies and the deformation amplitude. This figure shows that viscoelastic materials have very high damping, especially in the case of a magnetorheological elastomer. This behavior can be attributed to the influence of the applied magnetic field on the magnetizable particles within the elastomer. When a magnetic field is applied, the magnetizable particles inside the MRE align with the field, leading to an increase in the stiffness of the material. Consequently, the MRE layer of the beam becomes stiffer, resulting in higher natural frequencies in the sandwich structure. Additionally, under the influence of the magnetic field, MRE exhibits reduced damping properties, resulting in lower loss factors.

This reduction in damping means that less energy is dissipated as heat during the vibration cycles, allowing the vibrational energy to be sustained in the system for a longer duration. On the other hand, the increased stiffness of the MRE due to the magnetic field contributes to a decrease in the loss factor.

From this study, it is concluded that beams with viscoelastic cores are widely used as reliable and cost-effective passive damping treatments for noise and vibration attenuation in many technical applications. Such elastomers, due to their fixed parameters, are known to be effective in a limited frequency range.

On the other hand, beams with a magneto-rheological elastomer core (MRE, class of smart composite materials), present reversible and rapid variations in their dynamic properties under the application of an external magnetic field. MREs consist of micro-sized ferromagnetic particles pre-arranged or suspended in an elastomer matrix. Due to their fast response, the elastic moduli (storage modulus, loss modulus) of these flexible smart composites can be efficiently controlled in near real-time in response to variable external excitations. MREs thus offer greater potential for active vibration suppression in a wider frequency range. In addition, MREs have additional desirable features, such as low power consumption and fail-safe character.

4. CONCLUSION

Numerical simulation of free vibrations of magnetorheological and viscoelastic beams was carried out. The effects of various boundary conditions and magnetic fields on the vibration properties such as natural frequency and deformation modes were studied in this research. From the results, the following inference has been made:

Boundary conditions have a significant impact on the variation of natural frequencies.

Since the skins significantly enhance the overall bending stiffness of the sandwich beams, the MRE core improves and makes this stiffness adjustable by the interaction force between the ferromagnetic particles developed by the magnetic field.

The vibration properties of an MRE beam change with the change in rigidity due to the application of the magnetic field. On the other hand, these properties remain constant in the case of a viscoelastic beam.

The rapid increase in the rigidity of MRE beams with the increase in the magnetic field intensity leads to a decrease in vibratory waves and a reduction in damping.

Due to their fast response, the elastic moduli of these flexible smart composites can be efficiently controlled in real time in response to varying external excitations. MREs thus offer greater potential for active vibration suppression in a wider frequency range. In addition, MREs exhibit additional desirable features, such as low power consumption and high safety character.

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