# Signal Time-Shifting Effects on DFT Spectra

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*Abstract:* Accurate frequency estimation is crucial for researchers and engineers engaged in audio processing, communication, and vibration analysis. In the case of short signals containing low-frequency components and a non-integer number of periods, conventional frequency estimation algorithms, such as DFT, prove inadequate in precisely identifying signal frequencies due to inherent limitations in frequency resolution and the presence of the leakage phenomenon. Interpolation methods offer the potential to enhance frequency estimation by discerning frequencies at intermediate positions within the spectrum. Precisely determining amplitude values across spectral bins is critical when employing such methods. Remarkably, while the initial phase of a signal can significantly alter the spectrum, its influence on amplitude distribution remains an aspect yet to be thoroughly researched. In this study, we use Python to implement the DFT algorithm and analyze how the initial phase affects the results. We are conducting tests using generated sinusoidal signals and signals processed with Hamming and Hanning windowing functions to determine the extent of changes in amplitude. The findings indicate that a signal's time shift can influence the spectrum significantly, resulting in inaccurate frequency estimations and erroneous conclusions in signal analysis.

*Keywords:* - signal time-shifting, DFT, short signal, non-integer number of periods, accurate frequency estimation

## 1. INTRODUCTION

A precise frequency estimation is crucial in numerous research and engineering applications, among which we exemplify in this paper vibration-based damage detection. Vibration-based damage detection has emerged as a non-destructive approach, utilizing a structure's dynamic response to identify potential defects [1]. The most common method implies analyzing the natural frequency changes of the structure, typically made by recording vibration signals and extracting the frequency components. The Discrete Fourier Transform (DFT) is crucial in this process as it converts signals from the time domain to the frequency domain. Early damage detection is challenging due to the small changes in natural frequency caused by damage [2].

Even if DFT is a powerful tool for converting discrete signals from the time domain into the frequency domain, it has inherent limitations. One of the key assumptions of DFT is that signals are periodic. A challenge that arises in DFT when dealing with short signals with low-frequency components that do not contain an integer number of periods or when the number of samples in the signal is not a whole multiple of the targeted frequency is spectral leakage. Spectral leakage occurs when energy is spread to neighboring discrete spectral components, affecting the precision of frequency detection. Thus, because of the discrete nature of the DFT, the spectrum results in discrete frequency lines, potentially missing the target frequency [3].

Various methods have been developed to overcome these limitations. Techniques such as zero-padding can be used to increase the signal length and reduce the distance between the spectral bins [4]. Another method is signal trimming, which ensures a signal length adjustment to reposition the spectral bins, bringing one of them closer to the actual frequency [5].

The most convenient methods involve using interpolation on the peak amplitude and its one or two neighboring values, as these methods are simple and allow for finding the frequency at an inter-bin location. A review of these methods is given in [6]. Regardless of the interpolation method used, whether it involves two points [7-9] or three points [10-12], accurate amplitudes are required. Additionally, work on precision-enhancing methods, such as PyFEST, has shown that spectral interpolation can improve DFT accuracy, especially in scenarios with short signals and low-frequency components [13].

A class of frequency estimation methods developed in recent years involves artificial intelligence [14-15]. These methods also use amplitudes to estimate the frequency components of the signal.

Previous studies have examined the influence of various factors on the accuracy of DFT in terms of frequency estimation. For instance, research on spectral leakage has demonstrated how incomplete periods and non-periodic signals can lead to inaccuracies in frequency component detection [16]. However, as far as we know, there are no studies regarding the effect of the initial phase on the amplitudes calculated by DFT, even though these seriously affect the precision of interpolation methods.

In our recent studies, when using methods based on artificial intelligence, we encountered problems accurately estimating frequencies when the initial phase was not zero. The inaccuracies occur because these methods involve spectral amplitudes, which depend on the initial phase of the signal. In the present study, we highlight the effect of the initial phase on the spectral amplitudes and determine the limits of variation. This contributes to a better understanding of the phenomenon studied and shows how a complex database with amplitudes of signals and different initial phases can be developed.

## 2. METHODOLOGY AND EXPERIMENT

We conducted the experiment using Python. The function *generateSignal* was implemented to generate sinusoidal signals containing desired frequency components, with respective amplitudes and phase shifts, for a given sampling rate. Function *dft* was implemented to calculate the DFT straightforwardly, looping through the signal with two nested loops to compute the spectrum and dividing the values in the resulting array with the number of samples. Functions *hann* and *hamm* were implemented to apply Hanning and Hamming windows to the signal.

An experiment was designed to generate phase-shifted sinusoids, apply the window functions, and calculate the DFT for raw and windowed signals.

Frequency components and signal length were selected to produce sinusoids with integer and non-integer periods, and the results are displayed in charts for comparison.

### 2.1. Simulation data

Using the *generateSignal* script, we generate signals with a sampling rate of 1,000 samples per second. The signal frequency is 10 Hz, and the amplitude is 1. We consider two signal lengths, one of 1 second for which we use 1000 samples, and the second of 1.03 seconds for which we use 1030 samples. As expected, the first signal has a whole number of periods, while the second signal contains a non-integer number of periods. The two signals are subject to phase shift increments of $\pi/10$ until the shift achieves $\pi$. As a result, we obtain 10 signals with an entire number of periods and 10 with a non-integer number of periods. For these 20 signals, we calculated the DFT and plotted their graphs.

Next, we applied Hamming and Hanning windowing functions to these 20 signals, obtaining 40 new functions to be analyzed. We calculated the DFT and represented the spectra in new graphs. By examining these graphs and the peak values, one can determine if and to what extent deviations in the spectra occur due to phase shifts.

### 2.2. Experiment results

We illustrate the 10 generated sinusoids with a whole number of periods in Figure 1.a, and the 10 generated sinusoids with a non-integer number of periods in Figure 1.b.

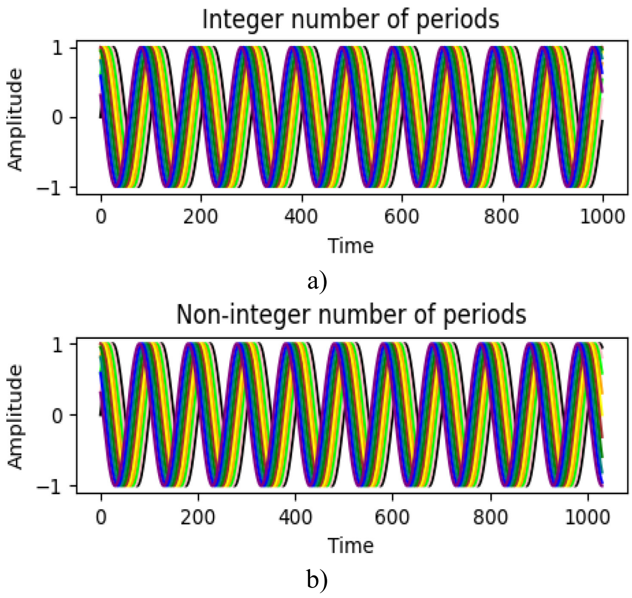


**Figure 1.** Generated sinusoidal signals: a) with 1000 samples, b) with 1030 samples.

The signals with Hanning windowing functions are shown in Figure 2, and those with Hamming windowing functions are shown in Figure 3.

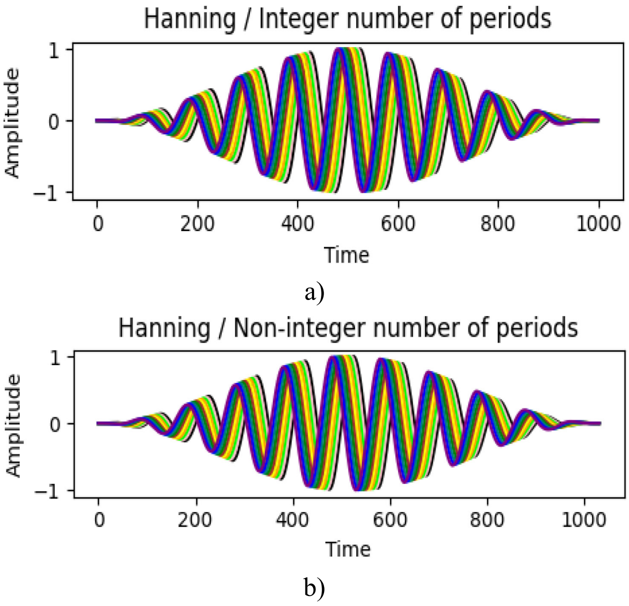


**Figure 2.** Generated signals with Hanning windows applied: a) with 1000 samples, b) with 1030 samples.

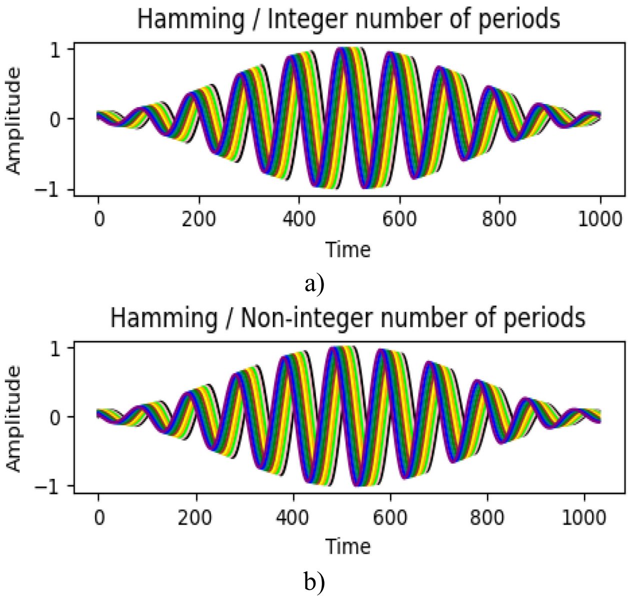


**Figure 3.** Generated signals with Hamming windows applied: a) with 1000 samples, b) with 1030 samples.

The DFT was applied to these signals, producing six collections with 20 spectra each. Figure 4.a shows the DFT spectra for the sinusoids with integer and non-integer number of periods, focusing on the 5-15 Hz part to examine any significant deviations around the target 10 Hz component. Figures 4.b and 4.c show the windowed signals' DFT spectra, again focusing on the 5-15 Hz frequency range.

Due to the distribution of spectral bins along the spectrum based on signal length, signals with a whole number of periods will always display a frequency of 10 Hz, and insignificant leakage occurs. On the other hand, for all signals with a non-integer number of periods, the frequency is displayed at 9.708 Hz, since the frequency resolution is 0.9708 Hz. Here, we can observe significant leakage.

At this zoom level, one can notice that the spectra for non-windowed signals get values close to the expected DFT amplitude of 500. Dissimilar, the windowed signals show a smaller amplitude, which is justified on the one hand by the energy loss due to windowing and, on the other hand, due to leakage.

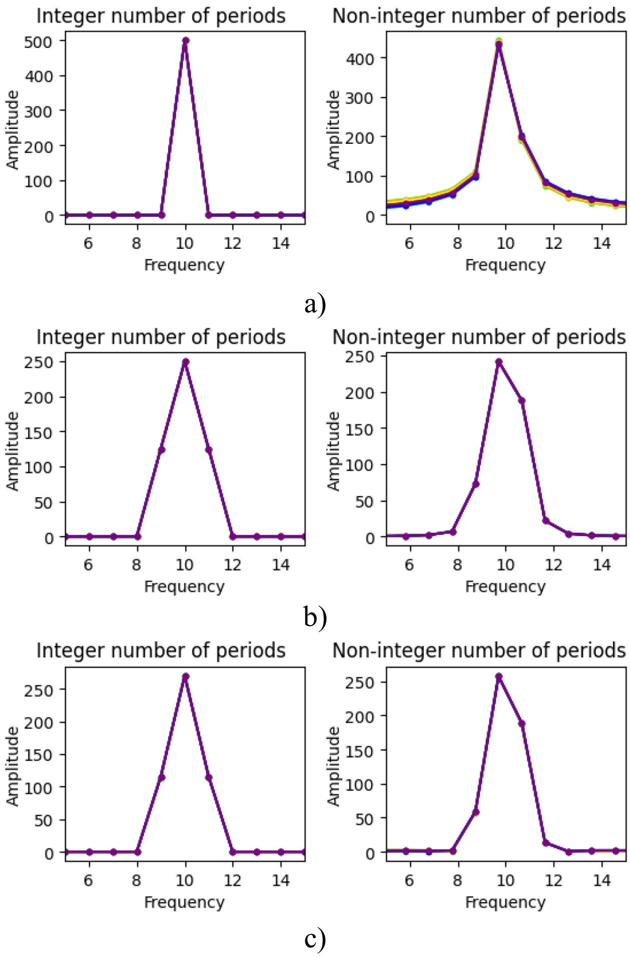


**Figure 4.** DFT spectra around 10 Hz: a) original signal, b) Hanning window, c) Hamming window

In Figure 5, we show a zoom around the peaks of the DFTs. Zooming in the charts reveals the minor spectra deviations for unwindowed time-shifted signals with an entire number of periods. All other signals present an alteration of the peak amplitude due to the initial phase. Table 1 lists all analyzed signals' maximum and minimum DFT amplitude values.

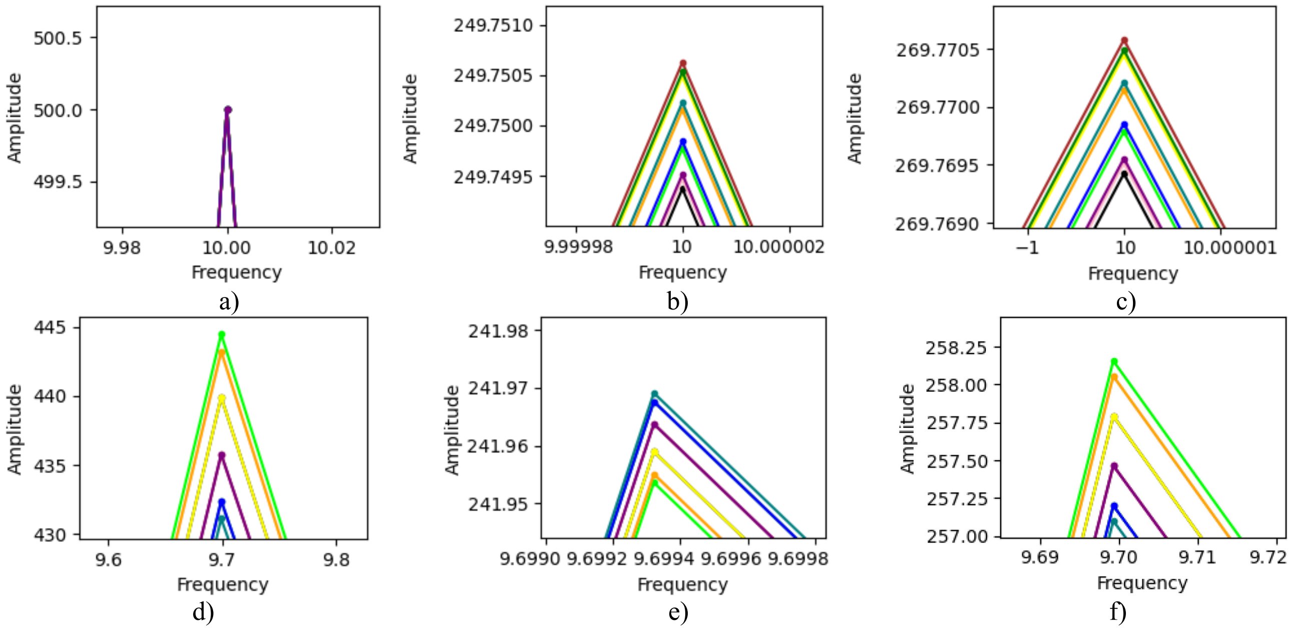


**Figure 5.** Zoom on the DFT peaks: a) integer periods, not windowed, b) integer periods, Hanning window, c) integer periods, Hanning window, d) non-integer periods, not windowed, e) non-integer periods, Hanning window, c) non-integer periods, Hanning window,

**Table 1.** Extreme values for the peak DFT-amplitudes of the analyzed signals

| No. of samples | Window applied | No. of periods | Frequency (Hz) | Values at closest bin to10 Hz | | | Difference to max ratio |
|---|---|---|---|---|---|---|---|
| | | | | min | max | Difference | |
| 1000 | None | integer | 10 | 500.000000 | 500.000000 | 0.000000 | 0.000000 |
| 1030 | None | non-integer | 9.708 | 435.548635 | 448.597372 | 13.048737 | 0.029088 |
| 1000 | Hann | integer | 10 | 249.749374 | 249.750626 | 0.001252 | 0.000005 |
| 1030 | Hann | non-integer | 9.708 | 242.681378 | 242.696525 | 0.015147 | 0.000062 |
| 1000 | Hamm | integer | 10 | 269.769424 | 269.770576 | 0.001152 | 0.000004 |
| 1030 | Hamm | non-integer | 9.708 | 258.124680 | 259.154644 | 1.029964 | 0.003974 |

## 3. DISCUSSION

Sinusoids with the same frequency but different time shifts produced different spectra. In the case of sinusoids with an integer number of periods, the observed difference between the extreme values is in the order of $10^{-12}$, and the actual DFT amplitude is correctly calculated. The frequencies are displayed in a spectral bin that fits the actual frequency.

For a sinusoid with a non-integer number of periods, the DFT amplitude decreases by more than 10%. The variation between the extreme values is also significant, namely 2.9%. Such variation induces uncertainty and leads to inaccurate frequency estimation in all methods based on analyzing the amplitudes in the spectrum. The DFT inaccurately calculates the frequencies for these sinusoids.

We obtain less amplitude variation by applying windowing functions to signals with an integer number of periods. It is 0.00125 or 0.005% when using the Hanning window and 0.00115 or 0.004% when using the Hamming window.

For the signals with non-integer numbers of periods, which contain 1030 samples, after applying Hanning and Hamming windowing functions, we reduced the deviation to 0.015147 or 0.06% and 1.029964 or 0.39%, respectively.

The slight amplitude variation obtained when windowing the signal is promising and may constitute an advantage that has to be proved in the following research approaches.

Even though the research focuses on a specific signal with a frequency of 10 Hz and an amplitude of 1, we obtain a general case by normalizing the DFT in terms of frequency and amplitude. This means that the conclusions drawn for the analyzed signal are of a general nature and apply to any sinusoidal signal.

## 4. CONCLUSIONS

The paper detailed an experiment aimed at determining the impact of time-shifting signals on the amplitudes within a DFT spectrum, particularly for signals comprising integer and non-integer numbers of periods. The study utilized a Python application to generate signals with specified length, amplitude, frequency, and initial phase for analysis. This study used raw sinusoids and sinusoids windowed with Hamming and Hanning functions. Because the chance to get signals with an integer number of periods is minimal when acquring real-life data, we focus in this section on commenting on signals with a non-integer number of periods. For the latter signals, we can conclude that:

- Windowing does not improve the accuracy in frequency estimation performed with standard methods (such as DFT);
- The phase shift produces changes in the amplitude distribution in the DFT spectrum and induces errors when advanced interpolation or AI-based methods are used for frequency estimation;
- The most extensive amplitude variation occurs for the raw sinusoids with a non-integer number of samples to which no windowing was applied;
- The slightest amplitude variation occurs for the signals with a non-integer number of samples to which a Hanning window was applied.

The actual research contributes to a better understanding of the effect of phase shift and highlights the capacity of the Hanning window to stabilize the amplitude values for signals with various lengths. Consequently, this window function is helpful in advanced frequency estimation techniques and contributes to a more accurate frequency estimation.

Future research can extend in several directions. One approach involves the development of mathematical models capable of forecasting the initial phase and adjusting amplitudes to align with a signal beginning with an initial phase of zero. Another approach involves creating a database using simulation results. This database can be utilized to estimate signal frequencies accurately through artificial intelligence, even for signals with an initial phase.

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