
Reliability Assessment of Centrifugal Pumps and State Water Supply Systems in Condition Monitoring and Diagnostics

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Abstract: - The study's primary goal is to explore various reliability measures that enhance the functionality of two systems: The Centrifugal Pump system, comprising the Impeller and Shaft, and the State Water Supply system, which has a single component. The Impeller is crucial in converting mechanical energy into kinetic energy, facilitating fluid movement with precision for optimal flow and system efficiency. Concurrently, the Shaft maintains alignment and stability while transmitting power from the motor to the Impeller. These systems function in parallel, remaining operational as long as at least one of them functions. They exhibit distinct failure and repair rates, with immediate access to repair servers in case of system failures. The study assumes exponential distributions for failure and repair times, with repairs being highly effective and all random variables being independent and uncorrelated. To evaluate reliability measures, the study employs the Regenerative Point Technique and a semi-Markov process. Understanding and assessing measures like Mean Time to System Failure (MTSF) and system availability empower engineers to optimize maintenance schedules, forecast system downtime, and use resources efficiently.

Keywords: - Parallel Systems; Reliability Analysis; State Water Supply System; Centrifugal Pumping System; Regenerative Point Technique (RPT); Semi-Markov Process; Reliability Analysis; Availability Modelling; Condition Monitoring.

1. INTRODUCTION

Agriculture and industries are integral components of India's economy, and water serves as a finite and essential resource for preserving livelihoods, manufacturing processes, power generation, crop processing, cleaning and sanitation, irrigation, and livestock. Recognizing the critical importance of water in both industry and agriculture, concerted efforts in research and innovation are required to develop and implement alternative reliable techniques for water supply. The current work provides a stochastic analysis of the water delivery system in which a model for a centrifugal pumping system connected in parallel with the state water supply system has been designed. The state government is responsible for sourcing raw water, treating it, and distributing drinkable water to homes, industries, and farms. However, to meet the enormous demand for water, alternative systems such as centrifugal pumps should be built

concurrently. Centrifugal pump systems effectively meet the demand for water by drawing, pressurizing, moving, and dispersing it to different areas in accordance with the needs of diverse applications and consumers. Therefore, to ensure a continuous water supply, it is important to analyze the reliability metrics of these systems. The shaft and impeller are two crucial components of a centrifugal pump. Any malfunction or failure in these components can lead to system downtime, decreased efficiency, and potentially costly repairs. Thus, understanding the functioning and ensuring the reliability of the impeller and shaft are paramount for maintaining the system's operational integrity. Several research papers have focused on the reliability of centrifugal pump systems. A. T. Funmilayo et al. [3] studied maintenance strategies of centrifugal pumps. O. A. Kolenchukov et al. [19] examined technological measures for centrifugal pump reliability for biomass processing products. M. V. Forteza et al. [18] proposed a design model for API centrifugal

pumps to predict the maximum expected time between failures. A. Olabisi et al. [25] examined reliability methodologies to determine centrifugal pumps' maintainability for petroleum products delivery.

In the current work, the centrifugal pump system that operates in parallel with the state water supply system is called a redundant system. Redundancy is a configuration in which extra units or components are added in parallel during the system-building process to enhance system effectiveness. Numerous researchers have created mathematical models to investigate availability, Mean Time to System Failure (MTSF), and reliability, aiming to comprehend the complexities of parallel-unit systems. Researchers focusing on the reliability metrics of parallel-connected identical units include Gitanjali et al. [12], Malik et al. [23], and Rathee et al. [21]. Alternatively, stochastic models for non-identical parallel units have been developed by Sweety et al. [24], Chopra et al. [11], Gitanjali et al. [13], and Sruthi et al. [15].

To broaden the focus, researchers have also investigated stochastic models of complex systems in which units are replaced by subsystems connected in parallel or series. Lado et al. [1], Taj et al. [26], Negi et al. [22], Singh et al. [27], Kumar et al. [8-9], and Jana et al. [7] developed models for repairable systems with subsystems connected in series, which are noteworthy contributions. Munjal et al. [2] and Kumar et al. [20] evaluated systems with parallel-connected subsystems, taking into account particular scenarios such as a k -out-of- n : G policy. Reliability analysis of systems with interacting Markov subsystems based on the k -out-of- n model with load-sharing components has been discussed by Wu et al. [6]. Sharifi et al. [17] proposed a matrix-based technique for load-sharing components to evaluate system inspection cost. Guo et al. [14] developed a reliability model for consecutive k -out-of- n : F systems with local load-sharing effect and shock processes. Nezakati et al. [10] developed a reliability model for the load-sharing k -out-of- n system and discussed the conditional distribution.

Other studies have investigated complex systems with subsystems and their reliability. Kumar et al. [4] and M. Saini et al. [16] studied systems such as the sludge digestion processing system and the load haul dump system using a Genetic Algorithm approach. A. Kumar et al. [5] conducted a RAMD study of the TIUP irrigation system, considering the impact of failure and repair rates on subsystems.

Unlike previous studies such as Wu et al. [6], Guo et al. [14], and Nezakati et al. [10], which analyzed reliability in k -out-of- n systems with load-sharing components, the reliability analysis of

centrifugal pumping systems operating in redundancy with a state water supply system remains relatively underexplored. Accordingly, the objective of this study is to develop a stochastic reliability framework for a centrifugal pumping system arranged in parallel with a state water supply system as shown in Figure 2. The study further aims to evaluate key reliability metrics such as Mean Time to System Failure (MTSF), availability, and profit.

STRUCTURE OF PAPER:

In this paper, Section 2: Methodology presents a flow chart (Figure 1) that provides a pictorial explanation of the approach used to analyze the proposed model. Section 3: Case study describes the specific application considered. Section 4: System description is explained in this section. Section 5: Analysis, notations, state description (as mentioned in Table 2), and calculations of transition probabilities, mean sojourn times, and various reliability measures are presented in this section. Section 6: Results obtained from all calculations and tables are discussed in this section. Section 7: This section demonstrates the conclusions drawn from the calculations and tables. Section 8: This section describes future scope and limitations of this work.

2. METHODOLOGY

This paper utilized a modeling approach to analyze a specific complex system outlined graphically in Figure 1

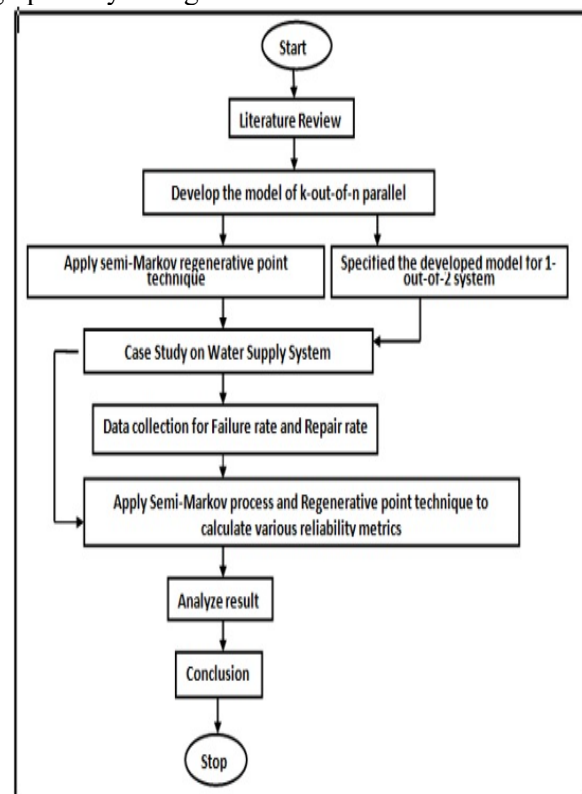


Figure 1. Methodology for estimating the reliability of a water supply system

- Conducted a comprehensive literature review on reliability models involving units, subsystems, and systems configured in parallel, series, series-parallel, and k-out-of-n configurations.
- Formulated a generalized k-out-of-n model, applied in the current study as a specific situation of a 1-out-of-2 configuration.
- Utilized the semi-Markov process-supported regenerative point technique.

Assumptions:

- The system is considered operational if at least one system remains operational.
- To ensure compatibility with the semi-Markov process and regenerative point technique, repair times are assumed to follow an exponential distribution.
- A constant repair rate allows closed-form evaluation of MTSF and availability.
- Upon failure, a single repair server is available to restore the failed unit, and the repair is assumed to be perfect.
- All failure and repair processes are assumed to be statistically independent.

Semi-Markov Process: A semi-Markov process is a continuous-time stochastic process in which transitions between states adhere to the transition probabilities of a Markov process. Additionally, the time interval between successive transitions is considered a random variable. The distribution of this random variable may be contingent upon both the current state from which the transition occurs and the subsequent state to which it leads. This concept is applied in various domains such as predicting traffic flows and communication networks.

Regenerative Point Technique: A regenerative process is a stochastic process characterized by time points at which, from probabilistic behavior, the process restarts itself. These time points may be determined by the evolution of the process. The process $(X(t), t \geq 0)$ is regenerative if there exists a sequence of time points $0 \leq T_0 < T_1 < T_2 \dots$ such that the post- T_k process $\{X(T_{0+t}): t \geq 0\}$ has the same distribution as the post-T process $\{X(t) : 0 \leq t < T_k\}$, and is independent of the pre- T_k process $\{X(t) : 0 \leq t < T_k\}$.

These epochs are called regenerative points. Smith (1955) introduced the regenerative stochastic process, which is crucial to the understanding of complex systems. To conduct an analysis of a state water supply system connected in parallel with a centrifugal pumping system, the present study

employed a semi-Markov process with the regenerative point technique.

3. CASE STUDY

The agricultural water supply system plays a vital role across various engineering sectors and households. The present study investigates the water supply system, comprising two parallel-connected systems: a centrifugal pumping system and the state water supply system as illustrated in Figure 2. For comprehensive reliability analysis, operational data on the failure rates and repair rates of the impeller and shaft have been gathered from literature sources, as cited in [3].

LAYOUTS OF WATER SUPPLY SYSTEM:

3.1 Centrifugal Pumping system (System V_1)

Centrifugal pumps are fluid transport devices widely used across various industries for pumping water, chemicals, sewage, oil, and other liquids. Among its components, the impeller and shaft are key components, each playing a crucial role in the pump's operation and performance. Thus, this study focuses on the stochastic analysis of these two crucial elements.

3.1 (a) Impeller (First unit of the system V_1)

The impeller function is to impart energy to the fluid by converting mechanical motion into fluid acceleration. As the impeller spins, it draws fluid into the center and then flings it outward at high velocity. The shape and arrangement of the impeller blades determine the pump's performance characteristics, such as flow rate and pressure.

3.1 (b) Shaft (Second unit of system V_1)

The shaft is a metallic component that transmits rotational energy from the motor to the impeller. It is designed to withstand the torque and bending forces generated during operation. The shaft passes through the center of the impeller and is connected to it. Moreover, the shaft is supported by bearings to ensure smooth rotation and minimize friction losses.

Together, the impeller and shaft form the core of the centrifugal pump, enabling it to efficiently transfer fluid from the inlet to the outlet by converting mechanical energy into fluid flow.

3.1.2 State Water Supply System (System V_2)

Governments hold primary responsibility for managing and allocating water resources to maintain an equitable, sustainable, and reliable water supply for agricultural and industrial activities. The state

water supply system starts with sourcing water from rivers, lakes, reservoirs, groundwater wells, or desalination plants. The intake system, comprising pipes, screens, and pumps, is used to collect raw water from these sources and transport it to the treatment plants. After treatment, water is stored in reservoirs, tanks, or elevated storage structures. Subsequently, a network of pipes and distribution infrastructure supplies water from storage facilities to consumers, including households, businesses, industries, and institutions.

4. SYSTEM DESCRIPTION:

The study analyzes a complex system comprising two non-identical systems: V_1 - Centrifugal pumping system and V_2 - the state water supply system, which are arranged in parallel. The centrifugal pumping system comprises two series-connected components: the impeller and the shaft, while the

state water supply system is a single unit as shown in Figure 2. The system operates if either of the two systems is operational but with less capacity. The system always has one server available. Upon failure, a single repairperson handles repairs. It is assumed that the repair time follows an exponential distribution and failure rates are constant.

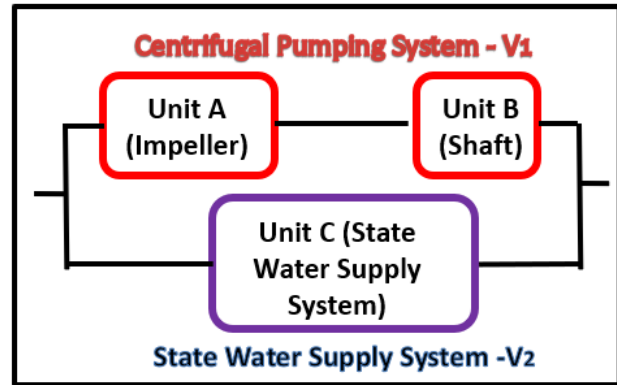


Figure 2. Block diagram of state water supply system

5. ANALYSIS

5.1 NOTATIONS:

Table 1. Notations description

NOTATIO NS:	Notations Description
V_1/V_2	Centrifugal Pumping system/State Water Supply system.
Unit-A/Unit-B/ Unit-C	Impeller/ Shaft/ State Water Supply System.
V_1O/V_2O	Centrifugal Pumping system/State Water Supply system is in operational state.
$\bar{V}_1AUR/\bar{V}_1BUR$	Centrifugal Pumping is under repair due to the failure of Unit A/Unit B.
$\bar{V}_2CUR$	State Water Supply system is under repair due to the failure of Unit C.
$\bar{V}_1AUR/\bar{V}_1BUR$	The centrifugal pumping system is under continuous repair from the previous state due to failure of unit A/unit B.
$\bar{V}_2CUR$	State Water Supply system is under continuous repair from the previous state due to failure of unit C.
$\bar{V}_2CWR$	State Water Supply system is waiting for repair due to failure of Unit C.
λ_1/λ_2	Constant failure rate of Impeller/ Shaft of the system V_1 .
λ_3	Constant failure rate of State Water Supply System V_2 .
α_1/α_2	Constant repair rate of Impeller/ shaft of system V_1 .
α_3	Constant repair rate of the State Water Supply System V_2 .
$g_1(t)/G_1(t)$	p.d.f/c.d.f. of the repair time for the Impeller of the system V_1 .
$g_2(t)/G_2(t)$	p.d.f/c.d.f. of the repair time for the Shaft of the system V_1 .
$g_3(t)/G_3(t)$	p.d.f/c.d.f. of the repair time of State Water Supply System V_2 .
$q_{ij}(t)/Q_{ij}(t)$	p.d.f. /c.d.f. of the transition period from regenerative state i to state j or to a non-functioning state j without passing any other regenerative state in $(0, t]$.
m_{ij}	Contribution to mean sojourn time μ_i in state S_i when the system transits straight to state S_i so that $\mu_i = \sum_j m_{ij}$ where $m_{ij} = \int t dQ_{ij}(t) = -q_{ij}^*(0)$
μ_i	Mean sojourn time in state S_i , which is given by $\mu_i = E(T_i) = \int_0^\infty P(T_i)dt = \sum_j m_{ij}$ where T_i denotes the time to failure.
**/*	Symbol for Laplace stieltjes transform/ Laplace transform.
$\otimes/\otimes$	Symbol for Stieltjes convolution/Laplace convolution.

5.2 DESCRIPTION OF STATES:

Table 2. State descriptions

STATE	STATE DESCRIPTION
s_0	An operative state where the Centrifugal Pumping system V_1 and State Water Supply system V_2 are operational
s_1	An operative state where Centrifugal Pumping V_1 is not operating due to failure of the impeller and is under repair, while V_2 is operative
s_2	An operative state where Centrifugal Pumping V_1 due to failure of the shaft is not operating and is under repair while V_2 is operative
s_3	An operative state where the State Water Supply system V_2 is not operating due to failure of unit C while V_1 is operative
s_4	A failed state where the State Water Supply system V_2 is not operating due to failure of unit C and is waiting to be repaired while V_1 is under repair continuously since an earlier state
s_5	A failed state where the State Water Supply system V_2 is not operating due to failure of unit C and is waiting to be repaired while V_1 is under repair continuously from the earlier state
s_6	A failed state where the centrifugal pumping system V_1 is not operating due to failure of the impeller and is waiting to be repaired while V_2 is under repair continuously from the earlier state
s_7	A failed state where the centrifugal pumping system V_1 is not operating due to failure of the shaft and is waiting to be repaired while V_2 is under repair continuously from the earlier state

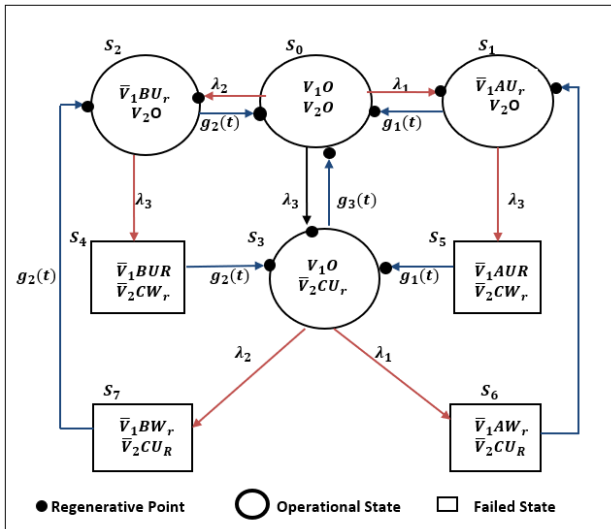


Figure 3. State transition model for water supply system.

5.3 STEADY STATE TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

Using Figure 3, we evaluate the steady-state probabilities p_{ij} where $i = 0, 1, 2, 3, j = 0, 1, 2, 3, 4, 5$ for the water supply system, transitioning from s_i to state s_j . These probabilities for various states as mentioned in Table 2, are computed as follows:

$$p_{ij} = Q_{ij}(\infty) = \int_0^{\infty} q_{ij}(t) dt \quad (1)$$

The rate of change of these probabilities is given by:

$$dQ_{ij}(t) = q_{ij}(t) dt. \quad (2)$$

Specifically, for $i=0$ and $j=1$, we have:

$$dQ_{01}(t) = q_{01}(t) dt = \lambda_1 e^{-(\lambda_1 + \lambda_2 + \lambda_3)t} dt \quad (3)$$

where λ_i and α_i , $i=1, 2, 3$ represent the failure rates and repair rates, respectively, as shown in Table 1. Taking the Laplace-Stieltjes transform (L.S.T.) of the above equation, we derive:

$$Q_{01}^{**}(s) = \int_0^{\infty} e^{-st} d[Q_{01}(t)] = \lambda_1 \int_0^{\infty} e^{-(\lambda_1 + \lambda_2 + \lambda_3 + s)t} dt = \frac{\lambda_1}{\lambda_1 + \lambda_2 + \lambda_3 + s} \quad (4)$$

Now, as $s \rightarrow 0$, we get the steady-state probability

$$p_{01} = \frac{\lambda_1}{\lambda_1 + \lambda_2 + \lambda_3} \quad (5)$$

Similarly, we can derive the following steady state transition probabilities:

$$p_{02} = \frac{\lambda_2}{\lambda_1 + \lambda_2 + \lambda_3}, p_{03} = \frac{\lambda_3}{\lambda_1 + \lambda_2 + \lambda_3} \quad (6)$$

$$p_{10} = \frac{\alpha_1}{\alpha_1 + \lambda_3}, p_{15} = \frac{\lambda_3}{\alpha_1 + \lambda_3} \quad (7)$$

$$p_{20} = \frac{\alpha_2}{\alpha_2 + \lambda_3}, p_{24} = \frac{\lambda_3}{\alpha_2 + \lambda_3} \quad (8)$$

$$p_{30} = \frac{\alpha_3}{\alpha_3 + \lambda_1 + \lambda_2}, p_{36} = \frac{\lambda_1}{\alpha_3 + \lambda_1 + \lambda_2}, p_{37} = \frac{\lambda_2}{\alpha_3 + \lambda_1 + \lambda_2} \quad (9)$$

$$p_{43} = p_{53} = p_{61} = p_{72} = 1. \quad (10)$$

This implies that:

$$\begin{aligned} p_{01} + p_{02} + p_{03} &= p_{10} + p_{15} = p_{20} + p_{24} = \\ p_{30} + p_{36} + p_{37} &= p_{10} + p_{1,3:5} = p_{20} + p_{2,3:4} = \\ p_{30} + p_{3,1:6} + p_{3,2:7} &= 1. \end{aligned} \quad (11)$$

This can be shown as:

$$\frac{\lambda_1}{\lambda_1 + \lambda_2 + \lambda_3} + \frac{\lambda_2}{\lambda_1 + \lambda_2 + \lambda_3} + \frac{\lambda_3}{\lambda_1 + \lambda_2 + \lambda_3} = 1 \quad (12)$$

$$p_{10} + p_{15} = \frac{\alpha_1}{\alpha_1 + \lambda_3} + \frac{\lambda_3}{\alpha_1 + \lambda_3} = 1 \quad (13)$$

$$p_{20} + p_{24} = \frac{\alpha_2}{\alpha_2 + \lambda_3} + \frac{\lambda_3}{\alpha_2 + \lambda_3} = 1 \quad (14)$$

$$p_{30} + p_{36} + p_{37} = \frac{\alpha_3}{\alpha_3 + \lambda_1 + \lambda_2} + \frac{\lambda_1}{\alpha_3 + \lambda_1 + \lambda_2} + \frac{\lambda_2}{\alpha_3 + \lambda_1 + \lambda_2} = 1 \quad (15)$$

Mean sojourn time in state S_i is given by:

$$\mu_i = E(T_i) = \int_0^{\infty} P(T_i) dt = \sum_j m_{ij} \quad (16)$$

where T_i denotes the time to failure.

$$\mu_0 = m_{01} + m_{02} + m_{03} = \frac{1}{\lambda_1 + \lambda_2 + \lambda_3}. \quad (17)$$

$$\mu_1 = m_{10} + m_{15} = \frac{1}{\alpha_1 + \lambda_3}. \quad (18)$$

$$\mu_2 = m_{20} + m_{24} = \frac{1}{\alpha_2 + \lambda_3}. \quad (19)$$

$$\mu_3 = m_{30} + m_{36} + m_{37} = \frac{1}{\alpha_1 + \lambda_1 + \lambda_2}. \quad (20)$$

$$\mu'_1 = \mu_3 = m_{10} + m_{1,3:5} = \frac{1}{\alpha_1}. \quad (21)$$

$$\mu'_2 = \mu_3 = m_{10} + m_{1,3:5} = \frac{1}{\alpha_1}. \quad (22)$$

$$\mu'_3 = m_{30} + m_{3,1:6} + m_{3,2:7} = \frac{1}{\alpha_3 + \lambda_1 + \lambda_2}. \quad (23)$$

5.4 MTSF

Let $\phi_i(t)$ represent the c.d.f. of the initial time interval between the regeneration state S_i , as mentioned in Figure 3, and any failure state. We discover the recurring relationships listed below.

$$\phi_0(t) = Q_{01}(t) \otimes \phi_0(t) + Q_{02}(t) \otimes \phi_2(t) + Q_{03}(t) \otimes \phi_3(t). \quad (24)$$

$$\phi_1(t) = Q_{10}(t) \otimes \phi_0(t) + Q_{15}(t). \quad (25)$$

$$\phi_2(t) = Q_{20}(t) \otimes \phi_0(t) + Q_{24}(t). \quad (26)$$

$$\phi_3(t) = Q_{30}(t) \otimes \phi_0(t) + Q_{36}(t) + Q_{37}(t). \quad (27)$$

Using the L.S.T. (as mentioned in Table 1) of the aforementioned equations, find $\phi_0^{**}(s)$, MTSF is given by:

$$\text{MTSF} = \lim_{s \rightarrow 0} \frac{1 - \phi_0^{**}(s)}{s} = \frac{N_0}{D_0}. \quad (28)$$

$$N_0 = \mu_0 + p_{01}\mu_1 + p_{02}\mu_2 + p_{03}\mu_3. \quad (29)$$

$$D_0 = 1 - p_{01}p_{10} - p_{02}p_{20} - p_{03}p_{30}. \quad (30)$$

5.5 AVAILABILITY

Let $A_i(t)$ be the probability that the system is in upstate at instant t , given that the system entered the regenerative state S_i as shown in Figure 3 at time $t = 0$. The recurrence relations for $A_i(t)$ are given as:

$$A_0(t) = M_0(t) + q_{01}(t) \otimes A_1(t) + q_{02}(t) \otimes A_2(t) + q_{03}(t) \otimes A_3(t). \quad (31)$$

$$A_1(t) = M_1(t) + q_{10}(t) \otimes A_0(t) + q_{1,3:5}(t) \otimes A_3(t). \quad (32)$$

$$A_2(t) = M_2(t) + q_{20}(t) \otimes A_0(t) + q_{2,3:4}(t) \otimes A_3(t). \quad (33)$$

$$A_3(t) = M_3(t) + q_{30}(t) \otimes A_0(t) + q_{3,1:6}(t) \otimes A_1(t) + q_{3,2:7}(t) \otimes A_2(t). \quad (34)$$

where,

$$M_0(t) = e^{-(\lambda_1 + \lambda_2 + \lambda_3)t}, M_1(t) = e^{-(\lambda_3)t} \overline{G_1(t)}, M_2(t) = e^{-(\lambda_3)t} \overline{G_2(t)}. \quad (35)$$

Taking the Laplace transform of the above equations and using Cramer's rule for $A_0^*(s)$.

The system's availability in steady state is as follows:

$$A = \lim_{t \rightarrow \infty} A_i(t) = \lim_{s \rightarrow 0} s A_0^*(s) = \frac{N_A}{D_A}. \quad (36)$$

where,

$$N_A = \mu_0(1 - q_{2,3:4}^* q_{3,2:7}^* - q_{1,3:5}^* q_{3,1:6}^*) + \mu_1(q_{01}^*(1 + q_{2,3:4}^* q_{3,2:7}^*) + q_{3,1:6}^*(q_{02}^* q_{2,3:4}^* + q_{03}^*)) + \mu_2((1 - q_{1,3:5}^* q_{3,1:6}^*) + q_{3,2:7}^*(q_{1,3:5}^* q_{3,1:6}^*)) + \mu_3(q_{1,3:5}^*(1 + q_{2,3:4}^* q_{3,2:7}^*) + (q_{02}^* + q_{03}^*)) \quad (37)$$

$$D_A = \mu_0(q_{10}^*(1 - q_{2,3:4}^* q_{3,2:7}^*) + q_{1,3:5}^*(q_{20}^* q_{3,2:7}^* + q_{30}^*)) + \mu'_1(q_{01}^*(1 - q_{2,3:4}^* q_{3,2:7}^*) + q_{3,1:6}^*(q_{02}^* q_{2,3:4}^* + q_{03}^*)) + \mu'_2(1 - q_{1,3:5}^* q_{3,1:6}^* + q_{3,2:7}^*(q_{03}^* + q_{1,3:5}^* q_{01}^*)) + \mu'_3(q_{1,3:5}^* q_{01}^* + q_{02}^* q_{2,3:4}^* + q_{03}^*) \quad (38)$$

where

$$\overline{G_i(t)} = 1 - G_i(t), i = 1, 2, 3. \quad (39)$$

$\mu_0, \mu_1, \mu_2, \mu_3, \mu'_1, \mu'_2$, and μ'_3 are already defined in equations (17-23).

5.6 SERVER BUSY PERIOD DUE TO REPAIR:

Let $B_i(t)$ represents the likelihood that the server is engaged in repair at an instant t given that the system reached the regenerative state S_i as shown in Figure 3 at $t=0$. $B_i(t)$ recursive relations are as follows:

$$B_0(t) = q_{01}(t) \otimes B_1(t) + q_{02}(t) \otimes B_2(t) + q_{03}(t) \otimes B_3(t) \quad (40)$$

$$B_1(t) = W_1(t) + q_{10}(t) \otimes B_0(t) + q_{1,3:5}(t) \otimes B_3 \quad (41)$$

$$B_2(t) = W_2(t) + q_{20}(t) \otimes B_0(t) + q_{2,3:4}(t) \otimes B_3(t) \quad (42)$$

$$B_3(t) = W_3(t) + q_{30}(t) \otimes B_0(t) + q_{3,2:7}(t) \otimes B_2(t) + q_{3,1:6}(t) \otimes B_1(t) \quad (43)$$

where,

$$W_1(t) = (e^{-\lambda_3 t} + \lambda_3 e^{-\lambda_3 t} \otimes 1) \overline{G_1(t)} \quad (44)$$

$$W_2(t) = (e^{-\lambda_3 t} + \lambda_3 e^{-\lambda_3 t} \otimes 1) \overline{G_2(t)} \quad (45)$$

$$W_3(t) = (e^{-(\lambda_1 + \lambda_2)t} + (\lambda_1 + \lambda_2) e^{-(\lambda_1 + \lambda_2)t} \otimes 1) \overline{G_3(t)} \quad (46)$$

$$\overline{G_i(t)} = 1 - G_i(t), i = 1, 2, 3. \quad (47)$$

Making use of Cramer's rule, solve for $B_0^*(s)$ (Table 1) by using the Laplace transform of the aforementioned set of equations. The period of time the server will be occupied while being repaired in a steady state is indicated by:

$$B_0(\infty) = \lim_{s \rightarrow 0} s B_0^*(s) = \frac{N_B}{D_A} \quad (48)$$

$$N_B = W_1^*(q_{01}^* - q_{01}^* q_{24}^* q_{37}^* + q_{02}^* q_{24}^* q_{36}^* + q_{03}^* q_{36}^*) + W_2^*(q_{02}^* - q_{02}^* q_{15}^* q_{36}^* + q_{01}^* q_{15}^* q_{37}^* + q_{03}^* q_{37}^*) + W_3^*(q_{01}^* q_{15}^* + q_{02}^* q_{24}^* + q_{03}^*), \quad (49)$$

where D_A is already established.

5.7 EXPECTED NUMBERS OF REPAIRS:

Given that the system reached the regenerative state Si as shown in Figure 3 at $t=0$, let $R_i(t)$ represent the anticipated number of subsystem repairs by the server in $(0, t]$.

The following provide the recurrence relations:

$$R_0(t) = Q_{01}(t) \otimes [R_1 + 1] + Q_{02} \otimes [R_2 + 1] + Q_{03} \otimes [R_3 + 1]. \quad (50)$$

$$R_1(t) = Q_{1,3:5}(t) \otimes R_3(t) + Q_{10}(t) \otimes R_0(t). \quad (51)$$

$$R_2(t) = Q_{2,3:4}(t) \otimes R_3(t) + Q_{20}(t) \otimes R_0(t). \quad (52)$$

$$R_3(t) = Q_{3,1:6}(t) \otimes R_1(t) + Q_{3,1:7}(t) \otimes R_2(t) + Q_{30}(t) \otimes R_0(t) \quad (53)$$

Making use of Cramer's rule, solve for $R_0^{**}(s)$ by using the L.S.T. (Table 1) of the aforementioned set of equations. Assuming that the server is in steady state, the anticipated number of visits per unit of time is given by:

$$R_0^{**}(\infty) = \frac{N_R}{D_A} \quad (54)$$

where

$$N_R = 1 - Q_{1,3:5}^* Q_{3,1:6}^* - Q_{2,3:4}^* Q_{3,2:7}^* \quad (55)$$

and D_A is already defined.

5.8 REVENUE ANALYSIS:

In steady state, the system model's profit can be calculated as:

$$P = R A_0 - C_0 B_0 - C_1 R_0. \quad (56)$$

where,

P is the profit of the system model.

R is the revenue per unit time of the system.

C_0 is the cost per unit for which the server is busy.

C_1 is the cost per unit for repair.

A_0 is the availability period of the system.

R_0 is the repair period of the system.

B_0 is the busy period analysis of the system.

For numerical illustration, the following values were adopted:

$R = 1000$ INR per hour

$C_0 = 100$ INR per hour

$C_1 = 200$ INR per hour

5.9 NUMERICAL REPRESENTATIONS OF DIFFERENT RELIABILITY MEASURES:

The study conducted numerical computations of reliability metrics, including Mean Time to System Failure (MTSF), availability, and the profit function. These results are crucial for assessment of system performance under varying repair rates across its components.

In the literature review in [3], we find an assessment of maintenance strategies driven by component-level failure and repair rates specific to the shaft and impeller. These rates are specified as 0.0277 hours for the repair rate and 1.27 E-04 hours for the failure rate. The utilization of this data in the current research enables evaluation of how these rates influence the system's reliability measures.

Table 3. Effect of repair rate of unit a (impeller) on MTSF.

α_1	$\alpha_2 = 0.0277$ $\alpha_3 = 0.3$ $\lambda_1 = 1.27E-04$ $\lambda_2 = 1.27E-04$ $\lambda_3 = 0.015$	$\alpha_2 = 0.0279$ $\alpha_3 = 0.3$ $\lambda_1 = 1.27E-04$ $\lambda_2 = 1.27E-04$ $\lambda_3 = 0.015$	$\alpha_2 = 0.0277$ $\alpha_3 = 0.32$ $\lambda_1 = 1.27E-04$ $\lambda_2 = 1.27E-04$ $\lambda_3 = 0.0153$	$\alpha_2 = 0.0277$ $\alpha_3 = 0.3$ $\lambda_1 = 1.29E-04$ $\lambda_2 = 1.27E-04$ $\lambda_3 = 0.015$	$\alpha_2 = 0.0277$ $\alpha_3 = 0.3$ $\lambda_1 = 1.27E-04$ $\lambda_2 = 1.29E-04$ $\lambda_3 = 0.015$	$\alpha_2 = 0.0277$ $\alpha_3 = 0.3$ $\lambda_1 = 1.27E-04$ $\lambda_2 = 1.27E-04$ $\lambda_3 = 0.017$
0.0 277	10361	10382	10411	10280	10280	9572
0.0 279	10382	10403	10432	10301	10301	9590
0.0 281	10403	10424	10453	10322	10321	9609
0.0 283	10423	10445	10474	10343	10342	9627
0.0 285	10444	10465	10495	10364	10362	9645
0.0 287	10465	10486	10516	10384	10383	9663
0.0 289	10485	10507	10537	10405	10403	9681
0.0 291	10506	10527	10558	10425	10423	9699

On another front, the data for failure and repair rates were obtained from a northern Indian industry. The specific industry cannot be disclosed due to confidentiality agreements; the identity of the

organization is not disclosed. In our analysis, we adopt a more general approach, setting these rates as negative exponentials, denoted as $g_1(t) = \alpha_1 e^{-\alpha_1 t}$, $g_2(t) = \alpha_2 e^{-\alpha_2 t}$, $g_3(t) = \alpha_3 e^{-\alpha_3 t}$ where $g_i(t)$ are p.d.f. of repair times and α_i repair rates for $i = 1, 2, 3$. These values are detailed in Table 1 for the respective components, allowing us to integrate them into our reliability assessment framework.

Table 4. Effect of repair rate of unit c (state water supply system) on MTSF.

α_3	$\alpha_1 = 0.02$ $\alpha_2 = 0.02$ $\lambda_1 = 1.27$ - 04 $\lambda_2 = 1.27$ - 04 $\lambda_3 = 0.01$	$\alpha_1 = 0.0279$ $\alpha_2 = 0.0277$ $\lambda_1 = 1.27E$ - 04 $\lambda_2 = 1.27E$ - 04 $\lambda_3 = 0.015$	$\alpha_1 = 0.0277$ $\alpha_2 = 0.0279$ $\lambda_1 = 1.27E$ - 04 $\lambda_2 = 1.27E$ - 04 $\lambda_3 = 0.015$	$\alpha_1 = 0.0277$ $\alpha_2 = 0.0277$ $\lambda_1 = 1.29E$ - 04 $\lambda_2 = 1.29E$ - 04 $\lambda_3 = 0.015$	$\alpha_1 = 0.0277$ $\alpha_2 = 0.0277$ $\lambda_1 = 1.27E$ - 04 $\lambda_2 = 1.29E$ - 04 $\lambda_3 = 0.015$	$\alpha_1 = 0.0277$ $\alpha_2 = 0.0277$ $\lambda_1 = 1.27E$ - 04 $\lambda_2 = 1.27E$ - 04 $\lambda_3 = 0.017$
0.3	10361	10382	10382	10280	10280	9572
0.32	10411	10432	10432	10330	10330	9618
0.34	10456	10477	10477	10375	10375	9659
0.36	10497	10518	10518	10415	10415	9696
0.38	10533	10555	10555	10451	10451	9730
0.4	10567	10589	10589	10485	10485	9760
0.42	10597	10620	10620	10515	10515	9788
0.44	10625	10648	10648	10543	10543	9814

Table 5. Effect of repair rate of unit a (impeller) on availability.

α_1	$\alpha_2 = 0.0277$ $\alpha_3 = 0.3$ $\lambda_1 = 1.27E$ - 04 $\lambda_2 = 1.27E$ - 04 $\lambda_3 = 0.015$	$\alpha_2 = 0.0279$ $\alpha_3 = 0.3$ $\lambda_1 = 1.27E$ - 04 $\lambda_2 = 1.27E$ - 04 $\lambda_3 = 0.015$	$\alpha_2 = 0.0277$ $\alpha_3 = 0.32$ $\lambda_1 = 1.27E$ - 04 $\lambda_2 = 1.27E$ - 04 $\lambda_3 = 0.01532$	$\alpha_2 = 0.0277$ $\alpha_3 = 0.3$ $\lambda_1 = 1.27E$ - 04 $\lambda_2 = 1.29E$ - 04 $\lambda_3 = 0.015$	$\alpha_2 = 0.0277$ $\alpha_3 = 0.3$ $\lambda_1 = 1.27E$ - 04 $\lambda_2 = 1.27E$ - 04 $\lambda_3 = 0.015$	$\alpha_2 = 0.0277$ $\alpha_3 = 0.3$ $\lambda_1 = 1.27E$ - 04 $\lambda_2 = 1.27E$ - 04 $\lambda_3 = 0.017$
0.0 277	0.996768	0.996787	0.9967735	0.996743	0.9967436	0.9965001
0.0 279	0.996787	0.996806	0.9967922	0.996762	0.9967623	0.9965200
0.0 281	0.996805	0.996824	0.9968106	0.996781	0.9967807	0.9965396
0.0 283	0.996824	0.996842	0.9968287	0.996799	0.9967988	0.9965588
0.0 285	0.996841	0.996860	0.9968464	0.996817	0.9968165	0.9965777
0.0 287	0.996859	0.996877	0.9968639	0.996835	0.9968340	0.9965962
0.0 289	0.996876	0.996895	0.9968810	0.996852	0.9968511	0.9966145
0.0 291	0.996893	0.996911	0.9968978	0.996869	0.9968679	0.9966324

Table 6. Effect of repair rate of unit c (state water supply system) on availability.

α_3	$\alpha_1 = 0.0277$ $\alpha_2 = 0.0277$ $\lambda_1 = 1.27E$ - 04 $\lambda_2 = 1.27E$ - 04 $\lambda_3 = 0.015$	$\alpha_1 = 0.0279$ $\alpha_2 = 0.0277$ $\lambda_1 = 1.27E$ - 04 $\lambda_2 = 1.27E$ - 04 $\lambda_3 = 0.015$	$\alpha_1 = 0.0277$ $\alpha_2 = 0.0279$ $\lambda_1 = 1.27E$ - 04 $\lambda_2 = 1.27E$ - 04 $\lambda_3 = 0.015$	$\alpha_1 = 0.0277$ $\alpha_2 = 0.0277$ $\lambda_1 = 1.29E$ - 04 $\lambda_2 = 1.29E$ - 04 $\lambda_3 = 0.015$	$\alpha_1 = 0.0277$ $\alpha_2 = 0.0277$ $\lambda_1 = 1.27E$ - 04 $\lambda_2 = 1.29E$ - 04 $\lambda_3 = 0.015$	$\alpha_1 = 0.0277$ $\alpha_2 = 0.0277$ $\lambda_1 = 1.27E$ - 04 $\lambda_2 = 1.27E$ - 04 $\lambda_3 = 0.017$
0.3	0.99676	0.99678	0.99678	0.99674	0.99674	0.99650
0.32	0.99677	0.99679	0.99679	0.99674	0.99674	0.99650
0.34	0.99677	0.99679	0.99679	0.99675	0.99675	0.99650
0.36	0.99678	0.99679	0.99679	0.99675	0.99675	0.99651
0.38	0.99678	0.99680	0.99680	0.99675	0.99675	0.99651
0.4	0.99678	0.99680	0.99680	0.99676	0.99676	0.99651
0.42	0.99678	0.99680	0.99680	0.99676	0.99676	0.99652
0.44	0.99678	0.99680	0.99680	0.99676	0.99676	0.99652

Table 7. Effect of repair rate of unit a (impeller) on profit.

α_1	$\alpha_2 = 0.0277$ $\alpha_3 = 0.3$ $\lambda_1 = 1.27E$ - 04 $\lambda_2 = 1.27E$ - 04 $\lambda_3 = 0.015$	$\alpha_2 = 0.0279$ $\alpha_3 = 0.3$ $\lambda_1 = 1.27E$ - 04 $\lambda_2 = 1.27E$ - 04 $\lambda_3 = 0.015$	$\alpha_2 = 0.0277$ $\alpha_3 = 0.32$ $\lambda_1 = 1.27E$ - 04 $\lambda_2 = 1.27E$ - 04 $\lambda_3 = 0.01532$	$\alpha_2 = 0.0277$ $\alpha_3 = 0.3$ $\lambda_1 = 1.29E$ - 04 $\lambda_2 = 1.27E$ - 04 $\lambda_3 = 0.015$	$\alpha_2 = 0.0277$ $\alpha_3 = 0.3$ $\lambda_1 = 1.27E$ - 04 $\lambda_2 = 1.29E$ - 04 $\lambda_3 = 0.015$	$\alpha_2 = 0.0277$ $\alpha_3 = 0.3$ $\lambda_1 = 1.27E$ - 04 $\lambda_2 = 1.27E$ - 04 $\lambda_3 = 0.017$
0.0 277	423.881	424.012	444.655	418.534	418.534	426.083
0.0 279	424.012	424.143	437.765	418.667	423.680	426.197
0.0 281	424.142	424.273	437.894	418.799	423.809	426.310
0.0 283	424.270	424.401	438.021	418.929	423.938	426.422
0.0 285	424.397	424.528	438.147	419.058	424.064	426.533
0.0 287	424.522	424.653	438.271	419.185	424.190	426.642
0.0 289	424.646	424.777	438.395	419.311	424.314	426.750
0.0 291	424.769	424.900	438.516	419.436	424.436	426.857

Table 8. Effect of repair rate of unit c (state water supply system) on profit.

α_3	α_1 = 0.0277 α_2 = 0.0277 λ_1 = 1.27E - 04 λ_2 = 1.27E - 04 λ_3 = 0.015	α_1 = 0.0279 α_2 = 0.0277 λ_1 = 1.27E - 04 λ_2 = 1.27E - 04 λ_3 = 0.015	α_1 = 0.0277 α_2 = 0.0279 λ_1 = 1.27E - 04 λ_2 = 1.27E - 04 λ_3 = 0.015	α_1 = 0.0277 α_2 = 0.0277 λ_1 = 1.29E - 04 λ_2 = 1.27E - 04 λ_3 = 0.015	α_1 = 0.0277 α_2 = 0.0277 λ_1 = 1.27E - 04 λ_2 = 1.29E - 04 λ_3 = 0.015	α_1 = 0.0277 α_2 = 0.0277 λ_1 = 1.27E - 04 λ_2 = 1.27E - 04 λ_3 = 0.017
0.3	0.99676	0.99678	0.99678	0.99674	0.99674	0.99650
0.32	0.99677	0.99679	0.99679	0.99674	0.99674	0.99650
0.34	0.99677	0.99679	0.99679	0.99675	0.99675	0.99650
0.36	0.99678	0.99679	0.99679	0.99675	0.99675	0.99651
0.38	0.99678	0.99680	0.99680	0.99675	0.99675	0.99651
0.4	0.99678	0.99680	0.99680	0.99676	0.99676	0.99651
0.42	0.99678	0.99680	0.99680	0.99676	0.99676	0.99652
0.44	0.99678	0.99680	0.99680	0.99676	0.99676	0.99652

6. INTERPRETATIONS OF RESULTS:

To study the established model practically, this section presents the numerical results of MTSF, Availability, and Profit.

a) EFFECT ON MTSF:

Table 3 and Table 4 present the pattern of MTSF with respect to repair rates of both systems V_1 and V_2 , respectively, for different values of other parameters. Numerical data in these tables illustrate that raising repair rates and lowering failure rates of both systems V_1 and V_2 cause the system's total MTSF to increase.

Detailed examination of Table 3:

When the repair rate α_2 is increased from 0.0277 to 0.0279, MTSF values increase by approximately 21. When α_3 rises from 0.3 to 0.32, the system's mean time to failure increases by around 51. A change in values λ_1 and λ_2 from 1.27 E-04 to 1.29 E-04 causes a decline of 80 and 82, respectively, in the failure-free operation period of the system. Nevertheless, as λ_3 rises from 0.015 to 0.017, the corresponding reliability metric drops by 797. The highest possible system downtime value of 10558 is achieved with an increased repair rate of $\alpha_3 = 0.32$.

Detailed examination of Table 4:

MTSF values increased by 22 when the values of α_1 and α_2 increased from 0.0277 to 0.0279, respectively. When λ_1 and λ_2 values increase from 1.27 E-04 to 1.29 E-04, the system's downtime falls by 82. However, the length of failure-free operation time reduces by 791 as λ_3 rises from 0.015 to 0.017. With respect to increasing repair rate α_3 , the maximum value for this metric is 10648, achieved at elevated repair rates α_1 and α_2 . When comparing the consequences of raising the repair rate of the State Water Supply system to that of the Centrifugal Pump system, it is evident that MTSF is significantly larger in the former scenario.

b. EFFECT ON AVAILABILITY

Detailed examination of Table 5:

Table 5 shows that system availability rises with respect to an increase in impeller repair rate α_1 . Availability rises once again when the shaft and state water supply system's repair rates α_2 and α_3 are rising. However, its values decrease when impeller, shaft, and state water supply system failure rates rise. When the repair rate α_2 increases from 0.0277 to 0.0279, the system's accessibility increases by 0.000019. When the repair rate α_3 goes from 0.3 to .32, the system's attainability is enhanced by 0.000005. Therefore, it is observed that the system's readiness is lower when the state water supply system's repair rate is increased compared to the centrifugal pump system. On the other hand, when λ_1 and λ_2 values change from 1.27 E-04 to 1.29 E-04, the system's reachability decreases by the same value, 0.000024. Nevertheless, when λ_3 increases from 0.015 to 0.017, the system's presence decreases by 0.00026. Furthermore, the system's maximum accessibility with an improved repair rate of $\alpha_2 = 0.0279$ is 0.99691191. Table 5 illustrates that an increase in shaft and impeller failure rates is correlated with the same drop in availability.

Detailed examination of Table 6:

An improvement in availability of about 0.000019 can be observed when repair rates α_1 and α_2 increase from 0.0277 to 0.0279. Repair rates λ_1 and λ_2 rise from 1.27 E-04 to 1.29 E-04, leading to an approximate 0.000025 reduction in system readiness. This value drops by 0.00026 when λ_3 shifts from 0.015 to 0.017. Furthermore, with respect to a continual rise in repair rates alpha sub 1 and alpha sub 2, the maximum accessibility of the system is 0.9968085. One thing to note is that Table

6 shows that an increase in shaft and impeller repair rates corresponds to the same increment in availability. Additionally, a rise in shaft and impeller failure rates results in the same decline in the system's readiness. It is clear from comparing the numerical figures of Tables 5 and 6 that the centrifugal pump's higher repair rate contributes more to the system's availability than does the calculation for the repair rate of the state water supply system.

c. IMPACT ON PROFITABILITY:

Tables 7 and Table 8 demonstrate the system's profit with respect to an increased repair rate of the impeller and the state water supply system, respectively. Different values for revenue per unit, busy server cost, and repair expenses are used to calculate profit. Both tables show that the system's profit progressively rises as repair rates rise and that it decreases as failure rates rise.

Detailed examination of Table 7:

Profit increases by 0.131 when the repair rate α_2 climbs from 0.0277 to 0.0279. With a jump in repair rate α_3 from 0.3 to .32, revenue increased by 14.628. This implies that the rate of repair of the state water supply system boosts the system's gain more than the rate of repair of the centrifugal pump system. On the other hand, the systems' return drops by the same value, 5.340 and 0.959, respectively, when λ_1 and λ_2 values shift from 1.27 E-04 to 1.29 E-04. This value drops by 2.144 as λ_3 rises from 0.015 to 0.017. Additionally, at an enhanced repair rate of $\alpha_3=0.32$, the greatest revenue value is 438.516. Table 7 illustrates that an increase in shaft and impeller failure rates is correlated with the same decrease in the system's yield. In a similar vein, increased availability is a consequence of increased shaft and impeller repair rates. One thing to note is that Table 7 shows that a rise in shaft and impeller failure rates results in the same decrease in the system's yield. Likewise, an increase in shaft and impeller repair rates causes the system yield to climb by the same amount.

Detailed examination of Table 8:

In this table, profit is calculated with respect to an increase in repair rate α_3 . For parameters $\lambda_1, \lambda_2, \lambda_3, \alpha_1, \alpha_2$ the trend of revenue values is the same as in Table 7. The maximum gain value of 530.761 is attained at an increased repair rate of $\alpha_2=0.0279$ w.r.t. increasing repair rate α_3 . This maximum profit value exceeds Table 7's highest

profit value. Moreover, when the system's profit is computed based on the impeller and shaft failure and repair rates, which are series-connected components, it stays constant.

All system states $s_0 - s_7$ are explicitly incorporated in the discussion of performance measures. State s_0 represents the fully operative condition and contributes directly to maximum availability and profit. States s_1, s_2 and s_3 correspond to partial operational modes under component repair and influence the gradual variation observed in MTSF and availability with changing repair and failure rates. The waiting states $s_4 - s_7$ capture prolonged downtime scenarios, which significantly reduce MTSF and profitability when failure rates increase. Thus, the numerical analysis reflects transitions among all regenerative states, ensuring that each state meaningfully contributes to the evaluation of system performance.

7. CONCLUSIONS:

The analysis reveals that the parallel system architecture significantly enhances various reliability measures, including Mean Time to System Failure (MTSF), Availability, and Profit. This improvement stems from the system's fault tolerance, scalability, performance, and efficiency compared to non-parallel configurations. The parallel setup effectively distributes the workload across multiple components, ensuring optimal resource utilization.

Furthermore, variations in failure and repair rates directly influence the system's profitability, availability, and MTSF. Higher repair rates lead to increased profitability, availability, and MTSF, while higher failure rates result in a decline in these indicators.

Upon thorough examination of the tables, it becomes evident that MTSF, availability, and profitability exhibit similar numerical values when computed concerning the repair rates of series-connected components. However, the impact of the repair rate on the State Water Supply system shows more significant enhancements in MTSF and profit, while, when compared to the Centrifugal Pump system, the influence of repair rate on the system availability of centrifugal pumps shows a noticeable improvement.

This analysis underscores the importance of effectively managing the failure rate of the State Water Supply system to achieve optimal outcomes in terms of MTSF and profitability in parallel system configurations.

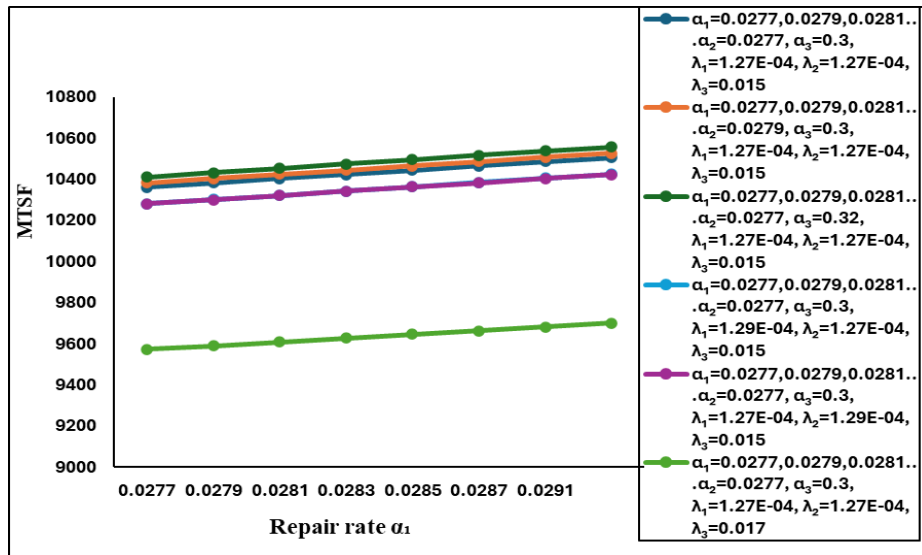


Figure 4. MTFS Vs repair rate α_1

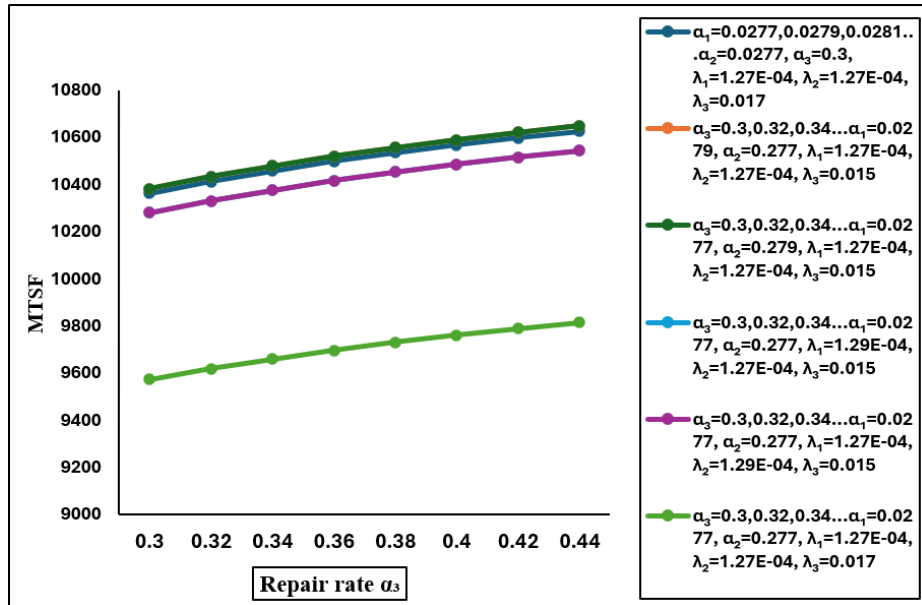


Figure 5. MTFS Vs repair rate α_3

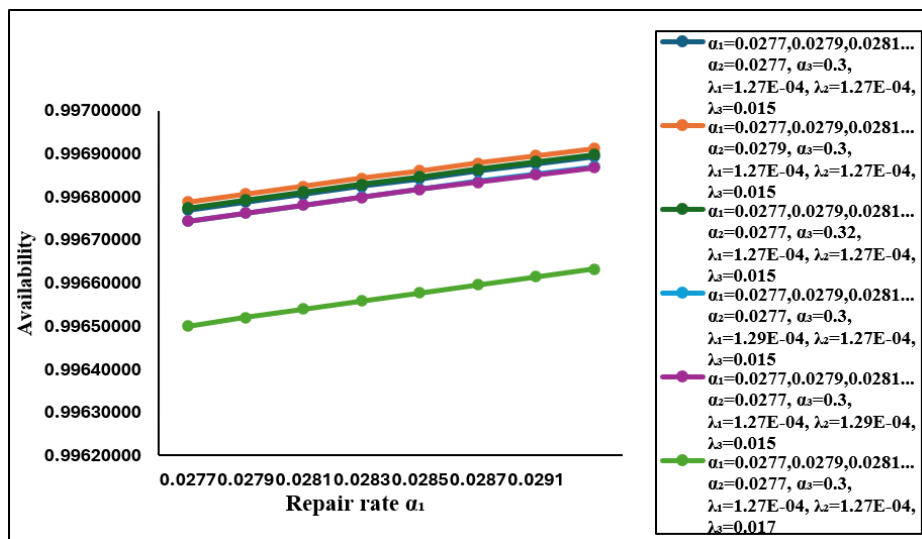


Figure 6. Availability Vs repair rate α_1

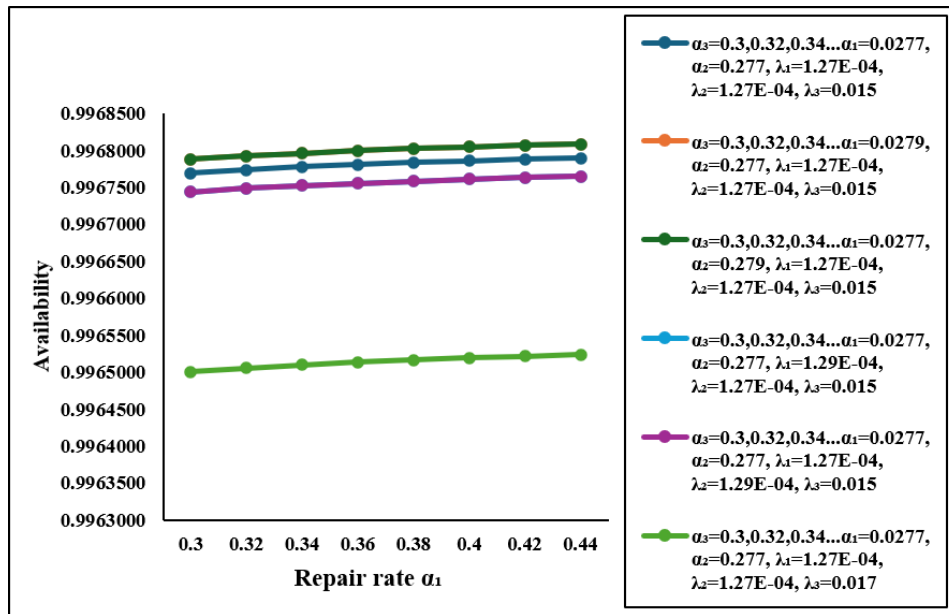


Figure 7. Availability Vs repair rate α_3

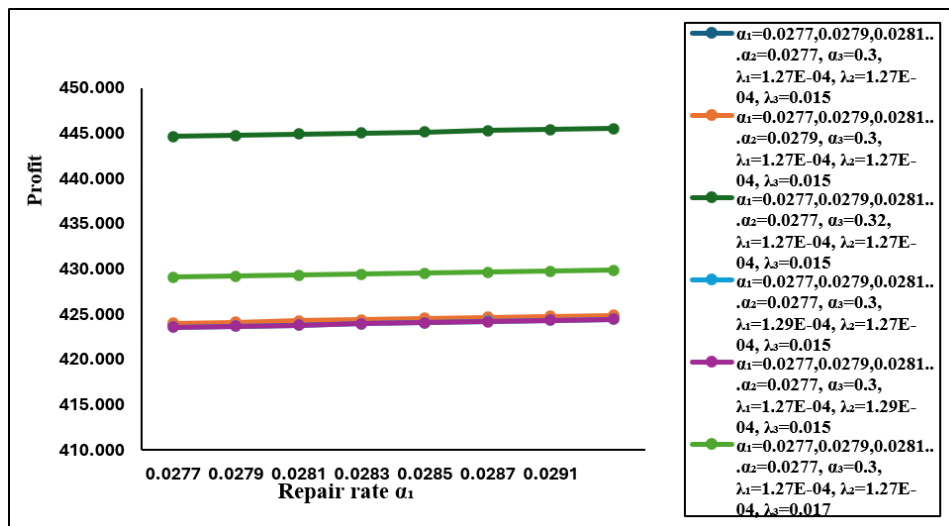


Figure 8. Profit Vs repair rate α_1

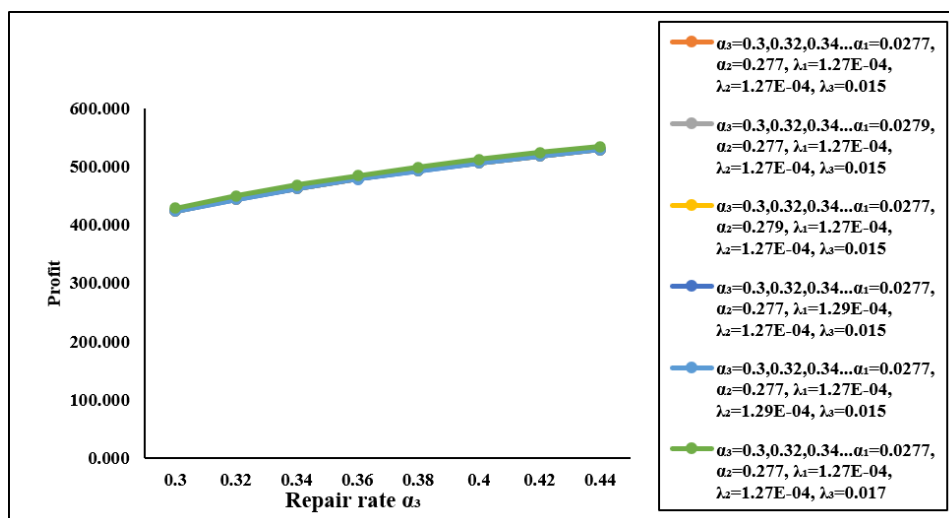


Figure 9. Profit Vs repair rate α_1

8 LIMITATIONS AND FUTURE SCOPE

Limitations: The present work was modeled using an exponential repair time distribution, which may not capture real maintenance behavior. Moreover, a single-server with perfect repair is considered, which restricts practical applicability.

The future scope of the conclusions drawn from the reliability analysis in parallel systems can encompass several areas of research and practical application:

- a. Further research can delve into developing optimization strategies that consider not only repair rates but also other factors like maintenance schedules, spare parts availability, and system configuration to maximize profitability, availability, and MTSF.
- b. Multi-server repair policies may be considered in future work to enhance profitability, availability, and MTSF.
- c. Exploring advanced modeling techniques such as machine learning algorithms or hybrid models combining Semi-Markov processes with other analytical methods can provide more accurate predictions and insights into system reliability under varying conditions.

DATA AVAILABILITY STATEMENT

We have no data to make available.

CONFLICT OF INTERESTS:

The authors declare that they have no conflicts of interest.

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