Frequency Response Analysis of Composite Aircraft Wing Using a Finite Element Euler-Bernoulli Beam Model

Ismail BENNAMIA
Laboratoire des Sciences Aéronautiques, Institut d'Aéronautique et des Etudes Spatiales, Université de Blida1, BP 270 Route de Soumâa Blida, Algérie.
Bennamia_ismail@yahoo.fr

Aimad-eddine BADEREDDINE
Laboratoire des Sciences Aéronautiques, Institut d'Aéronautique et des Etudes Spatiales, Université de Blida1, BP 270 Route de Soumâa Blida, Algérie.

Mansour YAHIA CHERIF
Institut d'Aéronautique et des Etudes Spatiales, Université de Blida1, BP 270 Route de Soumâa Blida,Algérie.

Toufik ZEBBICHE
Laboratoire des Sciences Aéronautiques, Institut d'Aéronautique et des Etudes Spatiales, Université de Blida1, BP 270 Route de Soumâa Blida, Algérie.

Abstract: - The purpose of this study is to show the influence of fiber orientation of laminated composites on bending-torsion coupling rigidity and Frequency analysis of aircraft wing model by using finite element composite beam model. The beam model is effective for preliminary design of aerospace structures (optimization, dynamic response and aeroelasticity, etc). New materials or composites have a high relationship between the bending and torsion modes due to their anisotropies. The bending-torsion coupling coefficient considered here is calculated using different ply angles of laminated composite materials. The Energy Method is used to derive the basic mass and rigidity matrices of the beam where the stiffness matrix contains terms of bending-torsion coupling. An application for free vibration analysis of aircraft wing is validated in this study and a frequency response using a MATLAB@ program is illustrated.

Keywords: - Aircraft wing, Composite Beam Model, Modal Analysis, Frequency Response

1. INTRODUCTION

Beam models have been used in preliminary design of high aspect ratio, all metal aircraft wings from the early days of aviation [5].

Analytical and numerical methods are adopted to determine approximate natural frequencies and mode shapes of uniform beams. Among these methods, the Dynamic Stiffness Method (DSM) and finite element method (FEM).

The finite element approach is the most widely used method for static and dynamic theoretical modeling of aircraft structures, providing the basic equations involving mass and stiffness terms for both aeroelastic and loads calculations (Jan R. Wright and E. Cooper [5]).

A sound pressure excitation produced by a speaker used by Romanu et.al [16] to determine the natural frequencies of beams. Tufoi et. al.[17] studies the influence of temperature changes on clamped-clamped Euler Bernoulli beams.

Arghir et Bere[18] studied the utilisation of composite materials in model aircraft structure. Free vibration analysis of composite beams and its application to aircraft wings using DSM method has been studied by Banerjee et. al.[2]. Li Jun et. al. [14] used the dynamic finite element method for free vibration analysis of generally laminated composite beams.

The coupled bending and torsional natural frequencies of uniform beams with an arbitrary cross-section are studied by Tanaka and Bercin using the finite element method in which non coincident shear centre and centroid of the beam cross-section is considered [1]. A finite element program is developed by Yildizet. et.al.[15] to analyze multi-layer composite beams and plates. A finite element beam elements structural model of a small flexible aircraft is updated using ground vibration test is studied by Gupta et. al.[6].

The purpose of the present study is to show the influence of the layers ply's angle on the frequency response of composite aircraft wing using Euler Bernoulli beam model. The Euler-Bernoulli beam theory based on the assumption that the plane normal to the neutral axis before deformation remains normal to the neutral axis after deformation (no effects of transverse shear deformation) [4].The Fast Fourier Transform (FFT) adopted in this study developed by Kwon, and Bang [4] using MATLAB computer programs.

Eight layers model the section of the beam symmetric about the midplane axis (see figure 1.). The laminated composite beam has three configurations as shown in figure 1,$[\theta/\theta/\theta/\theta]_S$, $[-\theta/-\theta/-\theta/-\theta]_S$ and $[\theta/-\theta/\theta/-\theta]_S$. θ is the ply angle orientation.

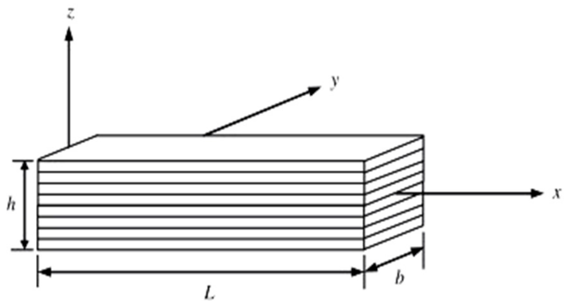


Figure 1. Eight layers composite beam

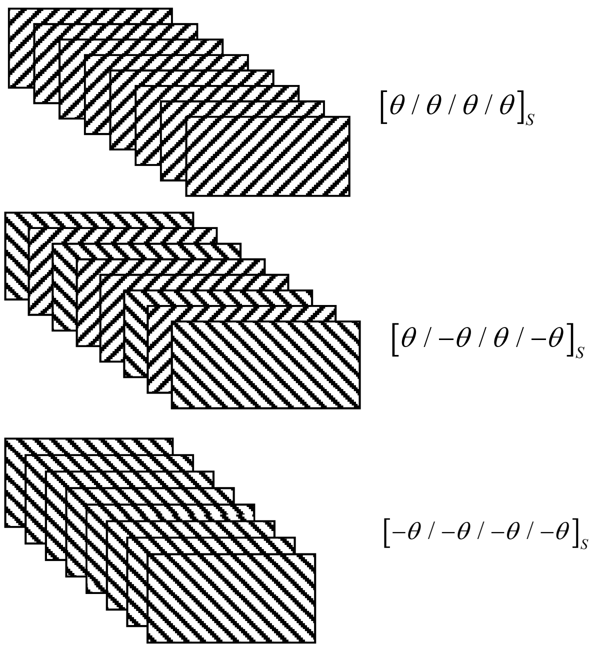


Figure 2. Three proposed configuration for the eight layers

2. The FEM Modeling

The finite element method is adopted to model the wing aircraft structure and deduce its mass and stiffness matrices [2].

2.1. The Kinetic Energy

The kinetic energy of bending-torsional layered composite beam given by [2]:

$$T = \frac{1}{2}\int \rho(\{\dot{w}\} + x_\alpha\{\Psi\})^2 dxdydz \qquad (1.a)$$

w and Ψ represent the movement in bending and torsion, respectively, ρ is the mass density and $x_\alpha$is the geometric coupling (distance between the mass axis and elastic axis of the wing).

The kinetic energy can be written as [9]:

$$T = \frac{1}{2}\{\ddot{q}\}^t[M_e]\{\ddot{q}\} \qquad (1.b)$$

$[M_e]$: Beam element mass matrix

2.2. The potential Energy

The potential energy of bending-torsional layered composite beam given by [21]:

$$U = U_{coupled} + U_{decoupled}$$

$$U_{decoup} = \frac{1}{2}\int EI\left(\frac{d^2w}{dx^2}\right)^2 dx + \frac{1}{2}\int GJ\left(\frac{d\Psi}{dx}\right)^2 dx \qquad (2.a)$$

$$U_{coupled} = \frac{1}{2}\int K\left(\frac{d^2w}{dx^2}\right)\left(\frac{d\Psi}{dx}\right) dx \qquad (2.b)$$

The kinetic energy can be written as [9]:

$$U = \frac{1}{2}\{q\}^t[K_e]\{q\} \qquad (2.c)$$

$[K_e]$: Beam element stiffness matrix

The governing partial differential equations of motion for the coupled bending-torsional free natural vibration of the composite beam are given by [2][13][12]:

$$EI\frac{\partial^4 w}{\partial x^4} + K\frac{\partial^3 \Psi}{\partial x^3} - mx_\alpha\frac{\partial^2 \Psi}{\partial t^2} + m\frac{\partial^2 w}{\partial t^2} = 0 \qquad (3.a)$$

$$GJ\frac{\partial^2 \Psi}{\partial x^2} + K\frac{\partial^3 w}{\partial x^3} - I_\alpha\frac{\partial^2 \Psi}{\partial t^2} + mx_\alpha\frac{\partial^2 w}{\partial t^2} = 0 \qquad (3.b)$$

EI is the bending stiffness, *GJ* is the torsional stiffness and *K* the stiffness of bending-torsion coupling, *m*, $I_\alpha$is the mass per unit in length and the moment of inertia about the axis elastic.

2.3. Modal Analysis

The matrix system obtained by the discrete equations of motion for a global structure without damping (conservative system) is given by [2][4][5][19]:

$$[M]\{\ddot{q}\} + [K]\{q\} = 0 \quad (4)$$

$[M]$ is the global mass matrix, $[K]$is the global stiffness matrix and $\{q\}$is The displacement vector.

2.4. Frequency Response Analysis

Most of the vibration systems can be characterized by their inherent frequency components which dictate both time and frequency response [4]. The governing differential equation of motion of forced system given as [8]:

$$[M]\{\ddot{q}\} + [K]\{q\} = \{F\} \quad (5)$$

$\{F\}$: vector force, $F = f_0 \cos(\omega t).\omega$: Frequency of excitation in *rad/s*.

3. LAMINATED COMPOSITE WING MODEL

The composite wing is simulated to clamped-free finite element composite beam, the wingspan is $L = 0,55m$ the width and thickness of the wing are: $d = 0,0605m$ and $t = 0,00424m$ with 8 layers [3]. The mechanical proprieties of the wing are[3]: $E_1 = E_2 = 10,2\,Gpa$, $G_{12} = 1.767\,Gpa$, $\nu_{12} = 0,1$, $m = 0,68\,Kg/m$,$I_\alpha = 2,75\ kg.m$ and x_α=0.0095m.

Different configurations are given to evaluate the frequency response of the proposed composite wing. For each configuration the bending, torsional and coupled stiffness rigidities are calculated for laminate layer by [7][20]:

$$EI = d\left(D_{22} - \frac{D_{12}^2}{D_{11}}\right) \quad (6.a)$$

$$GJ = 4d\left(D_{66} - \frac{D_{16}^2}{D_{11}}\right) \quad (6.b)$$

$$K = 2d\left(D_{22} - \frac{D_{12}\,D_{16}}{D_{11}}\right) \quad (6.c)$$

$[D]_{6\times 6}$ Called bending stiffness matrix of laminated composite beams [10][11][14].

4. RESULTS AND DISCUSSION

4.1. The effective rigidities evaluation for various ply angle orientation

Figures 3, 4 and 5 show the effectives rigidities as a function of ply angles, it appears clearly that a similar curves for bending and torsional rigidities for all configurations, where the coupling rigidity is not the same for all configurations.

Varying the ply angle from 0° to 90°, the material coupled rigidity Kequal to zero (decoupled case) for 0°, 90° and 45° angles, the bending rigidity is minimal in 45° and the same for 0° and 90° angles, where the torsional rigidity is maximal in 45°.

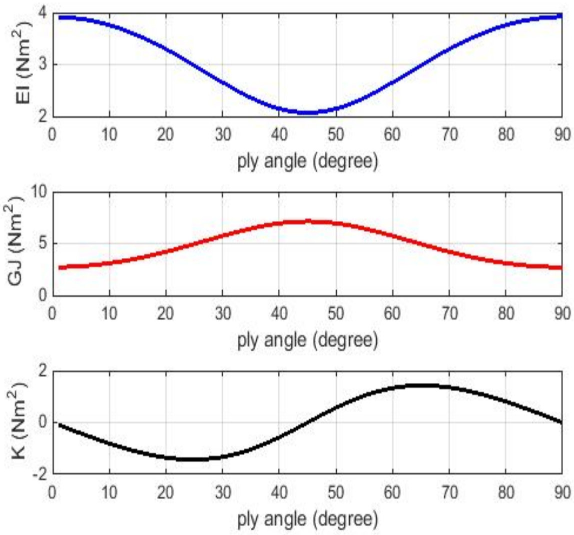


Figure 3. Effective rigidities for $[\theta/\theta/\theta/\theta]_s$

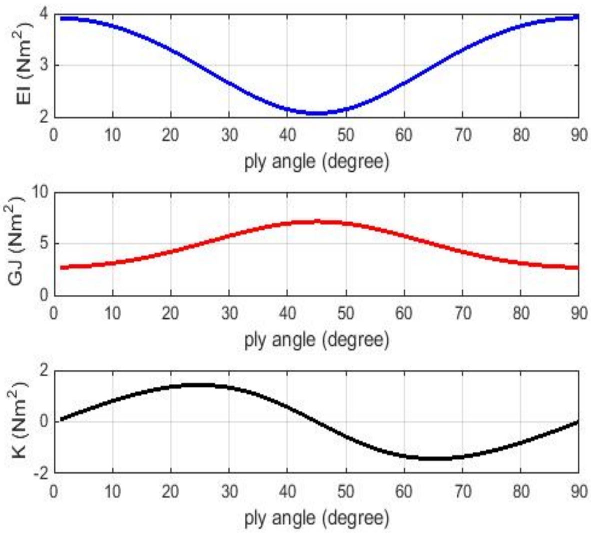


Figure 4. Effective rigidities for $[-\theta/-\theta/-\theta/-\theta]_s$

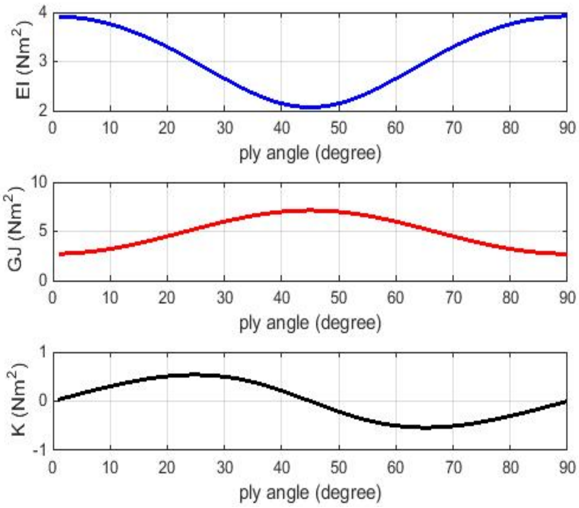


Figure 5. Effective rigidities for $[\theta/-\theta/\theta/-\theta]_s$

4.2. Validation

In order to validate the present beam model, the first and second frequencies results obtained from the beam model using MATLAB program are compared with those obtained by a composite plate free vibration analysis using ANSYS Software, for different configurations (see 6, 7 and 8 figures).

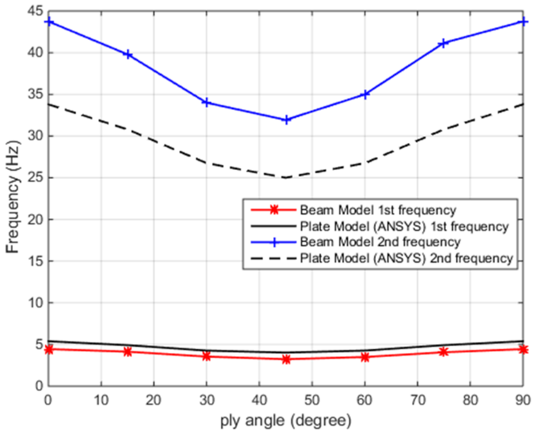


Figure 6. 1st and 2nd frequencies for Beam Model and Plate Model (ANSYS) for $[\theta/\theta/\theta/\theta]_S$

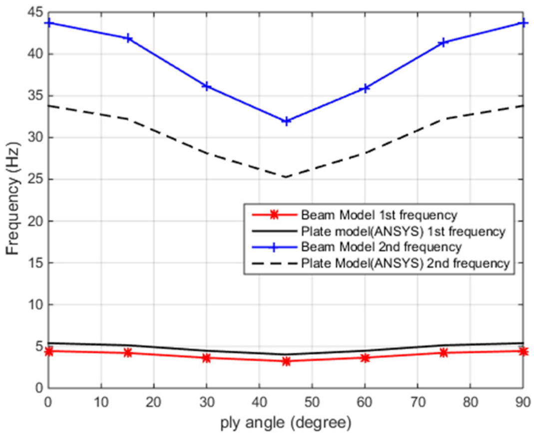


Figure 7. 1st and 2nd frequencies for Beam Model and Plate model (ANSYS) for $[\theta/-\theta/\theta/-\theta]_S$

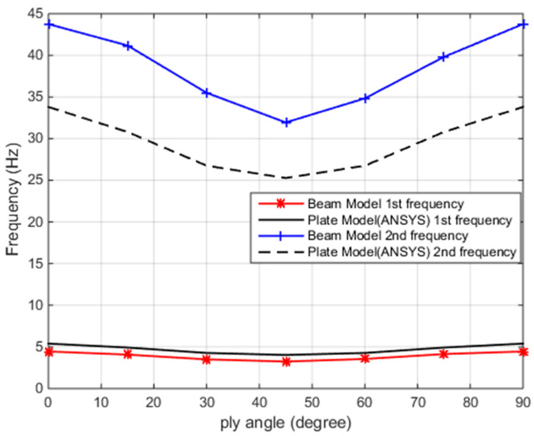


Figure 8. 1st and 2nd frequencies for Beam Model and Plate Model (ANSYS) for $[-\theta/-\theta/-\theta/-\theta]_S$

4.3. The three first frequencies for various ply orientation

In this part, the influence of ply angle in the three first frequencies is illustrated for the three proposed configuration: $[\theta/\theta/\theta/\theta]_S$, $[-\theta/-\theta/-\theta/-\theta]_S$ and $[\theta/-\theta/\theta/-\theta]_S$, the obtained curves shows the dimensionless frequencies of various ply's angle orientation. The obtained dimensionless frequencies are calculated by using the rate between the three first frequencies obtained by using modal analysis and the first three frequencies for ply angle of 0° as shown in figures 9, 10 and 11, of each ply angle. $\omega_1(1)$, $\omega_2(1)$ and $\omega_3(1)$ are the three first frequencies of 0° ply angle.

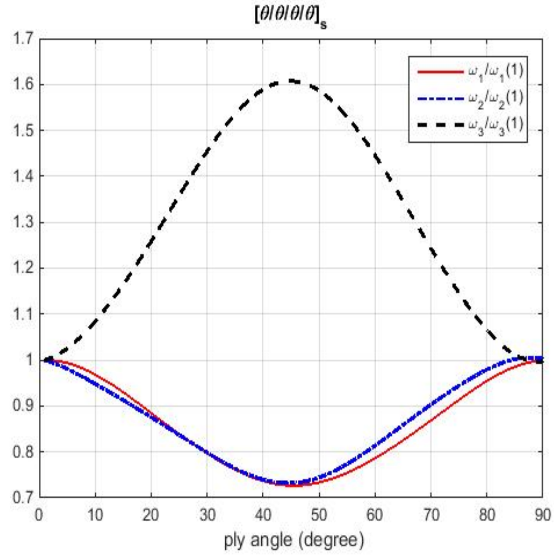


Figure 9. The Three first dimensionless frequencies for $[\theta/\theta/\theta/\theta]_s$

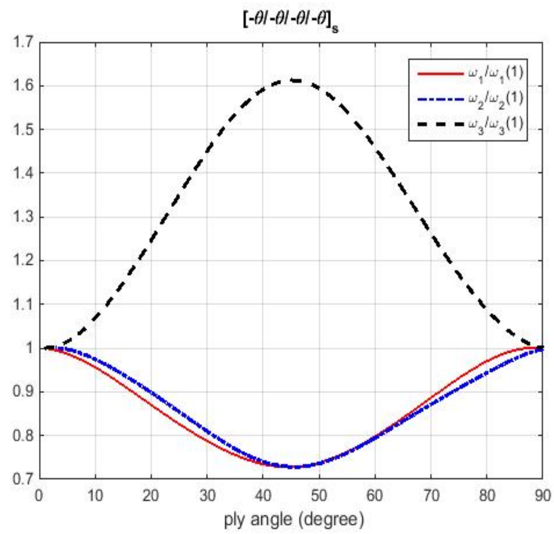


Figure 10. The Three first dimensionless frequencies for $[-\theta/-\theta/-\theta/-\theta]_s$

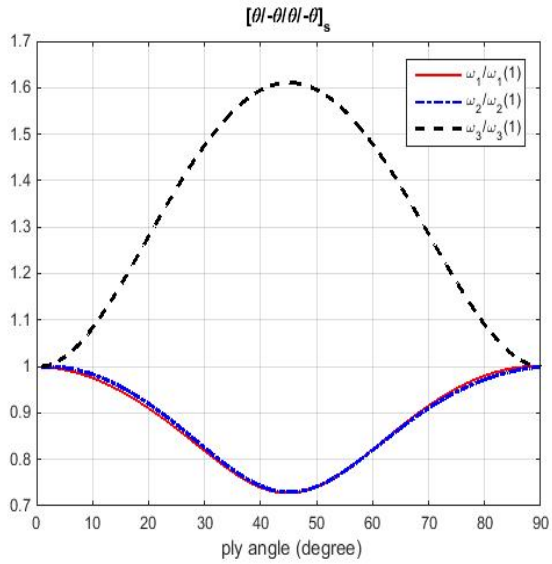


Figure 11. The three first dimensionless frequencies for $[\theta/-\theta/\theta/-\theta]_s$

It's appears clearly that the results obtained from varying ply's angle from 0 to 90°, the first and second dimensionless frequencies are almost similar (rate of 0,3), where the third dimensionless frequency varying with ply angle (rate of 1,6), and it's observed that these results are almost symmetric.

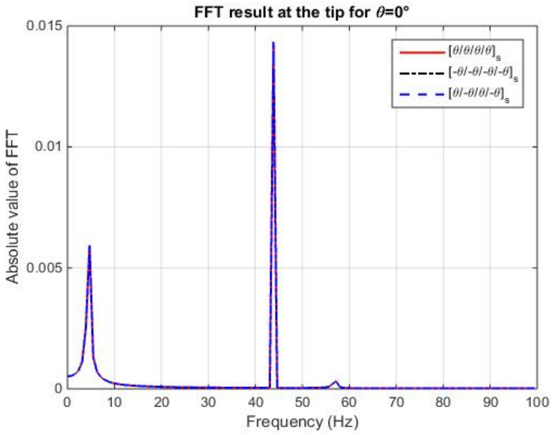
4.4 Frequency response

In order to further the present work, a spectrum response is investigated. The FFT result of the wing at the tip using an impulse force is presented. In this part, the various ply angles are given: 0°, 20°, 25°, 30°, 45°, 75° and 90°.

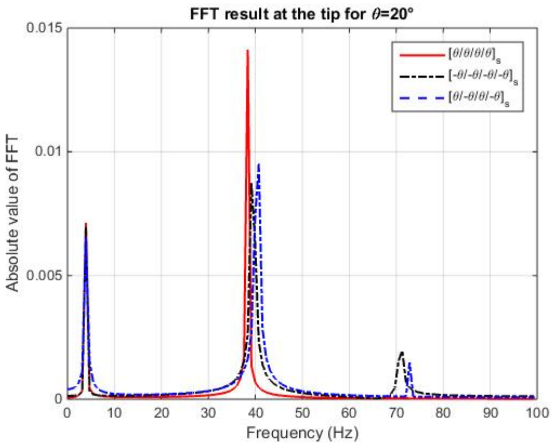
Figures 12(a), 12(e) and 12(h) show the FFT result of different ply angles of 0°, 45° and 90° respectively for all proposed configurations. The results show that there are two peaks of resonance of the 100 first frequencies range; each peak corresponds to the first and second frequencies. It observed that for these three-ply angles the curves are very similar for all configurations.

Figures 12(b), 12(c), 12(d), 12(f) and 12(g) show the FFT result for different configurations with ply angles of 20°, 25°, 30°, 60° and 75° respectively.

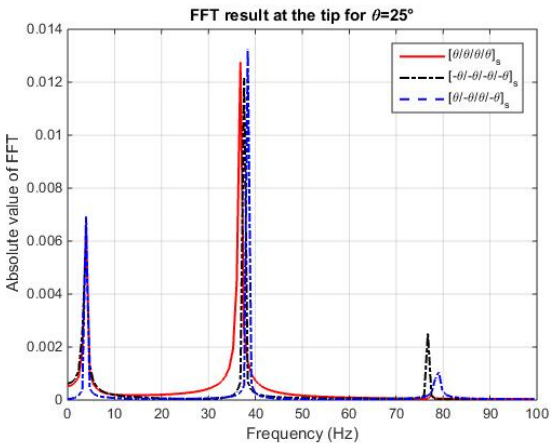
It's appears that there are three remarkable peaks of resonance in the frequencies range but the frequency response is not the same for each ply angle. For 20° ply angle, for example, the FFT value for the $[\theta/\theta/\theta/\theta]_S$ configuration is greater than the FFT value of the other configurations, where it' different for the other ply angles.



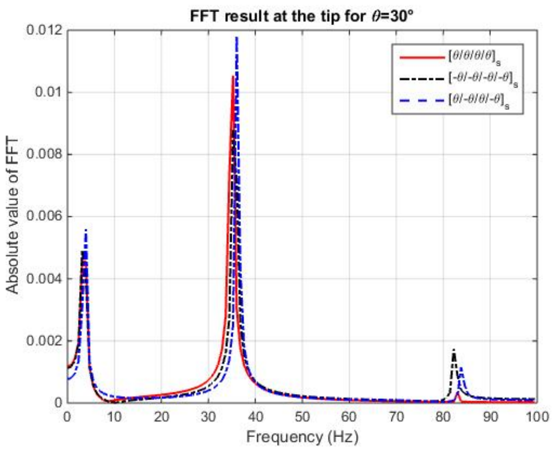
(a)



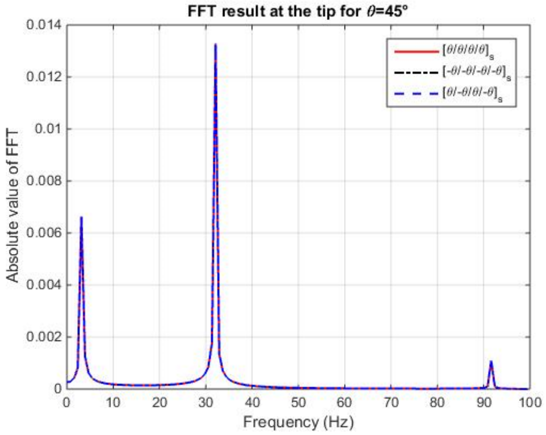
(b)



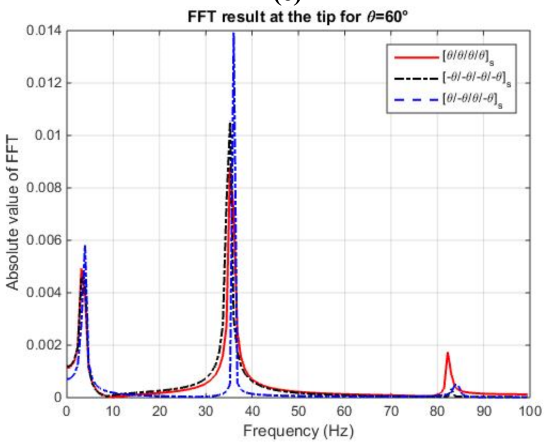
(c)



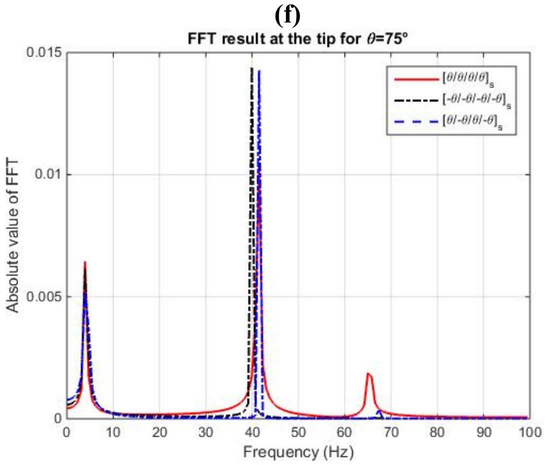
(d)



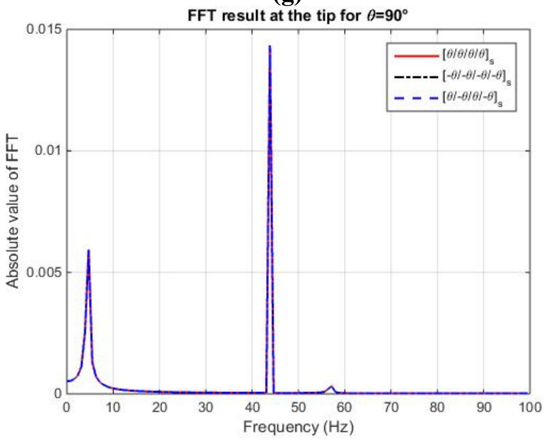
(e)



(f)



(g)



(h)

Figure 12. The FFT result for different configurations

5. CONCLUSIONS

In this paper, a finite element method is developed to illustrate the frequency response analysis of a composite aircraft wing using Euler-Bernoulli beam approximation.

The results obtained by free vibration analysis using the present beam model are validated with a composite plate free vibration analysis by ANSYS in first time. It's denoted that the first frequencies results obtained from both models for the different configurations are almost similar, where remarkable difference between models (beam model and plate results) in the second frequencies which explains the difference between the chosen model and the plate model.

The Absolute values of FFT (Fast Fourier Transform) result of an impulse response in MATLAB program for three different proposed configurations $[\theta/\theta/\theta/\theta]_S$, $[-\theta/-\theta/-\theta/-\theta]_S$ and $[\theta/-\theta/\theta/-\theta]_S$ is observed.

These three configurations are the most possible configurations for height layers with one ply angle value.

The obtained effective rigidities (bending, torsional and coupling) for all configurations have a symmetric axe at 45° for bending and torsional rigidities and symmetric point at 45° for material coupled rigidity.

The calculated coupling rigidity is influenced by ply angles, which explain the difference between metallic (isotropic) wing and laminated wing.

The obtained three first dimensionless frequencies from modal analysis curves show that the variation of ply angles has influence in third frequency, where the first and second have the same rate for all ply angles.

The FFT result at the tip for the three proposed configurations $[\theta/\theta/\theta/\theta]_S$, $[-\theta/-\theta/-\theta/-\theta]_S$ and $[\theta/-\theta/\theta/-\theta]_S$ of 8 various ply angles 0°,20°,25°, 30°,45°,60°,75°,90° demonstrate the frequency response of the wing. These results explain the influence of ply angles on coupled rigidities obtained by composite (laminated) materials, where the relationship between bending and torsion appears clearly in ply angles response results of 20°,25°, 30°,60°, 75°, and it gives a good argument for orthotropic angles 0° and 90°and for 45° ply angle. This last, can be explained that for this three angles (0°, 90° and 45°)the bending-torsional material coupling rigidity K equal to zero (decoupled case) for the three proposed configurations and the obtained frequencies from FFT are the same (no effects of ply's angle on frequencies results) .

The frequency response results obtained for the Euler-Bernoulli beam model for aircraft wing give an easy way to model a frequency response analysis for thick beams using Timoshenko beam model that includes the effect of transverse shear deformation.

REFERENCES

[1] Tanka M., Bercin A.N., Finite Element Bending of a Coupled Bending and torsional Free Vibration of Uniform Beams with an Arbitrary Cross-Section, *Applied Mathematical Modeling* Vol. 21, No. 6, 1997, pp. 339-344.

[2] Banerjee J.R., Su H., Jayatunga C., A Dynamic Stiffness Element for Free Vibration Analysis of Composite Beams and its Application to Aircraft Wings, *Computers and Structures* Vol. 86, No. 6, 2008, pp. 573-579.

[3] AhmedAbd Al-Hussain Ali, Mohammed Ismael Hamed, The Effect of Laminated Layers on the Flutter Speed of Composite Wing, *Journal of Engineering, Iraqi Academic Scientific Journals* Vol. 18, No. 8, 2012, pp. 924-934.

[4] Kwon W.Y., Bang H., *The Finite Element Method using Matlab*, CRC Presse, Taylor & Francis Group, 1997.

[5] Wright J. R., Cooper E., *Introduction to aircraft aeroelasticity and loads*, John Wiely & Sons, 2007.

[6] Gupta A., Moreno C. P., Pfifer H., Balas G. J., Updating a Finite Element Based Structural Model of a Small Flexible Aircraft, *AIAA Modeling and Simulation, Technologies Conference, AIAA SciTech Forum, (AIAA 2015-0903)*

[7] Taylor J. M., Butler R., Optimum Design and Validation of flat Composite Beams Subject to Frequency Constraints, *AIAA Journal*, Vol. 35, No. 3, 1997, pp. 540-545.

[8] Bassioni A.S., Gad-Elarb R.M., Elmahd T.H., Dynamic Analysis for Laminated Composite Beams, *Composite Structures,* Vol. 44, No. 2-3, 1999, pp. 81-87.

[9] Oz H.R., Calculation of the Natural Frequencies of a Beam Mass System Using Finite Element Method, *Mathematical and Computational Applications*, Vol. 5, No. 2, 2000, pp. 67-75.

[10] Özturk H., Sabunci M., Stability Analysis of a Cantilever Composite Beam on Elastic Supports, *Composites Science and Technology*, No. 65, No. 13, 2005, pp. 1982-1995.

[11] Attaran, D.L. Majid, S. Basri, A.S.Mohd Rafie, E.J. Abdullah, Structural Optimization of an Aeroelastically Tailored Composite Flat Plate Made of Woven fiberglass/epoxy, *Aerospace Science and Technology*, Vol. 15, No. 5, pp. 393-401.

[12] Guo S., Aeroelastic Optimization of an Aerobatic Aircraft Wing Structure, *Aerospace science and Technology,* Vol. 11, No. 5, 2007, pp. 396-404.

[13] Banerjee J.R., Explicite Analytical Expressions for Frequency Equation And Mode Shapes of Composite Beams, *International Journal of Solides and Structures*, Vol. 38, No. 14, 2001, pp. 2415-2426.

[14] Li Jun, Hua Hongxing, Shen Rongying, Dynamic Finite Element Method for Generally Composite Beams, *International Journal of Mechanical Science*, Vol. 50, No. 3, 2005, pp. 466-480.

[15] Yilidiz H., Sarikanat M., Finite Element Analysis of Thick Beams and Plates, *Composites Science and Technology*, Vol. 61, No, 12, 2001, pp. 1723-1727.

[16] Romānu B.I.G., Iancu V. , Gillich G.R., Determination of the Natural Frequencies of Beams Using Sound Pressure, *Romanian Journal of Acoustics and Vibration*, Vol. 5, No. 2, 2013, pp. 93-96.

[17] Tufoi M., Gillich G.R., Praisach Z.I., Iancu V., Furdui I., About the Influence of Temperature Changes on the Natural Frequencies of Clamped-Clamped Euler-Bernoulli Beams, *Romanian Journal of Acoustics and Vibration*, Vol. 6, No. 2, 2014, pp. 84-87.

[18] Arghir G., Bere P., Utilisation of Composite Materials in Model Aircraft Structure, *Acta Technical Napocensis, Applied Mathematics, Mechanics and Engineering*, Vol. 60, No. 1, 2017, pp. 19-26.

[19] Tizzi S., Numerical Procedure for the Dynamic Analysis of Three-Dimensional Aeronautical Structures, *Journal of Aircraft* Vol. 34, No. 1, 1997, pp. 120-129.

[20] Sheng S. H., Hwu C., On Line Measurement of Material Properties for Composite Wing Structures, *Composite Science and Technology*, Vol. 66 No. 7-8, 2006, pp. 1001-1009.

[21] Borneman S. R., *New Dynamic Finite Element Formulation with Application to Composite Aircraft Wings*, M.A.Sc. Thesis, Ryerson University, 2004.