

Investigation of the Effect of an Inconsistent Blade on Natural Frequencies of a Rotating Multi Blade System

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Abstract: - In this study, dynamic characteristics of a rotating multi blade system which contains an inconsistent blade are determined. The main goal is to investigate the effect of an inconsistent blade on the system natural frequencies. For this purpose, natural frequencies of a perfect system and a system containing one inconsistent blade have been compared to show how this inconsistency affects the natural frequencies of the system. It has been assumed that the inconsistent blade differs from the other in term of material property. The vibration frequency characteristics have been analyzed using assumed mode method along with Hamilton's law. Based on the results, it is demonstrated that natural frequencies related to some modes of the system are affected in case of inconsistent blade while for the most of the modes, they are not affected.

Keywords: - Natural frequency, Multi blade system, inconsistent blade, Assumed mode method

Nomenclature

Parameter	Definition
h	Cross section height
b	Cross section width
l	Beam length
α	Ratio of variation in Cross section height
β	Ratio of variation in Cross section width
r	Disk radius
s	Stretch of the beam
u	Axial displacement the beam
v	Chord-wise displacement of the beam
Ω	Rotating velocity
T	Kinetic energy
ρ	Density
A	Cross section area
U	Strain energy
E	Young's modulus
I	Second area moment of inertia
U_D	The energy term of the system which are caused by the flexibility of the disk
U_S	The energy term of the system which are caused by the flexibility of the shroud
a_D	The positions where the beam is connected to the disk
a_S	The positions where the beam is connected to the shroud
k_D	Disk stiffness variable
k_S	Shroud stiffness values
φ	Bending mode function of the beam
q	The generalized coordinate
μ	The number of coordinates
I_0	Second area moment of inertia of the cross section at the fixed end

1. INTRODUCTION

Plenty of engineering items, such as turbines, compressor, helicopter rotors, propellers of ships or planes, robot handlers, and structures for space whirling contain rotating beams. To create well-designed structures, the analysis of their vibration characteristics, including natural frequencies and mode shapes is of great importance. A considerable variation can be noticed in vibration characteristics of rotating structures compared to those in non-rotating ones.

In order to evaluate the natural frequency of a single rotating cantilever beam, Southwell and Gough developed a model [1], in which, according to the Rayleigh energy theory, natural frequencies of a rotating cantilever beam could be measured via a widely-used simple equation called Southwell. Then, to increase the accuracy of natural frequencies, Schilhansl introduced a relatively different linear equation, in which only the rotating beam bending motions are considered [2]. The mentioned method of vibration analysis was developed in numbers of studies in order to determine natural frequency of rotating beams under various conditions [3-9]. In one of these researches, a dynamic analysis method, including a lot of hybrid deformation variables, was proposed to reach equations of motion [10]. Due to the linearity of the existent equations in this method, no replacement procedure is needed to analyze the vibration and the equations can be utilized directly. Compared to the previous ones, this method is less complicated, more compatible, and more precise [11, 12].

Beyond vibration of single rotating beam, various studies have been conducted to analyze vibration characteristics of rotating multi blade system [13-15], in order to develop application of modal analysis in vital industrial equipment such as turbines. The majority of the mentioned studies are limited to the systems which contain consistent blades. However, equipment with multi blade system usually suffers from the existence of one or more inconsistent blades in their systems. In this case, the vibration behavior of the system such as natural frequencies will be changed. In order to keep the working frequencies away from the system natural frequencies, it is essential to be aware of the way of variation in system natural frequencies. Hence, the concept of vibration characteristics of the multi blade system containing inconsistent blade can be important. In this study, comparing the natural frequencies of a multi blade system with and without inconsistency will demonstrate the way of variation of system natural frequencies.

2. EQUATION of MOTION

The model of blade is considered as a slender beam which consists of rectangular and tapered cross section beam. This model is shown in Fig. 1.

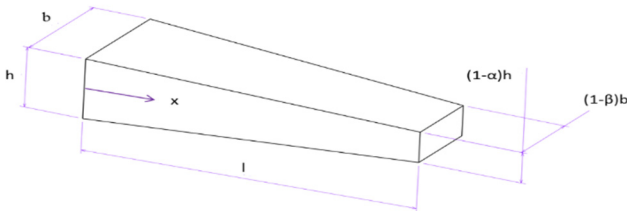


Figure 1. The schematic of the tapered blade model

The blade in this model contains homogeneous and isotropic material features. The shear and rotary inertia effects are neglected, since the beam has a slender shape. The rate of variation in the cross section dimensions is described by two dimensionless parameters α and β , as shown in Fig 1. Assuming a high value of out-of-plane rigidity for the blade, the out-of-plane motion is neglected and only the in-plane motion is considered. The schematic of deformed and un-deformed beams are presented in Fig. 2. The beam is considered as a cantilever of a length l which is fixed to a disk with a radius r . When the beam is deformed, the point P_0 placed at the position of point P . Hence, axial displacement u and the chord-wise displacement v can be used to define the deformation vector.

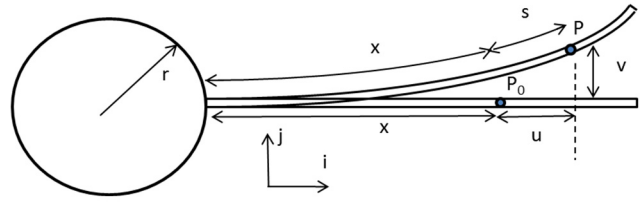


Figure 2. Configuration of rotating blade displacement

It was shown [5] that the relation between stretching of the beam (s), u and v is as below:

$$s = u + \frac{1}{2} \int_0^l \left(\frac{\partial v}{\partial \sigma} \right)^2 d\sigma \quad (1)$$

where σ is the dummy variable employed for integration. Using the defined deformation terms along with considering the beam rotating velocity (Ω) as a constant value ($\Omega=0$), the kinetic and strain energies of the beam can be expressed as below:

$$T = \frac{1}{2} \int_0^l \rho A(x) \left\{ \left(u' - \Omega v \right)^2 + \left(v' + \Omega(r+x+u) \right)^2 \right\} dx \quad (2)$$

$$U = \frac{1}{2} \int_0^l EA(x) \left(\frac{\partial s}{\partial x} \right)^2 dx + \frac{1}{2} \int_0^l EI(x) \left(\frac{\partial^2 v}{\partial x^2} \right)^2 dx \quad (3)$$

where E is the Young's modulus. $I(x)$ is the second area moment of inertia and $A(x)$ is the cross-section area which are functions of x . In case of multi blade systems such as turbine, stiffness of the shroud and disc results in additional terms of energy. In this study, the disc and shrouds are considered as the discrete springs with corresponding stiffness values. Hence, due to flexibility of the disc and shrouds, the energy terms can be written as:

$$U_D = \frac{1}{2} K_D \left(v^k (a_D) - v^{k-1} (a_D) \right)^2 + \frac{1}{2} K_D \left(v^{k+1} (a_D) - v^k (a_D) \right)^2 \quad (4)$$

and

$$U_S = \frac{1}{2} K_S \left(v^k (a_S) - v^{k-1} (a_S) \right)^2 + \frac{1}{2} K_S \left(v^{k+1} (a_S) - v^k (a_S) \right)^2 \quad (5)$$

In the above expressions, U_D and U_S are the energy terms of the k th blade of the system which are caused by the flexibility of the disk and the shroud. a_D and a_S are the positions where the beam is connected to the disk and the shroud, respectively, and K_D and K_S are the corresponding stiffness values. The superscript k , $k+1$ and $k-1$ above the displacement term are the blade numbers. When the strain energy and kinematic expressions are placed in Eq. 6, the equation of motion can be derived:

$$\int \delta(T - U) dt = 0 \quad (6)$$

A group of hybrid deformation variables can be approached by an assumed-mode method called the Rayleigh-Ritz. The bending displacement is approximated by mode functions as follows:

$$v(x, t) = \sum_{i=1}^{\mu} \varphi_i(x) q_i(t) \quad (7)$$

where φ_i is the bending mode function of the beam, q_i is the corresponding generalized coordinate, and μ is the number of coordinate q_i . Previously, it was demonstrated that the coupling effect between extensional and bending motions has a weak influence on the vibrational characteristics of the rotating blades [13]. Hence, these terms related to the coupling effects are ignored in this study. Using the expression of Eq. 7 for the bending displacement, the equations of the bending motion of the k th blade are derived as below (omitting the terms of coupling):

$$\sum_{j=1}^{\mu} \left[m_{ij}^{..k} q_j + \left(k_{ij}^B + \Omega^2 \left(k_{ij}^G - m_{ij} \right) \right) q_j^k \right] - k_{ij}^D \left(q_j^{k-1} - 2q_j^k + q_j^{k+1} \right) - k_{ij}^C \left(q_j^{k-1} - 2q_j^k + q_j^{k+1} \right) = 0 \quad (8)$$

where $i=1,2,3,\dots$, etc. and

$$m_{ij} = \int_0^l \rho(x) \varphi_i(x) \varphi_j(x) dx \quad (9)$$

$$k_{ij}^B = \int_0^l EI(x) \varphi_{i,xx}(x) \varphi_{j,xx}(x) dx \quad (10)$$

$$k_{ij}^G = \int_0^l g(x) \varphi_{i,x}(x) \varphi_{j,x}(x) dx \quad (11)$$

$$k_{ij}^D = K_D \varphi_i(a_D) \varphi_j(a_D) \quad (12)$$

$$k_{ij}^S = K_S \varphi_i(a_S) \varphi_j(a_S) \quad (13)$$

$$g(x) = \rho r(l-x) + \frac{\rho}{2l} (l - (\alpha + \beta)r) (l^2 - x^2) + \frac{\rho}{3l} (\alpha\beta r - \alpha l - \beta l) (l^3 - x^3) + \frac{\rho}{4l^2} \alpha\beta (l^4 - x^4) \quad (14)$$

Since, parametric study is preferred for different purposes such as system design, it is beneficial to transform equations of motion into a dimensionless form. Hence, employing the variables of Eq. 15, the equation of motion is rewritten in a dimensionless form as shown in Eq. 16.

$$\tau = \frac{t}{T}, \xi = \frac{x}{l}, \theta = \frac{q}{l}, T = \sqrt{\frac{\rho l^4}{EI_0}} \quad (15)$$

$$\sum_{j=1}^{\mu} [M_{ij}^{..k} \theta_j + \left(K_{ij}^B + \gamma^2 \left(K_{ij}^G - M_{ij} \right) \right) \theta_j^k] - K_{ij}^D \left(\theta_j^{k-1} - 2\theta_j^k + \theta_j^{k+1} \right) - K_{ij}^C \left(\theta_j^{k-1} - 2\theta_j^k + \theta_j^{k+1} \right) = 0 \quad (16)$$

where $i=1,2,3,\dots$, etc. and

$$M_{ij} = \int_0^1 (1 - \alpha\xi)(1 - \beta\xi) \varphi_i(\xi) \varphi_j(\xi) d\xi \quad (17)$$

$$K_{ij}^B = \int_0^1 (1 - \alpha\xi)^3 (1 - \beta\xi) \varphi_{i,\xi\xi\xi}(\xi) \varphi_{j,\xi\xi\xi}(\xi) d\xi \quad (18)$$

$$K_{ij}^G = \int_0^1 G(\xi) \varphi_{i,\xi}(\xi) \varphi_{j,\xi}(\xi) d\xi \quad (19)$$

$$K_{ij}^D = \beta_D \varphi_i(\xi_D) \varphi_j(\xi_D) \quad (20)$$

$$K_{ij}^S = \beta_S \varphi_i(\xi_S) \varphi_j(\xi_S) \quad (21)$$

$$\gamma = \Omega T, \delta = \frac{r}{l}, \beta_D = \frac{K_D l^3}{EI_0},$$

$$\beta_S = \frac{K_S l^3}{EI_0}, \xi_D = \frac{a_D}{l}, \xi_S = \frac{a_S}{l}$$

$$G(\xi) = \delta(1-\xi) + \frac{1}{2}(1-(\alpha+\beta)\delta)(1-\xi^2) + \frac{1}{3}(\alpha\beta\delta - \alpha - \beta)(1-\xi^3) + \frac{1}{4}\alpha\beta(1-\xi^4)$$

Now by assembling n (total number of blades) sets of equation of motion (Eq. 16) corresponding to n blades, the total equation can be written as Eq. 24.

$$[M] \left\{ \ddot{\theta} \right\} + \left([K] + \gamma^2 \left([K^G] - [M] \right) \right) \{ \theta \} = 0 \quad (24)$$

For the modal analysis of the system, the matrix $\{\theta\}$ can be assumed as:

$$\{ \theta \} = e^{-j\omega\tau} \{ \eta \} \quad (25)$$

Here, ω is the dimensionless natural frequency of the system. Employing Eq. 25, Eq. 24 can be written as Eq. 26 which is used for modal analysis of the multi blade system.

$$\omega^2 [M] \{ \eta \} = \left([K] + \gamma^2 \left([K^G] - [M] \right) \right) \{ \theta \} \quad (26)$$

3. NUMERICAL RESULTS

A code was developed in MATLAB software to determine the natural frequencies of the systems. In order to evaluate the code, natural frequencies of a single rotating beam with uniform section ($\alpha=\beta=0$) were determined and compared with those reported in Ref [10]. The natural frequency was determined in the case of $\delta=1$ for different values of γ . The obtained natural frequencies and the corresponding values from the literature are in good agreement as presented in Table 1.

Table 1. Comparison of the first natural frequencies (results of MATLAB code and those of literature) in the bending vibration

γ	Present	Cheng and Yan (2006)
1	3.889	3.889
2	4.834	4.834
3	6.082	6.084
4	7.476	7.481
5	8.942	8.951

In this study, the natural frequencies of a multi blade system shown in Fig. 3 have been determined. This system contains 4 packets and each packet consists of 6 blades (total number of 24 blades). The first five bending mode functions of a cantilever beam are considered as the assumed modes of the blades. It is worth mentioning that, in the assembly step, the term related to the stiffness of the shroud should not be considered for the blades at the ends of the pockets, as there is not any shroud connection between packets. Fig. 4 shows the dimensionless natural frequencies which are plotted versus the dimensionless angular speed. The values of dimensionless parameters employed for the numerical analysis are given in Table 2.

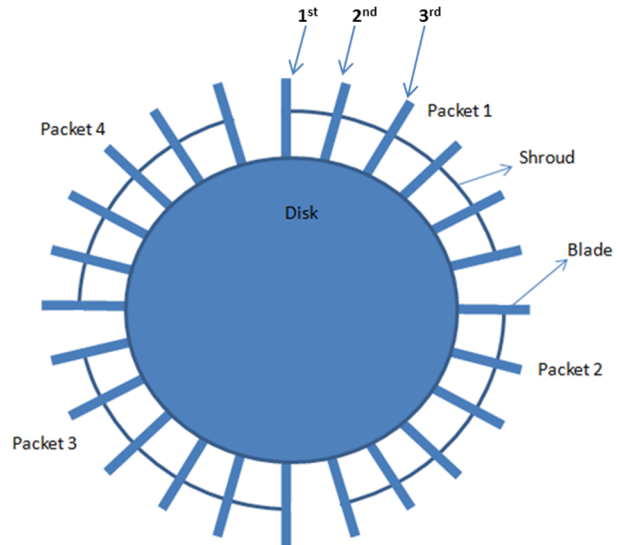


Figure 3. Configuration of the multi blade system

Table 2. The values of dimensionless parameters employed in the numerical analysis

α	β	δ	β_D	β_S	ξ_D	ξ_S
0.3	0.3	4	2e6	10	0.1	1

It is worth mentioning that ξ_D and ξ_S are the dimensionless terms related to the positions where the beam is connected to the disk and the shroud. β_D and β_S are the dimensionless stiffness values of disk and the shroud. δ is the ratio of the disk radius to the blade length (Eq. 22).

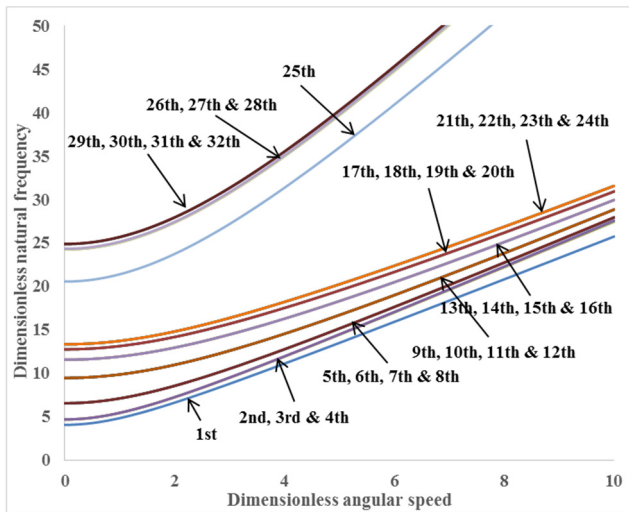


Figure 4. Dimensionless natural frequencies of the perfect multi blade system versus the dimensionless angular speed (γ)

In order to investigate the effect of an inconsistent blade on the natural frequency of the system, the dimensionless material properties of the one of the blades were changed. In case of defects such as crack, void and etc. in a blade, mechanical properties as well as Young's modulus would be significantly reduced. Addition to that, thermal shock would be other phenomena, which affect material properties of the blades. According to these kinds of material property variations, it is assumed that the Young's modulus of the inconsistent blade has been reduced to the 10% of the initiate value. Therefore, the values of β_D and β_S were considered to be $2e7$ and 100 , respectively (while these values were $2e6$ and 10 for perfect blades). Different conditions are considered in terms of the position of inconsistent blade in the system (1^{st} , 2^{nd} and 3^{rd} blade as shown in Fig. 3) and rotating speed of the system ($\gamma=0, 5$ and 10). It is worth mentioning that other positions for the inconsistent blade are similar to the considered positions due to symmetrical geometry. In Table 3, natural frequencies of the system for the mentioned conditions are presented.

Table 3. Natural frequency of the system without inconsistency and the systems with one inconsistent blade in the position of 1^{st} , 2^{nd} and the 3^{rd} blade (see Fig. 3)

$\gamma=0$				
Mode	Perfect	1^{st}	2^{nd}	3^{rd}
1	4.067	4.067	4.067	4.067
2	4.685	4.685	4.685	4.685
3	4.685	4.686	4.687	4.687
4	4.696	4.696	4.696	4.697
5	6.553	6.553	6.553	6.553
6	6.553	6.553	6.553	6.553
7	6.553	6.556	6.556	6.555
8	6.556	7.087	6.836	6.585

9	9.464	9.464	9.464	9.464
10	9.464	9.464	9.464	9.464
11	9.464	9.464	9.464	9.464
12	9.464	10.339	9.464	10.048
13	11.561	11.561	11.561	11.561
14	11.561	11.561	11.561	11.561
15	11.561	11.561	11.561	11.561
16	11.561	12.322	11.841	11.993
17	12.757	12.757	12.757	12.757
18	12.757	12.757	12.757	12.757
19	12.757	12.757	12.757	12.757
20	12.757	13.249	13.146	12.890
21	13.341	13.341	13.341	13.341
22	13.341	13.341	13.341	13.341
23	13.341	13.341	13.341	13.341
24	13.341	15.324	15.874	15.875
25	20.556	20.556	20.556	20.556

$\gamma=5$				
Mode	Perfect	1^{st}	2^{nd}	3^{rd}
1	13.505	13.505	13.505	13.505
2	14.459	14.458	14.459	14.459
3	14.459	14.462	14.462	14.462
4	14.488	14.488	14.489	14.489
5	15.156	15.156	15.156	15.156
6	15.157	15.157	15.157	15.157
7	15.157	15.163	15.163	15.162
8	15.166	15.396	15.282	15.174
9	16.706	16.706	16.706	16.706
10	16.706	16.706	16.706	16.706
11	16.706	16.706	16.706	16.706
12	16.706	17.314	16.706	17.104
13	18.316	18.316	18.316	18.316
14	18.316	18.316	18.316	18.316
15	18.316	18.316	18.316	18.316
16	18.316	19.055	18.576	18.722
17	19.531	19.531	19.531	19.531
18	19.531	19.531	19.531	19.531
19	19.531	19.531	19.531	19.531
20	19.531	20.125	19.995	19.686
21	20.243	20.243	20.243	20.243
22	20.243	20.243	20.243	20.243
23	20.243	20.243	20.243	20.243
24	20.243	23.660	25.022	25.024
25	35.992	35.992	35.992	35.992

$\gamma=10$				
Mode	Perfect	1^{st}	2^{nd}	3^{rd}
1	25.748	25.748	25.748	25.748
2	27.469	27.466	27.468	27.469
3	27.469	27.481	27.481	27.482
4	27.579	27.578	27.580	27.582
5	27.922	27.925	27.925	27.923
6	27.932	27.931	27.931	27.931
7	27.932	27.950	27.948	27.936
8	27.958	28.071	28.010	27.960
9	28.852	28.852	28.852	28.852
10	28.853	28.853	28.853	28.853
11	28.853	28.853	28.853	28.853

12	28.853	29.249	28.853	29.109
13	29.957	29.957	29.957	29.957
14	29.957	29.957	29.957	29.957
15	29.957	29.958	29.958	29.958
16	29.958	30.534	30.155	30.268
17	30.933	30.933	30.933	30.933
18	30.933	30.933	30.933	30.933
19	30.933	30.933	30.933	30.933
20	30.933	31.467	31.346	31.068
21	31.579	31.579	31.579	31.579
22	31.579	31.579	31.579	31.579
23	31.579	31.579	31.579	31.579
24	31.579	35.950	38.775	38.780
25	62.115	62.115	62.115	62.115

As observed in Table 3, natural frequencies of the first 7 mode are not changed, which indicates that any inconsistency in one of the blades of a multi blade system will not affect system dynamic characteristics. Variation of the natural frequencies of the system due to inconsistent blade only can be observed for the 8th, 12th, 16th, 20th and 24th modes of the system. In order to have a better presentation of the variation in natural frequencies, the values corresponding to the 8th, 12th, 16th and 24th modes are presented in Table 4.

Table 4. Variation of the natural frequencies of the system due to inconsistent blades for the 8th, 12th, 16th and 24th mode (IBN: Inconsistent Blade Number, NF: Natural Frequency, NFP: Natural Frequency of Perfect system, PV: Percentage of Variation in natural frequency due to inconsistent blade)

	$\gamma=0$			
	IBN	1 st	2 nd	3 rd
8 th mode	NF	7.087	6.836	6.585
	NFP	6.556	6.556	6.556
	PV	7.482	4.092	0.436
12 th mode	NF	10.339	9.464	10.048
	NFP	9.464	9.464	9.464
	PV	9.247	0	6.174
16 th mode	NF	12.322	11.841	11.993
	NFP	11.561	11.561	11.561
	PV	6.575	2.421	3.731
20 th mode	NF	13.249	13.146	12.89
	NFP	12.757	12.757	12.757
	PV	3.711	2.956	1.034
24 th mode	NF	15.324	15.874	15.875
	NFP	13.341	13.341	13.341
	PV	14.864	18.989	18.995

	$\gamma=5$			
	IBN	1 st	2 nd	3 rd
8 th mode	NF	15.396	15.282	15.174
	NFP	15.166	15.166	15.166
	PV	1.492	0.762	0.052
12 th mode	NF	17.314	16.706	17.104
	NFP	16.706	16.706	16.706
	PV	3.64	0	2.377
16 th mode	NF	19.055	18.576	18.722
	NFP	18.316	18.316	18.316
	PV	4.036	1.42	2.219
20 th mode	NF	20.125	19.995	19.686
	NFP	19.531	19.531	19.531
	PV	2.949	2.319	0.785
24 th mode	NF	23.66	25.022	25.024
	NFP	20.243	20.243	20.243
	PV	16.876	23.606	23.616

	$\gamma=10$			
	IBN	1 st	2 nd	3 rd
8 th mode	NF	28.071	28.01	27.96
	NFP	27.958	27.958	27.958
	PV	0.404	0.185	0.008
12 th mode	NF	29.249	28.853	29.109
	NFP	28.853	28.853	28.853
	PV	1.373	0	0.887
16 th mode	NF	30.534	30.155	30.268
	NFP	29.958	29.958	29.958
	PV	1.923	0.657	1.037
20 th mode	NF	31.467	31.346	31.068
	NFP	30.933	30.933	30.933
	PV	1.696	1.318	0.434
24 th mode	NF	35.95	38.775	38.78
	NFP	31.579	31.579	31.579
	PV	13.843	22.788	22.804

As illustrated in Table 4, the highest values of variation in natural frequencies are observed for 24th mode. Among the other four modes, the 12th and 16th modes demonstrate higher values of variation. The natural frequencies of the system mostly are increased due to the inconsistent blades. As observed in Table 4, in case of the 1st blade as the inconsistent one, the values of variations are more than other cases, except for the 24th mode. However, for the 24th mode, the values of variation become significant, in case of placing inconsistent blade in the position of 2nd or 3rd blade. Comparison of the values of variation for different angular speeds of the system, lead to the fact that higher variation values in the natural frequencies of the system are observed for the lower angular

speed ($\gamma=0$), except for the 24th mode in which lower variation values are related to the lower angular speed. For better presentation of the way of variation, the natural frequencies of the systems with the inconsistency in the position of 1st, 2nd and 3rd blades are plotted versus system angular speeds in Fig. 5, Fig. 6 and Fig. 7.

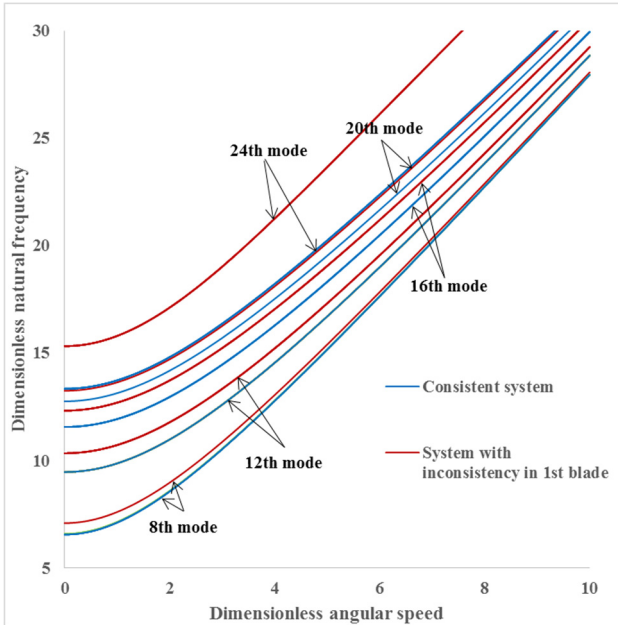


Figure 5. Dimensionless natural frequencies of a multi blade system without any inconsistency (perfect system) and a system containing one different blade in material parameters ($\beta_D = 2e7$ and $\beta_S = 100$) in the position of the 1st blade (see Fig.3) versus the dimensionless angular speed (γ)

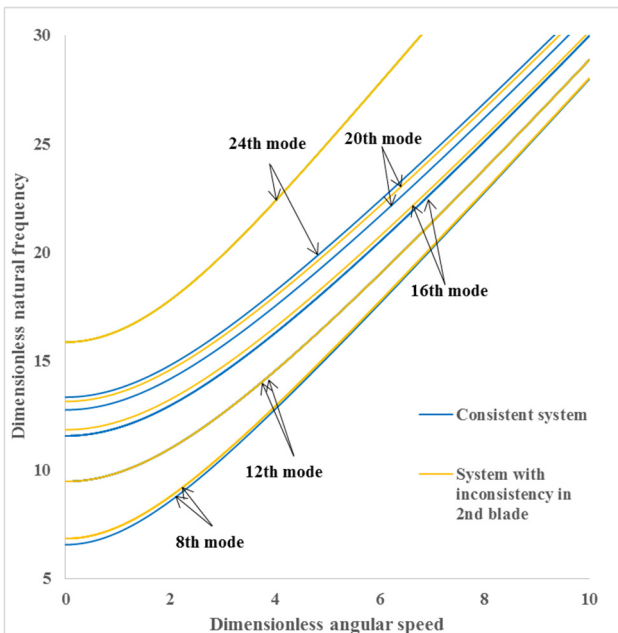


Figure 6. Dimensionless natural frequencies of a multi blade system without any inconsistency (perfect system) and a system containing one different blade in material parameters ($\beta_D = 2e7$ and $\beta_S = 100$) in the position of the 2nd blade (see Fig.3) versus the dimensionless angular speed (γ)

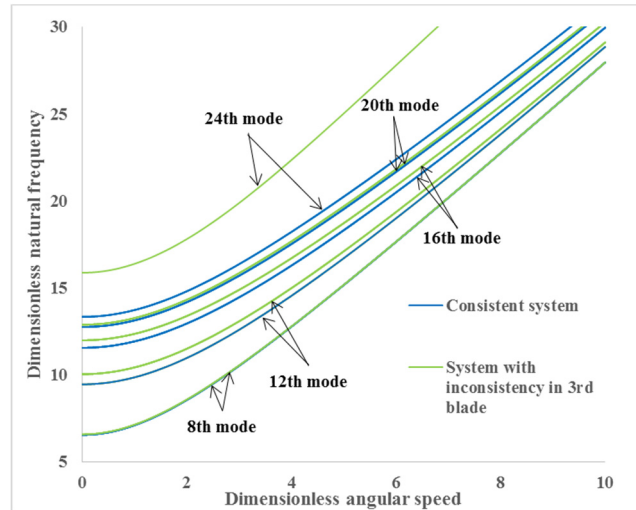


Figure 7. Dimensionless natural frequencies of a multi blade system without any inconsistency (perfect system) and a system containing one different blade in material parameters ($\beta_D = 2e7$ and $\beta_S = 100$) in the position of the 3rd blade (see Fig.3) versus the dimensionless angular speed (γ)

As shown in Fig. 5, there is a significant jump in the natural frequency of the 24th mode. This jump in natural frequencies of the 24th mode seems to be more for the higher values of angular speed. On the other hand, for the other four modes, the rate of variation in natural frequencies due to the inconsistency is decreased while the angular speed is increased. As same as observed in Fig. 5, the highest values of natural frequency jump are observed for the 24th mode in Fig. 6 which relates to the case of 2nd blade as the inconsistent blade. In this case (Fig. 6), the natural frequencies of the 12th mode are not affected due to inconsistency which can be attributed to the mode shapes of the blades. Similar to those of Fig. 5 and Fig. 6, again 24th mode shows the highest jump in natural frequency curves of Fig. 7 which relates to the system with inconsistency in the 3rd blade. This time, the 8th mode is not affected due to the inconsistency which again can be attributed to the mode shapes. In order to investigate the effect of mode shape on the way of variation in natural frequencies, the 8th and 12th mode shapes for consistent multi blade system (considering the first assumed bending mode as the mode of deflection) are plotted in Fig. 8 and Fig. 9.

The 3rd blade of the system has very low displacement in the 8th mode shape plotted in Fig. 8. This is the reason why the 8th mode rarely affected by the inconsistency in the position of 3rd blade. Additionally, as shown in Fig. 9, since 2nd blade in 12th mode shape is not displaced, this mode is not affected by the inconsistency of the system in the position of 2nd blade. These information about the way of variation in system natural frequencies due to inconsistency can be employed to avoid the multi

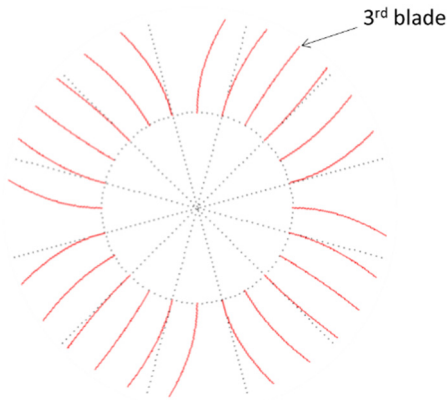


Figure 8. The 8th mode shape of the consistent multi blade system

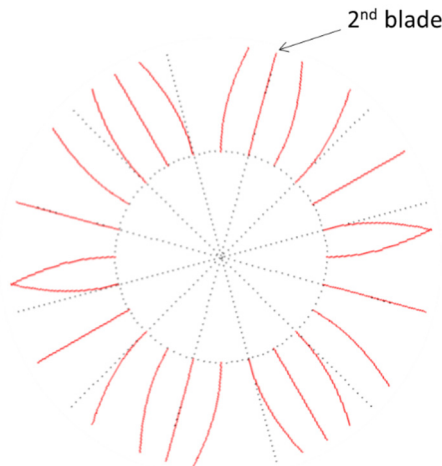


Figure 9. The 12th mode shape of the consistent multi blade system

blade systems to work in the frequencies close to their natural frequencies. This can be applicable, especially for the systems, in which, there is a high possibility to have an inconsistent blade. In the other word, natural frequencies of the multi blade systems, which have the possibility of system inconsistency, should be counted as an important issue in the dynamic behavior of the system.

4. CONCLUSIONS

In this study, the effect of the existence of an inconsistent blade on the natural frequencies of a rotating multi blade system was investigated. For this purpose, the natural frequencies of a perfect blade system and a system containing one different blade in material property have been compared for a domain of dimensionless angular speed. Based on the comparison results, although the majority of the natural frequencies (including first seven modes) did not show significant variation, however, some of the

natural frequencies (here, 8th, 12th, 16th, 20th and 24th modes) were affected (natural frequency of the system is increased) in case of an inconsistent blade in different possible positions. Increasing in natural frequency was more significant for the 25th mode. The obtained information about the way of variation in system natural frequencies due to inconsistency can be employed to keep the multi blade systems working frequencies away from the system natural frequencies, especially for the systems, in which, there is a high possibility to have an inconsistent blade.

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