Acoustic Fields of Circular Cylindrical Hydroacoustic Systems with a Screen Formed From Cylindrical Piezoceramic Radiators

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Abstract: - The "through" problem of radiation of sound by a screened circular cylindrical hydroacoustic system was solved by the method of coupled fields in multiply connected regions. At the same time, the specific values of the electric excitation voltages of the radiators are put in correspondence with the sound pressures formed by the system at any point of the environment surrounding the system. Analytical expressions describing the acoustic, mechanical and electrical fields of the system are obtained. These expressions take into account the interaction of the elements of the system in the external environment during the formation of the acoustic field and the interaction of these physical fields in piezoceramic radiators with circular polarization when they convert electric energy to acoustic. A violation of the radial symmetry of the acoustic loading of the radiators in the system is established and quantitated, while maintaining the radial symmetry of their electric loading. A consequence of this is the expansion and enrichment of the spectrum of natural frequencies of the radiators in the system. Resonance frequencies appear, which are 1.5-3 times smaller than the natural frequency of piezoceramic envelopes of radiators with comparable radiation efficiency.

Keywords: - Acoustic fields; Circular system with a screen; Cylindrical piezoceramic radiator; Radial symmetry.

1. INTRODUCTION

Circular cylindrical hydroacoustic systems of radiators have found wide application in practical hydroacoustics. In this case, as the radiators, they often use cylindrical piezoceramic transducers located near the acoustic screen placed in the internal cavity of the systems. The maximum energy efficiency of such systems in the radiation regime is ensured only if they work in the frequency range of the mechanical resonance of the converters forming the systems.

In the classical formulation of problems on the radiation of sound by systems of oscillating bodies, one usually proceeds from the assumption that the normal components of the field of vibrational velocities on the surfaces of bodies are given [1] [2] [3]. After solving the problems, interesting and important local and integral characteristics of the acoustic fields are calculated.

However, this formulation has a serious flaw. Setting the values of the vibrational velocities fundamentally excludes the possibility of taking into account the variation of these velocities due to the interaction of the elements of the systems along the acoustic field through the external medium when the sound is radiated. Also, the possibility of taking into account the interactions of electric, mechanical and acoustic fields in radiators during the process of their conversion of electric energy into acoustic energy and the reaction of the surrounding medium to excitation of sound waves in it is excluded. Such a simplified approach to solving sound radiation problems can be justified when the transducers are spaced a considerable distance apart, and their own mechanical impedances significantly exceed the radiation impedances. However, in real systems, the distances between their elements are small, and the intrinsic mechanical impedances in the resonance frequency region are comparable with the radiation impedances. Therefore, the interaction of the elements of the system through the external medium, caused by the multiple exchange of radiated and reflected waves between them, will really affect the vibrational speed of the radiators in the system. In turn, the occurrence of the interaction of physical fields in the radiators of the system in the process of energy transformation by them also leads to a change in the values of the vibrational velocities, different for different radiators of the system. This is due to the fact that in cylindrical piezoceramic transducers with a radially symmetrical method of their electric loading, only one eigenmode of their oscillations is excited and energy is "pumped" into all converters of the system only on this one-zero mode of their oscillations. At the same time, the presence of an acoustic shield in the composition of a circular system disrupts the radial symmetry of the acoustic loading of its radiators. In transducers with "broken" radial symmetry, there are, in addition to zero mode, subsequent modes of their oscillations whose amplitudes are comparable with the amplitude of the zero mode [4] [5] [6]. This means that under these conditions there is an effective redistribution of energy "pumped" into the system at the zero mode of oscillations of its radiators, between the subsequent forms of their oscillations. Naturally, this leads to a change in the values of the vibrational velocities of the radiators of the antenna. The real oscillatory velocities of the radiators in the system will be determined by the level of the electric energy supplied to each of them, the value of the working frequency, the composition of the piezoceramics used, the mutual arrangement of the elements in the system, and other factors [6] [7] [10].

As a consequence, this will lead to a change in the properties of the acoustic fields of hydroacoustic systems.

The aim of this work is to study the regularities of the formation of acoustic fields by cylindrical hydroacoustic systems with a screen taking into account the interaction of physical fields in their radiators.

2. CALCULATED RELATIONS

Let us determine the acoustic fields of the radiators system with an acoustic screen in the internal cavity of the system. The system (see Fig. 1) consists of N parallel circular cylindrical radiators 1 and an acoustically soft screen 2.

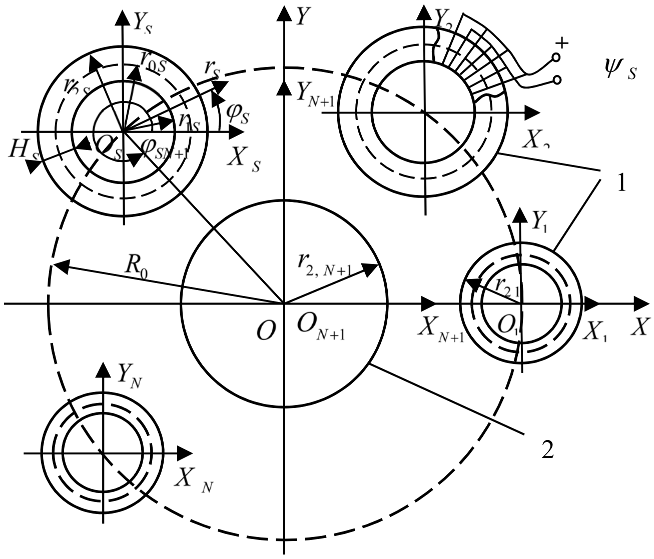

Figure 1. Normal section of the cylindrical system of radiators with a screen

The radiators with an average radius r_{0S} and thickness h_S ($S = 1,\ldots,N$) are formed from M_S electrically shunt piezoceramic prisms with circumferential polarization, rigidly glued together. To the prism electrodes a harmonically varying electric voltage $\psi_S = \psi_{0S}e^{-i\omega t}$ with a frequency ω (i – imaginary unit) is brought in time. The inner cavity of radiators is filled with a medium with density ρ_S and speed of sound c_S. The system of the radiators whose longitudinal axes are arranged along the circumference by a radius R_0, and the screen with an external radius r_{2N+1} is located in a medium with a density ρ and a speed of sound c. The coordinate systems used to solve the problem are shown in Fig. 1.

Acoustic fields of a circular cylindrical system of radiators with a screen can be determined by a joint solution of a system of differential equations consisting of:

- equations of forced electrostatics for piezoceramics: $\vec{E}_S = -grad\psi_S$; $div\vec{D}_S = 0$; S=1,…,N; equations of motion of thin piezoceramic shells of radiators with circular polarization in displacements [8] [9]:

$$\left(1+\beta_S\right)\frac{\partial^2 u_S}{\partial \varphi_S^2}+\frac{\partial w_S}{\partial \varphi_S}-\beta_S\frac{\partial^3 w_S}{\partial \varphi_S^3}=a_S\gamma_S\frac{\partial^2 u_S}{\partial t^2},$$

$$-\frac{\partial u_S}{\partial \varphi_S}+\beta_S\left(\frac{\partial^2 u_S}{\partial \varphi_S^2}-\frac{\partial^4 w_S}{\partial \varphi_S^4}\right)-w_S+ \qquad (1)$$

$$+\frac{\varepsilon_{33\,S} r_{OS}}{C^{E}_{33\,S}} r_{0\,S} E_{\varphi S} + \frac{a_S}{h_S} q_{r\,S} = a_S \gamma_S \frac{\partial^2 w_S}{\partial t^2};$$

S=1,…,N;

- the Helmholtz equation, describing the motion of media inside and outside the radiators of the system and outside its screen: $\Delta\Phi_{iS} + k^2_{iS}\Phi_{iS} = 0$; S=1,…,N; here $\vec{E}_S$ and $\vec{D}_S$ the vectors of intensity and induction of the electric field of the S-th radiator; Δ is the Laplace operator; Φ_{iS} – speed potential of the S-th radiator inside ($\Phi_{iS} = \Phi_{1\,S}$) and outside it ($\Phi_{iS} = \Phi_S$); k_{iS} – the wave numbers of the media inside ($k_{iS} = k_S$) and outside ($k_{iS} = k$) of the S-th radiator; u_S and w_S – the circumferential and radial components of the displacement vector of the points of the middle surface of the piezoceramic shell of the S-th radiator; $\beta_S = h^2_S \big/ 12 r^2_{0\,S}\left(1 + e^2_{33\,S} \big/ C^E_{33\,S}\varepsilon^0_{33\,S}\right)$; $a_S = r^2_{OS} \big/ C^E_{33\,S}$; q_{rS} – external load of the S-th radiator; $C^E_{33\,S}$, $\varepsilon^0_{33\,S}$, $e_{33\,S}$, γ_S – respectively, the elastic modulus at zero electrical tensity, the dielectric constant at zero strain, the piezoelectric constant and the material density of the piezoceramic shell of the S-th radiator of the system.

Acoustic boundary conditions include the Sommerfeld conditions, the conditions for the absence of singularities in the internal regions of all radiators of the system, and the condition on the surface of the screen $r_{2\,N+1}$ in the form:

$$\Phi\left(r_{N+1}, \varphi_{N+1}\right) = 0;\ 0 \le \varphi_{N+1} \le 2\pi;\ r_{N+1} = r_{2\,N+1}, \quad (2)$$

where $\Phi\left(r_{N+1}, \varphi_{N+1}\right)$ is the total acoustic field of a circular cylindrical system with a screen in the surrounding space, expressed in local coordinates of the screen.

The electric boundary conditions consist in specifying the electric field strength in the piezoceramic shell of each of the S=1,...,N radiators of the system in the form:

$$E_{\varphi_S} = -\frac{\psi_{0S} M_S}{2\pi r_{0S}}. \quad (3)$$

We divide the entire multiply connected region of existence of the physical fields of a circular system with a screen into a number of partial regions (Fig. 1).

In this case, in order to ensure completeness, the system of initial relations of problem Eqs. (1)–(3) must be supplemented by the kinematic and dynamic conditions for conjugation of fields at the interfaces of the partial regions:

$$-\frac{\partial\Phi_{1S}\left(r_S, \varphi_S\right)}{\partial r_S} = \frac{\partial w_S}{\partial t},\ 0 \le \left|\varphi_S\right| \le \pi,$$

$$r_S = r_{1S} = r_{0S} - \frac{h_S}{2},$$

$$-\frac{\partial\Phi\left(r_S, \varphi_S\right)}{\partial r_S} = \frac{\partial w_S}{\partial t},\ 0 \le \left|\varphi_S\right| \le \pi, \quad (4)$$

$$r_S = r_{2S} = r_{0S} + \frac{h_S}{2},$$

$$q_{r_S} = -\left(\frac{\rho\partial\Phi}{\partial t} - \frac{\rho_S \partial\Phi_{1S}}{\partial t}\right),\ 0 \le \left|\varphi_S\right| \le \pi,\ r = r_{0S},$$

S=1,…,N,

where $\Phi = \sum_{S=1}^{N+1} \Phi_S$ is the velocity potential of the total acoustic field, expressed in local coordinates of the S-th radiator.

Let us express the mechanical u_S, w_S and acoustic Φ_S, Φ_{1S} fields of all the S=1,...,N radiators and the screen S=N+1 of the circular system in the form of expansions of series on the angular and wave functions of the circular cylinder:

$$u_S = \sum_n u^{(S)}_n e^{in\varphi_1};\ w_S = \sum_n w^{(S)}_n e^{in\varphi_S}, \quad (5)$$

S=1,…,N;

$$\Phi_S = \sum_n A^{(S)}_n H^{(1)}_n\left(k\, r_S\right) e^{in\varphi_S},\ S=1,\dots,N+1;$$

$$\Phi_{1S} = \sum_n B^{(S)}_n J_n\left(k_S\, r_S\right) e^{in\varphi_S},\ S=1,\dots,N. \quad (6)$$

Eqs. (6) use the traditional notation of cylindrical functions. The unknown coefficients $u^{(S)}_n$, $w^{(S)}_n$, $A^{(S)}_n$, $B^{(S)}_n$ appearing in them are determined from the functional Eqs. (1), the boundary conditions Eqs. (2), and the conjugation conditions for the fields Eqs. (4). The necessary translation of coordinate systems is carried out on the basis of addition theorems for cylindrical wave functions [1]:

$$H_m^{(1)}\left(kr_q\right)e^{im\varphi_q}=\sum_n J_n\left(kr_S\right)H_{m-n}^{(1)}\left(kr_{qS}\right)e^{i(m-n)\varphi_{qS}}e^{in\varphi_S}, \quad (7)$$

where r_{qS}, φ_{qS} are the polar coordinates of the origin of the coordinate system O_S in the coordinates of the q-th system.

Algebraization of systems of functional Eqs. (1), (2), (4) using relations Eqs. (3), (5), (6), (7) and completeness properties and orthogonality of systems of angular functions on the interval [0, 2π] allows to obtain to determine the unknown coefficients of the expansions Eqs. (5) and (6), an infinite system of linear algebraic equations of the form:

$$B_n^{(S)}J_n'\left(k_S r_{1S}\right)+ic_S w_{nS}=0, \; S=1,\ldots,N, \; n=-\infty,0,\infty;$$

$$icw_{nS}-\left[\begin{array}{r}A_n^{(S)}H_n^{(S)'}\left(kr_{2S}\right)+\\ +\sum\limits_{\substack{q=1\\q\neq S}}^{N+1}\sum\limits_m A_m^{(q)}J_n'\left(kr_{2S}\right)H_{m-n}^{(1)}\left(kr_{qS}\right)e^{i(m-n)\varphi_{qS}}\end{array}\right]=0,$$

$$S=1,\ldots,N; \quad (8)$$

$$A_n^{(N+1)}H_n^{(1)}\left(kr_{2N+1}\right)+ +\sum_{q=1}^{N}\sum_m A_m^{(q)}J_n'\left(kr_{2N+1}\right)H_{m-n}^{(1)}\left(kr_{qN+1}\right)e^{i(m-n)\varphi_{qN+1}}=0,$$

$$n=-\infty,0,\infty;$$

$$R_n^{(S)}w_{nS}+\frac{\alpha_S}{h_S}i\omega\rho\left[\begin{array}{r}A_n^{(S)}H_n^{(1)'}\left(kr_{2S}\right)+\\ +\sum\limits_{\substack{q=1\\q\neq S}}^{N+1}\sum\limits_m A_m^{(q)}J_m\left(kr_{2S}\right)H_{m-n}^{(1)}\left(kr_{qS}\right)e^{i(m-n)\varphi_{qS}}\end{array}\right]-$$

$$-\frac{\alpha_S}{h_S}i\omega\rho_S B_n^{(S)}\left(k_S r_{1S}\right)J_n\left(k_S r_{1S}\right)=-\frac{e_{33S}}{C_{33S}^E}\frac{M_S\psi_{0S}}{2\pi}\int_0^{2\pi}e^{jn\varphi}d\varphi,$$

$$n=-\infty,0,\infty, \; S=1,\ldots,N;$$

where

$$R_n^{(S)}=n^2\left(1+\beta_S n^2\right)^3\frac{\left(1+\beta_S n^4-\alpha_S\gamma_S\omega^2\right)\times\left(n^2+\beta_S n^2-\alpha_S\gamma_S\omega^2\right)}{\left(1+\beta_S\right)n^2-\alpha_S\gamma_S\omega^2},$$

the prime denotes the derivative of the function.

3. ANALYSIS OF NUMERICAL RESULTS

Let us apply the formulated equations for a quantitative evaluation of the frequency and angular characteristics of the acoustic fields of circular cylindrical hydroacoustic systems with a screen in the circular radiation mode.

In the calculations, the following values of the parameters of the radiators, systems and media were taken: piezoelectric ceramics with a density γ=7210 kg/m^3; piezoelectric constant $e_{33}=d_{33}\cdot C_{33}^{\mathrm{E}}$, where d_{33}=286·10^{-12} (C/N) is the piezoelectric modulus; dielectric permeability $\varepsilon_{33}^0=1280\cdot 8{,}85\cdot 10^{-12}$ F/m and modulus of elasticity $C_{33}^E=13{,}6\cdot 10^{-10}$ N/m^2; average radius of the piezoceramic shell $r_0=0{,}068$ m at wall thickness $h=0{,}008$ m, number of prisms $M_S=48$; the number of identical radiators in the N=3 system when they are arranged in a circle $R_0=0{,}147$ m uniformly; outer radius of the screen $r_{2N+1}=0{,}072$ m; excitation voltage $\psi_0=200$ V for all S; the medium filling the external space of the system – water (ρc=1,5·10^6 kg/m^{2}s); The internal cavity of the radiators is evacuated ($\rho_S c_S=0$ for all S).

The frequency dependences of the amplitudes and phases of the acoustic fields of the radiators of the system, calculated on the surfaces of the radiators at points with coordinates ($r_S=r_{2S}, \varphi_S=0°, 120°, 240°$, for S=1,2,3), are shown in Fig. 2 and Fig. 3.

Curves 2 in Fig. 2 shows the frequency dependences of the amplitudes and phases of the radiator without a screen. Their analysis in the pre-resonance (0-7000 Hz), resonance (7000-14000 Hz) and postresonance (14000-20000 Hz) regions suggests the following. In the circular radiation of a circular system with a screen, all its radiators are in the same conditions from the point of view of interaction over the acoustic field, which is reflected by the curves in Fig. 2. In addition, with a small number of radiators in such a system, the main effect on the formation of its acoustic field is the exchange of acoustic waves between the radiators and the screen. In the pre-resonance region this acoustic interaction causes a number of new additional resonance frequencies, the values of which are 1.5-3 times lower than the values of the fundamental resonance frequency of the piezoceramic envelope of the radiators. In this case, the amplitudes of the acoustic pressures at these additional resonance frequencies are comparable or much larger than the pressure amplitude of a single unshielded radiator (curve 2 in Fig. 2). In the resonance region, the acoustic interaction between the radiators and the screen transforms the smooth course of the frequency dependence of the amplitude of a single radiator without a screen into a curve with several dips, one of which falls precisely on the resonance frequency of a single radiator. In this case, the amplitude of the pressure at the frequency of the central resonance of the radiators in the system under study is several times higher than the resonance amplitude of the pressure of a single radiator. In the postresonance region, the acoustic interaction of the radiators with the screen in the system again transforms the smoothly decreasing curve of a single radiator without a screen into a multiresonance frequency dependence with amplitudes both significantly larger and considerably smaller in comparison with this radiator.

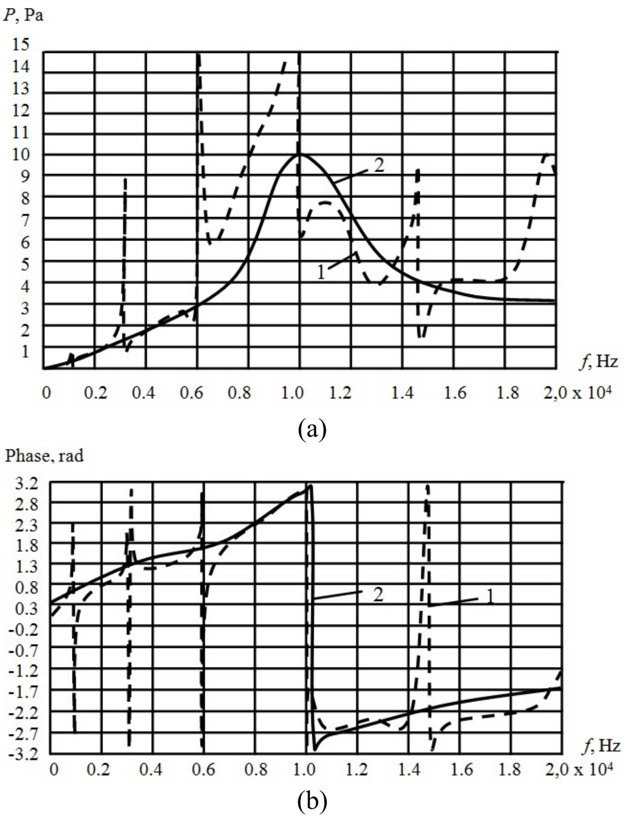

Figure 2. Frequency dependences of the amplitudes (a) and phases (b) of the acoustic pressure of vacuum radiators in the circular radiation of a circular system with a screen: 1 - radiator in the system; 2 - single radiator

Analysis of the phase characteristics of the radiators in the circular antenna with the shield (see Fig. 2b) indicates that they cross the frequency axis several times, which indicates a multiple change in the character of the total mechanical impedance of the radiators from elastic to inertial and back. Since the frequencies where the phase characteristic crosses the abscissa axis are the eigenfrequencies of the system "radiators-screen-environment", it can be asserted that acoustic interaction leads to the expansion and enrichment of the eigen-frequency spectrum of the radiating system under consideration. Of particular practical interest is the fact that without changing the dimensions of the radiating system, it becomes possible for it to radiate signals at resonance frequencies that are 1.5-3 times smaller than the intrinsic resonance of the piezoceramic shells of the radiators of the system, but with an efficiency close to the efficiency of these shells at the natural frequency.

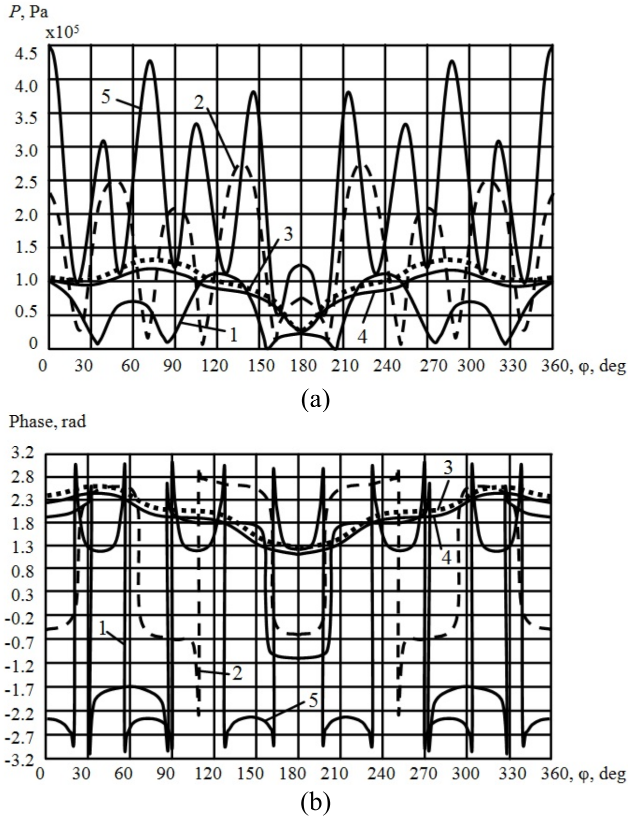

Figure 3. Angular distribution of the amplitudes (a) and phases (b) of the acoustic pressure on the surface of the vacuumed radiator in the circular radiation of a circular system with a screen at different frequencies:
1 – 3130 Hz, 2 – 6130 Hz, 3 – 8120 Hz, 4 – 8370 Hz ,
5 – 9990 Hz

Of considerable interest is the definition of the physical mechanism of the interaction of the elements of the circular system with the screen along the acoustic field on the formation of its acoustic field. First of all, we note that with the selected method of electric excitation of piezoceramic radiators with circumferential polarization, their electric fields are radially symmetrical irrespective of whether the radiators work in the system or as single ones. The latter significantly influences the nature of their acoustic loading due to the radiation of sound. For a single circular cylindrical radiator, the conditions of its radiation loading by the surrounding medium are radially symmetric. The presence of radial symmetry of electric and acoustic loads causes the possibility of "pumping" energy into the mechanical field of a piezoceramic radiator only at the zero mode of oscillations of its cylindrical shell. When radiators work as part of a circular system, the radial symmetry of their electrical loading does not change, so the energy is “pumped” into their mechanical fields, as before, only at the zero mode of their oscillations. At the same time, the appearance of acoustic interaction between elements of a circular system with a screen disrupts the radial symmetry of the acoustic loading of its radiators. In quantitative terms, the degree of this disorder depends on many physical factors. This is evidenced by the curves of the angular distribution of the acoustic pressure on the surfaces of the radiators (Fig. 3).

Analysis shows that the presence of acoustic interaction is the physical reason for changing the uniform angular distribution of the acoustic pressure on the surface of a single radiator to a very inhomogeneous one in the case of placing this radiator in the composition of a circular system with a screen. This inhomogeneity occurs for any frequency in the entire frequency range under study. Here we distinguish a number of features. First, the greatest degree of inhomogeneity in the angular distribution of acoustic pressure occurs at the frequencies to which resonances of the shells correspond. Secondly, in the region of angles that adjoin the acoustic screen of the system, the acoustic pressure on the surfaces of the radiators is one of the lowest.

Thus, a violation of the radial symmetry of the acoustic loading of the radiators has been established and quantitatively determined because of the acoustic interaction of the circular system with the screen while maintaining it under electric loading. This symmetry breaking caused the appearance of the piezoceramic shells following the zero mode of vibration in the mechanical fields of the radiators of a circular system with a screen and the effective redistribution of energy “pumped” into the radiators at the zero mode of their oscillations between these modes [2].

4. CONCLUSIONS

By the method of coupled fields in multiply connected domains, analytical expressions are obtained describing the acoustic fields of circular cylindrical systems with a screen in the internal cavity, formed from circular cylindrical piezoceramic radiators. The expressions obtained make it possible to match the electric stresses exciting the piezoceramic radiators of the system with the values of the acoustic field pressures in the surrounding media.

It is shown that taking into account the interaction of the elements of the circular system with the shield through the surrounding medium leads to a violation of the radial symmetry of the acoustic loading of the cylindrical radiators while maintaining the radial symmetry of their electric loading.

It is established that when excitation of piezoceramic radiators of a circular system with a screen of the same voltage, acoustic interaction leads to the expansion and enrichment of the spectrum of the natural frequencies of the radiators in the system.

Of practical interest is the appearance of resonances of radiators with frequencies 1.5-3 times smaller than the frequencies of the intrinsic resonances of piezoceramic envelopes of radiators, with comparable effectiveness of their radiation levels.

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