Effects of External Excitation Frequency Variations on the Vibrations of a Cubic Nonlinear Equation

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Abstract: - The effect of external excitation frequency variation on the vibrations of a damped cubic nonlinear equation is investigated. Three types of frequency variations are considered: Decaying type variation, built-in type variation, and harmonic variation. Approximate solutions are constructed using the Method of Multiple Scales. Steady-state deterministic solutions are possible in the first two cases for long time scales. In the harmonic variation case, steady-state solutions are not possible, and the amplitudes and phases do not tend to a constant value. Direct integration of the model depicts the chaotic behavior of the system.

Keywords: - forced vibrations, nonlinear dynamics, frequency variation, perturbation methods.

1. INTRODUCTION

Mechanical and electrical systems often display oscillatory behavior. Vibrations of continuous systems such as strips and beams are examples of mechanical vibratory systems. Alternating currents are examples of electrical vibratory systems. In modeling nonlinear oscillatory behavior, Duffing-type equations frequently appear. They may be directly derived from the discrete systems, such as the vibrations of a concentrated mass attached to a nonlinear spring, or can be reduced from a partial differential equation after the discretization process, such as the vibrations of continuous systems. Duffing equations may possess several different types of nonlinearities. The nonlinearities may be of cubic type [1-9], of mixed cubic-quintic type [8, 10-14], of only quintic type [8], of mixed quadratic-cubic type [8, 9, 15, 16], of cubic-quintic-septic type [17-19], or of a functional type [20, 21]. Since the models do not possess exact analytical solutions, approximate analytical solutions appeared in the mentioned literature. These include variants of the perturbation methods such as the Lindstedt-Poincaré method, the Method of Multiple Scales, the Multiple Scales Lindstedt Poincaré Method, Harmonic Balance Method, Shift-Perturbation Method, and Perturbation Iteration method. Other approximate techniques include Homotopy Analysis Method, Taylor Wavelets, Differential Transform Method, Fourier coefficients, Jacobi Elliptic functions, Laplace Decomposition Algorithm, etc. In this work, the Method of Multiple Scales has been employed in search of approximate analytical solutions. The method has proven to be most effective in analyzing nonlinear dynamical systems [22, 23].

In all the previous work [1-23], the physical parameters remain constant during vibrations. In a recent study [24], the variations of damping, nonlinearity coefficient, and external excitation amplitude were investigated using the Method of Multiple Scales. However, the variations in the external excitation frequency were not addressed in that work. The main goal, therefore, in this study is to treat such variations and their effects on the dynamics of the system. Three different mathematical functions are considered to express the variations in the external excitation frequencies: The decaying type, the built-in type, and the harmonic type. In the decaying type excitations, the steady state solutions tend to constant values, as can also be numerically verified. In experimental applications, it is a usual practice to increase or lower the excitation frequency. In this first case investigated, the aim is to search for transient and steady-state solutions when the excitation frequency is gradually lowered from a given constant value. In the built-in type excitations, the steady state solutions tend to harmonic solutions with amplitudes and phases retaining constant values. This second case of an exponentially increasing frequency can occur in electronic systems that are turned on, and takes some time before the excitation frequency reaches its nominal value. In signal processing applications, one usually encounters frequency shifts due to noise, which has to be eliminated [25]. Harmonically varying excitation frequencies about a mean frequency may be modeled to incorporate the noise in such systems. The response behavior is the most complex one compared to the previous two cases, leading to chaotic-type behavior. The amplitudes and phases do not tend to constant values and remain variable. The direct numerical solutions are also expressed in the differential space, which is proposed as a recent tool to display the nonlinear behavior [26].

Nonlinear oscillating circuits corresponding to a similar model of the Duffing-Holmes equation were shown experimentally to exhibit chaotic behavior [27].

In summary, the three models used in expressing the variations in the external excitation frequencies were not studied openly in the previous work, and the goal in this study is to exhibit the dynamic behavior in such cases. While the first two cases of decaying frequencies and built-in type frequencies lead to deterministic solutions, the third case of harmonically varying frequency may lead to more complex behavior, such as chaotic motions.

2. DECAYING EXTERNAL EXCITATION FREQUENCY

The mathematical model consists of forced vibrations of a damped cubic nonlinear system

$$\ddot{x} + \omega_0^2 x + \varepsilon\mu\dot{x} + \varepsilon x^3 = \varepsilon f cos(\Omega(t)t) \ , \quad (1)$$

with the frequency variation being of a decaying type

$$\Omega = \Omega_0 e^{-\varepsilon\lambda t} \ . \quad (2)$$

The Method of Multiple Scales, known to effectively analyse such systems and produce solutions compatible with experimentation, has been used [22, 23] in search of approximate analytical solutions. Primary resonances are considered in the subsequent analysis. They are the most dangerous resonances leading to the highest response amplitudes, which occur when the external excitation is near to one of the natural frequencies. In accordance, the damping, the nonlinearity, and the external excitation amplitudes are re-ordered such that their effects balance each other. To ensure small decay rates, the perturbation parameter ε is incorporated in (2) also. Instead of employing several different perturbation parameters, it is better to use a single perturbation parameter [28].

Using two different time scales in the perturbation expansion, the approximate response is

$$x(t;\varepsilon) = x_0(T_0, T_1) + \varepsilon x_1(T_0, T_1) + \cdots, \quad (3)$$

where $T_0 = t$ and $T_1 = \varepsilon t$ are the usual fast and slow time scales in the Method of Multiple Scales [22]. The time derivatives in terms of the new time scales are

$$\frac{d}{dt} = D_0 + \varepsilon D_1 + \cdots,$$

$$\frac{d^2}{dt^2} = D_0^2 + 2\varepsilon D_0 D_1 + \cdots,$$

$$D_0 = \frac{\partial}{\partial T_0}, \ D_1 = \frac{\partial}{\partial T_1} \quad (4)$$

Inserting (2)-(4) into (1) and separating at each order of approximation yields

$$D_0^2 x_0 + \omega_0^2 x_0 = 0 \quad (5)$$

$$D_0^2 x_1 + \omega_0^2 x_1 = -2D_0 D_1 x_0 - \mu D_0 x_0 \\ -x_0^3 + f cos\left(\Omega_0 e^{-\lambda T_1} T_0\right). \quad (6)$$

The first-order solution is

$$x_0 = A(T_1)e^{i\omega_0 T_0} + cc \\ = a(T_1)cos(\omega_0 T_0 + \beta(T_1)), \quad (7)$$

where cc represents complex conjugates and $A = \frac{1}{2}ae^{i\beta}$ are complex amplitudes.

For primary resonances, the external excitation frequency is near to the natural frequency

$$\Omega_0 = \omega_0 + \varepsilon\sigma \ , \quad (8)$$

with σ being a parameter to express the nearness of the frequencies. Inserting (7) into the right-hand side of (6), the equation is

$$D_0^2 x_1 + \omega_0^2 x_1 = -2i\omega_0 D_1 A e^{i\omega_0 T_0} \\ -\mu i\omega_0 A e^{i\omega_0 T_0} - A^3 e^{3i\omega_0 T_0} \\ -3A^2\bar{A}e^{i\omega_0 T_0} + \frac{f}{2}e^{i\Omega_0 e^{-\lambda T_1} T_0} \ . \quad (9)$$

Elimination of secular terms, which are responsible for unphysical blow-up solutions, yield the complex amplitude equation in view of (8)

$$2i\omega_0 D_1 A + \mu i\omega_0 A + \\ 3A^2\bar{A} - \frac{f}{2}e^{i\sigma T_1 e^{-\lambda T_1}} = 0 \quad (10)$$

Using the polar form

$$A = \frac{1}{2}ae^{i\beta} \ , \quad (11)$$

The complex amplitude equation (10) is separated into its imaginary and real parts

$$\omega_0 D_1 a + \frac{1}{2}\mu\omega_0 a - \frac{f}{2}sin\gamma = 0 \quad (12)$$

$$-\omega_0 a D_1\beta + \frac{3}{8}a^3 - \frac{f}{2}cos\gamma = 0 \ , \quad (13)$$

where

$$\gamma = \sigma T_1 e^{-\lambda T_1} - \beta$$

$$D_1\beta = \sigma(1-\lambda T_1)e^{-\lambda T_1} - D_1\gamma \, . \quad (14)$$

Hence, the amplitude and phase variations are governed by

$$D_1 a = -\frac{1}{2}\mu a + \frac{f}{2\omega_0} sin\gamma \quad (15)$$

$$D_1\gamma = \sigma(1-\lambda T_1)e^{-\lambda T_1} - \frac{3}{8\omega_0}a^2 + \frac{f}{2\omega_0 a}cos\gamma \quad (16)$$

From (7), (8), (11), and (14), the unsteady solution is

$$x(t) = acos\left[\Omega_0 t - \varepsilon\sigma t\left(1-e^{-\varepsilon\lambda t}\right) - \gamma\right], \quad (17)$$

where the real amplitudes and phases are variable and governed by (15) and (16).

For steady state solutions, due to degeneracy, the analysis has to be repeated from the beginning. Since the external excitation frequency decays, the external excitation term becomes a constant force in the long run, and the model to be solved is

$$\ddot{x} + \omega_0^2 x + \varepsilon\mu\dot{x} + \varepsilon x^3 = \varepsilon f \, , \quad (18)$$

Repeating a similar analysis, the equations to be solved are

$$D_0^2 x_0 + \omega_0^2 x_0 = 0 \quad (19)$$

$$D_0^2 x_1 + \omega_0^2 x_1 = -2D_0 D_1 x_0 - \mu D_0 x_0 - x_0^3 + f \quad (20)$$

The first-order solution is again given in (7). Inserting and eliminating secular terms at the next order yields

$$2i\omega_0 D_1 A + \mu i\omega_0 A + 3A^2\bar{A} = 0 \quad . \quad (21)$$

For the steady state case, $D_1 A = 0$ and from (21), the complex amplitude $A = 0$, which means the real amplitude at the first order also vanishes, i.e., $a = 0$ or $x_0 = 0$. From (20), $x_1 = \frac{f}{\omega_0^2}$ and substituting all into the approximate expansion (3), the steady state solution is a constant solution

$$x = \frac{\varepsilon f}{\omega_0^2} \quad . \quad (22)$$

This can be verified by numerical simulations of the original equations (1) and (2). In all the numerical simulations, MATLAB is used with the subroutine ODE45 when integration of the ordinary differential equations is necessary. In Figure 1, after the transient oscillations decay, the numerical solution tends to a constant value which is almost equal to the expression given in (22).

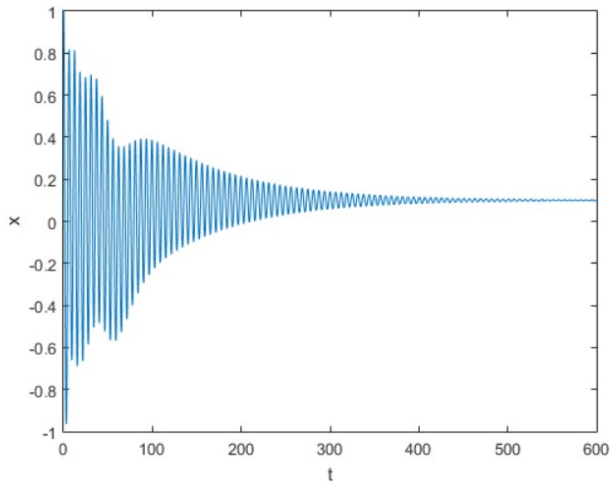

Figure 1. Numerical simulation of the decaying excitation frequency model

(μ=0.2, f=1, ω_0=1, Ω_0=1.05, ε=0.1, λ=0.5)

3. BUILT-IN EXTERNAL EXCITATION FREQUENCY

The frequency variation in this case is

$$\Omega = \Omega_0(1 - e^{-\varepsilon\lambda t}) \quad (23)$$

Proceeding in a similar way to Section 2, the equations governing the amplitude and phases are

$$D_1 a = -\frac{1}{2}\mu a + \frac{f}{2\omega_0} sin\gamma \quad (24)$$

$$D_1\gamma = \sigma(1-e^{-\lambda T_1}) + \lambda\sigma T_1 e^{-\lambda T_1} - \frac{3}{8\omega_0}a^2 + \frac{f}{2\omega_0 a}cos\gamma, \quad (25)$$

where

$$\gamma = \sigma T_1(1-e^{-\lambda T_1}) - \beta \quad . \quad (26)$$

For steady state solutions, $D_1 a = 0$, $D_1\gamma = 0$, $\lim_{T_1\to\infty} e^{-\lambda T_1} = 0$, and eliminating the phases, the frequency response relation turns out to be

$$\Omega_0 = \omega_0 + \varepsilon\left\{\frac{3}{8\omega_0}a^2 \pm \frac{1}{2}\sqrt{\frac{f^2}{\omega_0^2 a^2} - \mu^2}\right\}. \quad (27)$$

Figure 2 displays the response amplitudes corresponding to the frequencies.

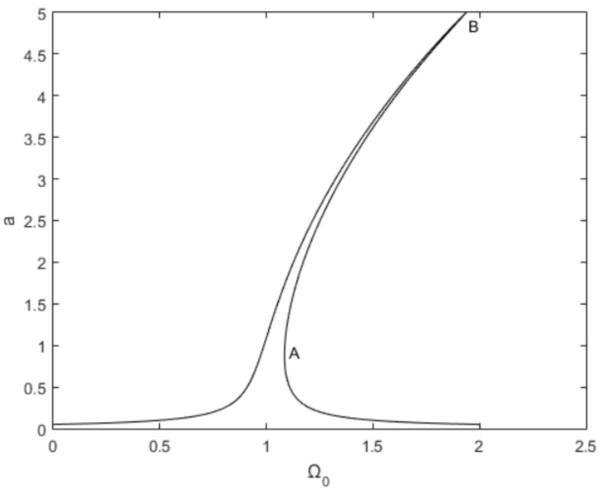

Figure 2. Frequency response curve (ε=0.1, μ=0.2, ω_0=1, f=1)

Such frequency response curves were analysed extensively in the previous literature [22, 23]. The points A and B correspond to saddle-node bifurcation points. The curve AB is unreachable both when decreasing the frequencies or increasing them gradually. Thus, the portion AB is unstable, whereas the rest of the curves correspond to stable solutions. This is the well-known jump phenomenon observed in hardening-type nonlinearities. When the frequency is gradually increased, the response reaches the point B, and then a sudden decrease to the bottom curve is observed when a further increase is done. When the frequencies are decreased, the curve below is followed up to point A, and a sudden jump to the higher curve is observed for an incremental decrease in the frequencies. For hardening nonlinear systems, the gradual increases of frequencies lead to a higher jump compared to the case of gradual decreases.

Effects of other parameters on the frequency response curves are given in Figures 3-5. In Figure 3, as the damping is increased, the responses are lower. Figure 4 shows the effect of external excitation amplitude on the frequency response curves. As the excitation amplitude increases, the responses increase with a more profound effect on the jump region. To isolate the effects of cubic nonlinearity, one has to fix εf and $\varepsilon\mu$ and vary ε only.

In Figure 5, the effect of the cubic nonlinearity coefficient can be seen. Since the behaviour is of a hardening type, the frequency response curves bend more to the right as the cubic nonlinearity increases without a change in the maximum amplitudes.

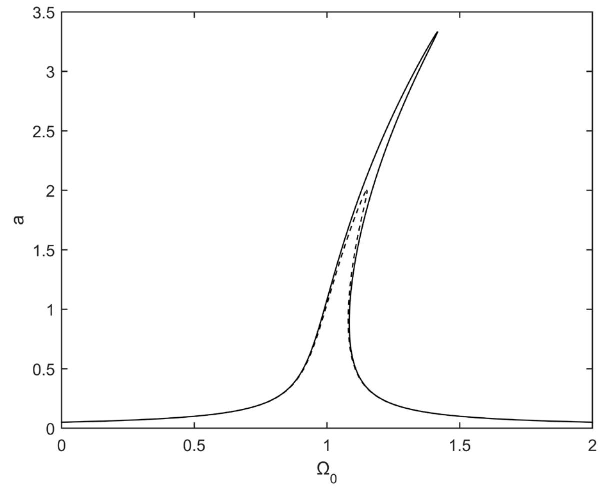

Figure 3. Frequency response curves for changes in damping coefficient μ=0.3(solid), μ=0.5(dashed) (ε=0.1, ω_0=1, f=1)

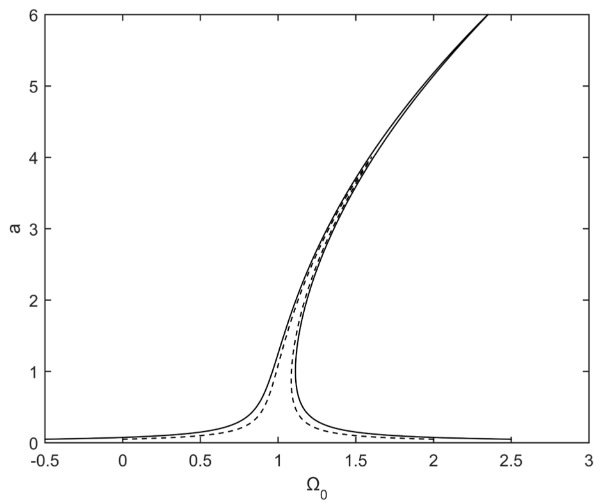

Figure 4. Frequency response curves for changes in excitation amplitude f=1.5(solid) and f=1(dashed) (ε=0.1, ω_0=1, μ=0.25)

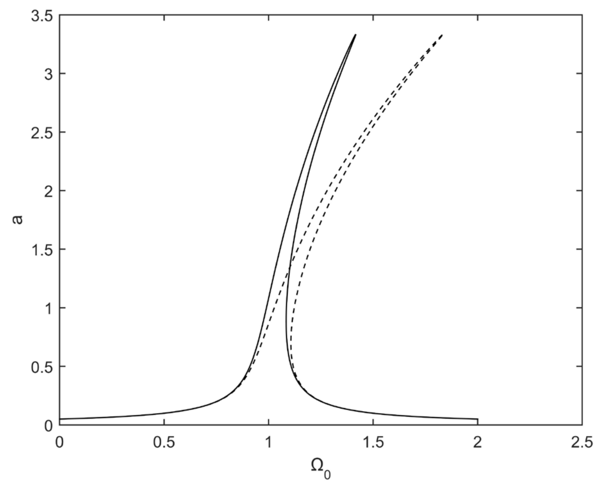

Figure 5. Frequency response curves for changes in cubic nonlinearity ε=0.1 (solid) and ε=0.2 (dashed) (ω_0=1, $\varepsilon\mu$=0.03, εf=0.1)

The transient solutions are

$$x(t) = acos\left[\Omega_0 t - \varepsilon\sigma t e^{-\varepsilon\lambda t} - \gamma\right], \quad (28)$$

where the amplitudes and phases are governed by (24) and (25). For sufficiently long time scales, $\lim_{t\to\infty} x(t) = acos[\Omega_0 t - \gamma]$ with the amplitudes and phases retaining steady state values. In summary, the built-in frequency leads to the classical constant frequency solution for long time scales, but the transient response differs from the classical solution.

4. HARMONIC EXTERNAL EXCITATION FREQUENCY

Real systems may have some fluctuations about the mean average frequency. The variation can be written as

$$\Omega = \Omega_0 + \varepsilon\Omega_1 \cos(\varepsilon\omega t) \quad . \quad (29)$$

Proceeding in a similar way with Section 2, assuming primary resonances, i.e., Equation (8) for expressing the nearness of mean external frequency to the natural frequency, the equations governing the amplitudes and phases are

$$D_1 a = -\frac{1}{2}\mu a + \frac{f}{2\omega_0}\sin(\gamma + \Omega_1 T_1 \cos(\omega T_1)) \quad (30)$$

$$D_1 \gamma = \sigma - \frac{3}{8\omega_0}a^2 + \frac{f}{2\omega_0 a} cos(\gamma + \Omega_1 T_1 \cos(\omega T_1)) \quad (31)$$

where

$$\gamma = \sigma T_1 - \beta \quad . \quad (32)$$

In terms of the original time variable, the equations read

$$\dot{a} = -\frac{1}{2}\varepsilon\mu a + \frac{\varepsilon f}{2\omega_0}\sin(\gamma + \varepsilon\Omega_1 t\cos(\varepsilon\omega t)) \quad (33)$$

$$\dot{\gamma} = \varepsilon\sigma - \frac{3\varepsilon}{8\omega_0}a^2 + \frac{\varepsilon f}{2\omega_0 a} cos(\gamma + \varepsilon\Omega_1 t\cos(\varepsilon\omega t)) \quad (34)$$

Since the harmonic terms in the equations do not tend to constant values, one cannot speak of steady state solutions. Equations (33) and (34) are numerically integrated for the initial conditions $a(0) = 2, \gamma(0) = 0$ (Figure 6).

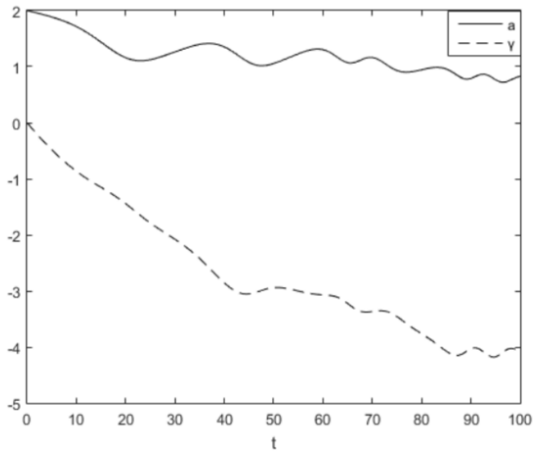

Figure 6. Amplitude and phase variations in time (ε=0.1, μ=0.2, ω_0=1, f=1, Ω_1 =0.8, ω=1.2, σ=0.2)

The amplitudes and phases do not tend to constant steady state values. The approximate solution is

$$x(t) = acos[\Omega_0 t - \gamma], \quad (35)$$

where the amplitudes and phases are governed by (33) and (34).

A new way of expressing the nonlinear dynamic behaviour for differential equations has been suggested recently. In the new representation, which is called the differential space representation [26], three dimensional solution curves are drawn with coordinates being the solution function and its first and second derivatives. Equation (1) is directly integrated subject to the frequency variation (29), and the result is given in Figure 7.

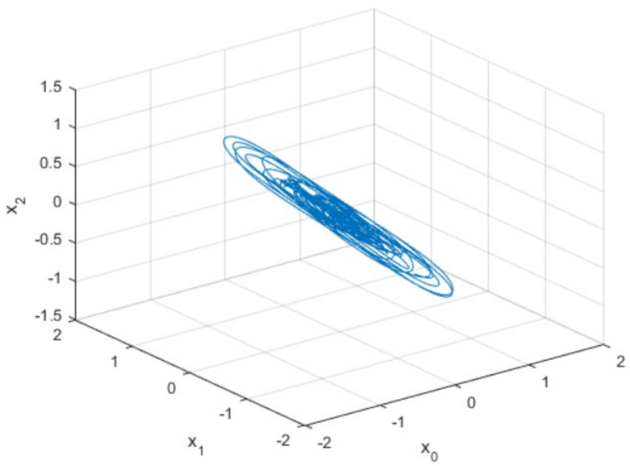

Figure 7. Differential space representation of the solution curve (ε=0.1, μ=0.2, ω_0=1, f=1, Ω_0 =1.05, Ω_1 =0.8, ω=1.2)

The initial conditions used are $x(0)=1$, $\dot{x}(0)=0$. The coordinates represent the function and its derivatives $x_0 = x, x_1 = \dot{x}, x_2 = \ddot{x}$. The chaotic behaviour is observed for the solution. Figure 7 depicts solutions for primary resonances. A secondary resonance type is when Ω_0 is near to three times the natural frequency [22]. For such secondary resonances, a completely different chaotic solution can also be obtained (Figure 8).

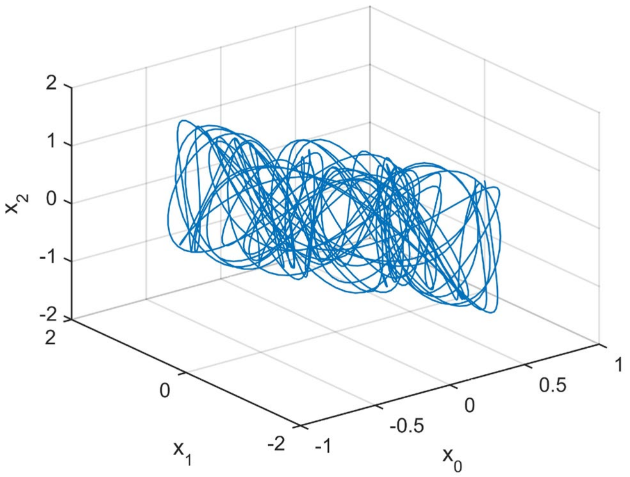

Figure 8. Differential space representation of the solution curve (ε=0.1, μ=0.2, ω_0=1, f=1, Ω_0 =3.05, Ω_1 =0.8, ω=1.2)

5. CONCLUSIONS

The external excitation frequency may change during oscillations of the nonlinear systems. Three different variation cases are considered in this study with primary resonances of the system. Approximate analytical solutions are derived using the Method of Multiple Scales. The decaying and built-in type variations lead to steady state solutions, while the harmonic fluctuations about a mean frequency lead to chaotic solutions. Therefore, it is concluded that the harmonic fluctuations of the excitation frequency are more dangerous and unpredictable than the continuous gradual decrease/increase of the excitation frequency.

In this work, cubic nonlinearities are considered. Other types of nonlinearities may also be considered. A detailed analysis of the sensitivity of all the parameters involved may also be performed. Control algorithms to suppress the unwanted vibrations may also be investigated. An extensive treatment of the chaotic behaviour and the range of parameters for such motions to occur are left for further studies.

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