
Numerical Study of the Stiffness Degradation Caused by Branched Cracks and its Influence on the Natural Frequency Drop

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Abstract: - This paper focuses on the accuracy of detecting the location of a complexly shaped damage present in a cantilever beam. The study is carried out by involving the finite element method. At first, we have parametrically described the geometry of a branched crack and designed its discrete model. As a further step, we performed simulations to observe the changes in the static and dynamic behavior of the beam that occur in the presence of damage. Several types of Y-shaped cracks have been taken for these analyses. We have shown the beam has a diminished capacity to store energy and consequently the frequency decreases. The frequency shift phenomenon for the first mode of vibration was found to be in concordance with the changes in the deflection of beam's free end. We derived on this basis a correction coefficient that considers the beam deflections in the intact and damaged state. This coefficient permits predicting the frequency shift due to a given crack. As the last step, we have successfully compared the frequencies achieved directly using the FEM with those for the intact beam to which the correction coefficient was applied.

Keywords: - stiffness degradation, deflection, natural frequency, cantilever beam, branched crack.

1. INTRODUCTION

Damages reduce the capacity of the beams to store energy because the slices where damage is present are subject to stiffness decrease. As a consequence the natural frequencies of damaged beams decrease [1]-[5]. The frequency decrease depends on the reduction of the cross-sectional area, hence on the crack depth, but also on the crack position [6]-[8]. For transverse cracks, either open or breathing, mathematical relations which permit predicting the frequency drop if the crack depth and position are known exist and are widely presented in the literature [9]-[12]. Most mathematical relations were derived empirically, from the fracture mechanics theory, and are applicable just for particular cases [13]-[14]. Our

research has been focused on finding a mathematical relation with a large degree of generality, and we have succeeded in creating a relationship that can be applied to any beam-like structure if it is subject to a transverse crack [15]-[17].

Our latest research focuses on branched cracks, which have a more complex geometry, and therefore their effect on the natural frequency changes is influenced by more parameters, increasing the approaches difficulty.

This paper presents a numerical study destined to find the dynamic response of beams with Y-shaped cracks, in particular the effect of the position of the transverse crack component relative to the longitudinal crack component.

2. MATERIALS AND METHODS

2.1. The test structure

The research aimed finding the effect of a branched crack on the natural frequencies of beam-like structures. To this aim, a steel beam, fixed on the left end and free at the other one (i.e. a cantilever beam) is considered in the study. It has the following key dimensions: the length $L=1000\text{ mm}$, the width $B=50\text{ mm}$ and the thickness $H=5\text{ mm}$. Thus, the intact beam has a cross-sectional area $A=250\text{ mm}^2$ and the moment of inertia $I=52083.33\text{ mm}^4$.

Table 1. Physical-mechanical properties of the structural steel used in the study

Mass density [kg/m ³]	Young modulus [N/m ²]	Poisson ratio [-]	Tensile strength [MPa]	Yield strength [MPa]	Min. elongation [%]
7850	$2 \cdot 10^{11}$	0.3	470-630	355	20

The relevant physical-mechanical properties, presented in Table 1, are extracted from the ANSYS library for a structural steel.

2.2. Branched crack geometry

For the damaged beam cases we considered the Y-shaped crack geometry with two components: (*) in the transverse and (**) in the longitudinal direction. The transverse crack component extends from the upper surface to the longitudinal crack component, as presented in figure 1. We consider the particular case of a longitudinal crack extent (i.e. $\alpha=\beta=90^\circ$) of length $l=50\text{ mm}$, located at depth $a=2.5\text{ mm}$ in the beam. The left end of the longitudinal crack component is located at distance $x=20\text{ mm}$ from the fixed end. The values chosen for this parameter are indicated in table 2, along with the other main dimensions of the damage.

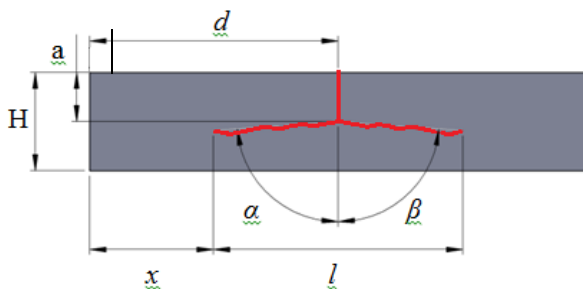


Figure 1. The geometry of a branched crack of Y-shape

The transverse crack component is located at distance d from the fixed end, the limits being: $d_{\min} = x$ and $d_{\max} = x + l$.

Table 2. Dimensions of the Y-shaped crack

a [mm]	l [mm]	α [°]	β [°]	x [mm]
2.5	50	90	90	20

The transverse crack component is iteratively removed between the limits d_{\min} and d_{\max} .

2.3. The Finite Element Model of the test structure

The boundary conditions for the fixed end at the left extremity of the analyzed beam are imposed by applying the *fixed support constraint*. For the static analysis we applied a gravitational acceleration of 9.81 m/s^2 .

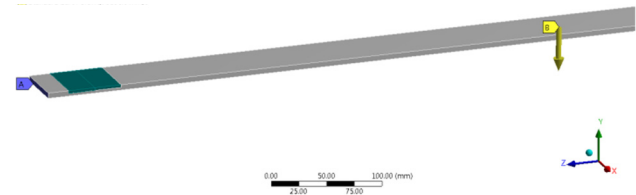


Figure 2. View on the intact beam with applied boundary conditions and gravitation

A fine mesh was chosen for the steel beam, the hexahedral elements having defined the maximum size of an edge by 2 mm , see Fig. 3. In consequence, the intact beam has resulted in a model containing 43587 elements and 231639 nodes.

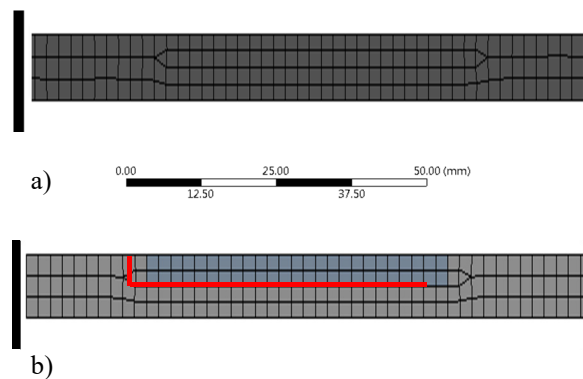


Figure 3. A zoom on the left beam end, for the intact beam (a) and damaged beam (b), highlighting the similar prismatic element's distribution

As it can be remarked from figure 3.b, the prismatic elements are deformed in the interface zone of the intact beam and damage boundary element resulting in an even finer mesh.

In addition, the defect location was replaced by a segment of reduced thickness $h = H / 2$, thus a smaller stiffness (see Fig. 4).

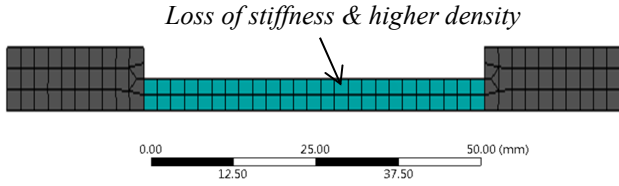


Figure 4. A zoom on the segment with reduced thickness

To compensate the loss of mass we increased the mass density of the segment to $\hat{\rho} = 15700 \text{ kg} / \text{m}^3$.

2.4. Test procedure

The beam static and dynamic behavior is analyzed by means of the finite element method (FEM). Two types of simulations were performed:

- static analysis, involving the STATIC STRUCTURAL MODULE, made in order to find the deflection of the free end;
- dynamic analysis, employing the MODAL MODULE, made in order to find the natural frequencies for the weak-axis bending vibration for the first mode.

For all studies we have iteratively removed the transversal crack component by a step of $d = 5 \text{ mm}$ along the longitudinal crack component. The first position is at left side of the longitudinal crack component, at $d_{\min} = d_1 = 20 \text{ mm}$, while the last one is at its right end, at $d_{\max} = d_{11} = 70 \text{ mm}$, as shown in figure 5. Also, the beam with reduced cross-section, nominated herein as the beam with a gap, is implied in this study.

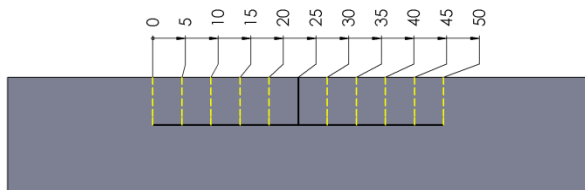


Figure 5. Positions of the transversal crack

The acquired results are deflections of the free end under dead load δ and frequencies f . From [18] and [19], the deflection δ_D achieved at the free end of the damaged beam is:

$$\delta_D = \frac{\rho \cdot g \cdot A \cdot L^4}{8EI_{eq}^*} \quad (1)$$

while for the intact beam it is:

$$\delta_U = \frac{\rho \cdot g \cdot A \cdot L^4}{8EI} \quad (2)$$

The beam, in accordance to the Euler-Bernoulli theory [20], has the frequencies of the bending vibration modes $i = 1 \dots n$ expressed as:

$$f_i = \frac{\lambda_i^2}{2\pi} \sqrt{\frac{EI}{\rho AL^4}} \quad (3)$$

After substituting Eq. (1) and (2) in Eq. (3), and performing simple mathematical calculus, one obtains:

$$f_{iU} = \frac{\lambda_i^2}{2\pi} \sqrt{\frac{g}{8\delta_U}} \quad (4)$$

and

$$f_{iD} = \frac{\lambda_i^2}{2\pi} \sqrt{\frac{g}{8\delta_D}} \quad (5)$$

The natural frequency of the damaged beam results, from Eq. (4) and Eq. (5), as:

$$f_{iD} = f_{iU} \sqrt{\frac{\delta_U}{\delta_D}} = \kappa(a, l) \cdot f_{iU} \quad (6)$$

where $\kappa(a, l)$ is the correction coefficient.

In consequence, the relation of the normalized frequency shift can found as:

$$\Delta \bar{f}_i = \frac{f_{iU} - f_{iD}}{f_{iU}} = \frac{\sqrt{\delta_D} - \sqrt{\delta_U}}{\sqrt{\delta_D}} = \gamma(a, l) \quad (7)$$

which is, in fact, the damage severity if the damage is located at the fixed end of the cantilever beam.

Knowing the deflection of the free end for both healthy and damaged beam, we calculated the correction coefficient and afterward the frequency of the damaged beam in respect to the natural frequency of the intact beam, in accordance to Eq. (6). The results were compared with the frequencies of the damaged beam obtained directly from the FEM analysis, in order to prove that it is possible to use the correction coefficient $\kappa(a, l)$ and the severity $\gamma(a, l)$ also for Y-shaped cracks.

3. Results and discussions

To profoundly understand the correlation between the frequency shift phenomena and the changes in the deflection of beams that occur due to different damage types, we compared for several scenarios of Y-shaped damages the frequencies obtained from the modal analysis with those from Eq. (6) supported by static analysis.

As a further step in the research, we compared the outcomes for the Y-shaped crack with the results achieved from a beam having a rectangular gap with dimensions: depth a and length l , equal to that of the crack.

3.1. Deflection at the free end

The deflection of the free beam end is calculated for the dead load mass, meaning a linearly distributed mass $\bar{m} = \rho A$. The results are presented in Table 3.

Table 3. Deflection of the beam's free end

d [mm]	δ_U [mm]	δ_D [mm]	$\kappa(a,l)$ [-]
20	22.948	50.698	0.6727
25		50.675	0.6729
30		50.673	0.6729
35		50.673	0.6729
40		50.673	0.6729
45		50.674	0.6729
50		50.677	0.6729
55		50.68	0.6729
60		50.685	0.6728
65		50.714	0.6726
70		50.715	0.6726
gap		50.874	0.6716

3.2. Comparison of frequencies achieved by FEM and after applying the correction term

The frequencies for the first mode of vibration were obtained by means of finite element analysis and compared to those calculated by using the original relations between deflection and natural frequency changes for the 11 positions of the transversal crack. The results, as well as the attained errors are presented in Table 4.

Table 4. Comparison of frequencies obtained directly from FEM and calculated involving the hybrid method

d [mm]	f_{iU} [Hz]	f_{iD_FEM} [Hz]	f_{iD_hybrid} [Hz]	Error [Hz]
20	4.088	2.7351	2.7503	0.55%
25		2.7359	2.7509	0.55%
30		2.736	2.7510	0.55%
35		2.7359	2.7510	0.55%
40		2.7362	2.7510	0.54%
45		2.7361	2.7510	0.54%
50		2.7361	2.7509	0.54%
55		2.7358	2.7508	0.55%
60		2.7358	2.7507	0.54%
65		2.7352	2.7499	0.54%
70		2.7339	2.7498	0.58%
gap		2.7311	2.7455	0.53%

The diagram obtained employing both the FEM and the hybrid FEM-analytical methods are presented in figure 6.

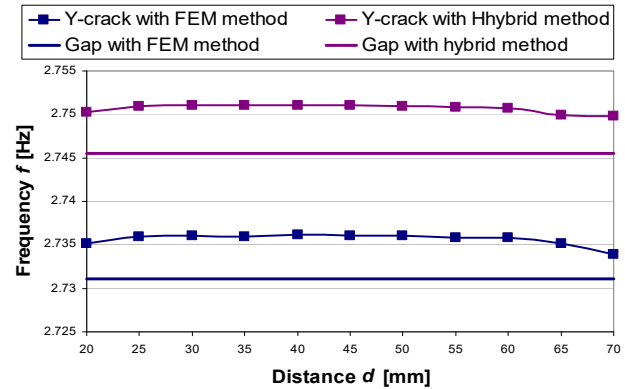


Figure 6. Comparison of frequencies obtained by FEM modal analysis and involving the hybrid method

Further, we compared the modal FEM analysis and calculated results of the different damage position with that of the gap study. The results are presented in Table 5.

Table 5. Comparison of frequencies obtained for the beam with a gap and subject of Y-shaped cracks

FEM			Calculated		
f_{i_GAP} [Hz]	f_{iD} [Hz]	Err. [%]	f_{i_GAP} [Hz]	f_{iD} [Hz]	Err. [%]
2.7311	2.7351	0.15	2.7455	2.7503	0.17
	2.7359	0.18		2.7509	0.20
	2.736	0.18		2.7510	0.20
	2.7359	0.18		2.7510	0.20
	2.7362	0.19		2.7510	0.20
	2.7361	0.18		2.7510	0.20
	2.7361	0.18		2.7509	0.19
	2.7358	0.17		2.7508	0.19
	2.7358	0.17		2.7507	0.19
	2.7352	0.15		2.7499	0.16
	2.7339	0.10		2.7498	0.16

From Table 4 and Fig. 6 one can observe that comparing the frequencies obtained directly from FEM and calculated involving the hybrid FEM-analytical method the errors are less than 0.6%. This proves the precision achieved by using the correction coefficient $\kappa(a,l)$ and qualifies the method for prediction of frequency changes due to Y-shaped cracks with known parameters.

Table 5 shows that the beam with a gap instead of the Y-shaped crack can substitute the crack in modeling the dynamic behavior of damaged beams. The error noticed here is less than 0.2%, both for the FEM and the hybrid methods. From prior research we know the frequency shift curves for beams with a gap [21], which can be successfully used also for Y-shaped cracks.

4. CONCLUSIONS

In this paper, we analyze the effect of a Y-shaped crack with two components (one transverse and the other longitudinal) on the free end deflections and natural frequencies of a cantilever beam. Also, a beam with reduced stiffness and increased mass density (nominated gap in this paper), which compensates for the loss of mass due to cross-sectional area reduction, is considered. The position of both discontinuities is the same, these being located near the fixed beam end. The analysis was performed by means of numerical simulation.

We found that the Y-shaped crack produces a significant frequency drop compared to the transverse crack of same depth. Regarding the position of the transverse crack component relative to that of the longitudinal crack component, this has a limited influence on the natural frequencies and free end deflections. We derived on this basis the relation for a compensation coefficient that permits evaluating the frequency change with high precision (error less than 0.5%)

Moreover, we found the beam with a gap having the same depth and position as the Y-shaped crack produces similar deflection and frequency drop, respectively. This qualifies it to be used for assessing Y-shaped cracks.

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