Vibration Analysis of a Mechanical System Composed of Two Identical Parts

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Abstract: - The aim of the paper is to analyze the free vibration of a symmetrical, spatial structure made up of bars, obtained by joining two identical subsystems. It is a real structure, encountered in civil engineering. In the literature, there are papers that have established vibration properties of structures with symmetry. In the paper, we propose to see if these properties apply to a structure made up by two identical, mirror symmetrical substructures and linked by common elements.

Keywords: Vibration, Eigenmode, Bar, Symmetry

1. INTRODUCTION

Engineering practice requires the use of structures containing identical components or parts. The existence of identical parts is useful from several points of view: less information is needed to describe the system, design is made quicker and easier, components are made faster than a complex assembly, and finally the time to achieve of the structure and cost of manufacturing decreases. Also, the subsequent maintenance of the system becomes easier and cheaper.

For the system that presents the symmetries or consists of identical elements, certain properties have been established in the literature regarding their eigenfrequencies of vibrations and some properties concerning the forced vibrations. Thus, for discrete systems that describe torsional vibrations of symmetric systems, results are presented in [6], [17], [18], [19], [22]. In the case of continuous systems, modeled with the finite element method, the analysis of symmetry systems resulted in contributions presented in the papers [1], [2], [5], [7], [8]. For continuous bar systems, studies were done in [20], [21]. For all these types of systems, similar results were obtained.

The conclusion drawn from the above-mentioned papers (drived by various applications) is that the existence of structural symmetries allows for the reduction of the computational effort to determine its eigenvalues. This leads to a decrease in design time and finally to a decrease the costs associated with the physical model of the structure. The present paper is applied on a structure made up of two identical parts, symmetrical in the mirror, linked by common element. The results confirm the above mentioned papers.

2. MECHANICAL MODEL

The structure consists of a series of trusses rigidly connected by welding (Fig.1). The dimensions are shown in the figure. The structure has 20 nodes and 48 bars. The median plane of the figure links the identical substructures to the left and right of these nodes. The four legs of the structure were fixed to the fixed space by means of elastic connections with high rigidity.

Figure 1. Real structure

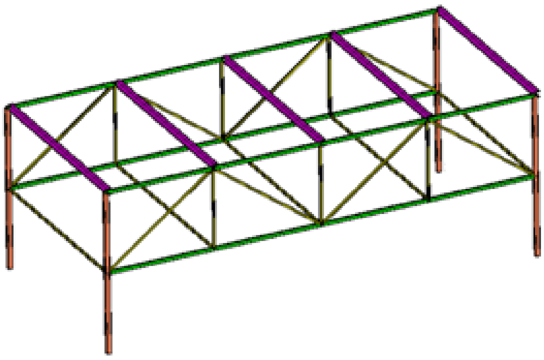

Figure 2. Structure sketch

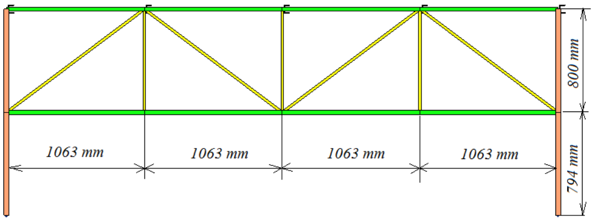

Figure 3. Side view of the structure

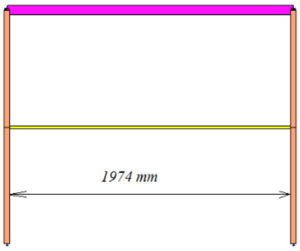

Figure 4. Front view of the structure

If we neglect the structural damping, the motion equations of the free vibration are ([3], [9]- [16]):

$$\begin{bmatrix} M_{11} & 0 & M_{12} \\ 0 & M_{11} & M_{12} \\ M_{12}^T & M_{12}^T & M_{22} \end{bmatrix} \begin{Bmatrix} \ddot{\Delta}_{1s} \\ \ddot{\Delta}_{1d} \\ \ddot{\Delta}_{2} \end{Bmatrix} + \\ + \begin{bmatrix} K_{11} & 0 & K_{12} \\ 0 & K_{11} & K_{12} \\ K_{12}^T & K_{12}^T & K_{22} \end{bmatrix} \begin{Bmatrix} \Delta_{1s} \\ \Delta_{1d} \\ \Delta_{2} \end{Bmatrix} = \{0\} \tag{1}$$

where:

M_{11} - the inertial matrix for the structure presented in Fig.5;

M_{12} - the inertial coupling matrix between the two identical parts and the liaisons elements;

M_{22} - the inertial matrix for the liaisons elements;

K_{11} - the rigidity matrix for the structure presented in Fig.5;

K_{12} - the rigidity matrix between the two identical parts and the liaisons elements;

K_{22} - the inertial matrix for the liaisons elements;

Δ_{1s} - the vector of the independent coordinates of the left part of the structure;

Δ_{1d} - the vector of the independent coordinates of the right part of the structure;

Δ_{2} - the vector of the independent coordinates of the liaisons elements.

In a compact form (1) becomes:

$$[M]\{\ddot{\Delta}\} + [K]\{\Delta\} = \{0\} \tag{2}$$

Consider now only a half part of the structure (see Fig.5). The equations of motion for this substructure are:

$$[M_{11}]\{\ddot{\Delta}_{1s}\} + [K_{11}]\{\Delta_{1s}\} = \{0\} \tag{3}$$

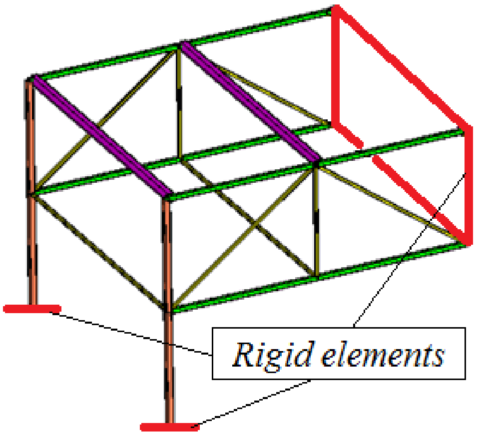

Figure 5. Half part of the structure

The mechanical model was realized using Finite Element Analysis. To perform the calculus we have used ABAQUS.

3. EIGENMODES OF VIBRATION

For the entire structure to solve the eigenvalues problem returns to solve the characteristic equation [4],[12],[17]:

$$\begin{vmatrix} K_{11}-p^2M_{11} & 0 & K_{12}-p^2M_{12} \\ 0 & K_{11}-p^2M_{11} & K_{12}-p^2M_{12} \\ K_{12}^T-p^2M_{12}^T & K_{12}^T-p^2M_{12}^T & K_{22}-p^2M_{22} \end{vmatrix} = 0 \quad (4)$$

In a similar way the eigenvalues problem for the structure of Fig.5, returns to solve the characteristic equation:

$$\det\left([K_{11}]-p^2[M_{11}]\right)=\{0\} \quad (5)$$

Now, we consider the generic matrix

$$M=(m_{ij})_{i,j\in\{1,2,..,n\}} \quad (6)$$

and

$$U=\left(m_{ij}\right)_{i\in\{i1,i2,..,ik\}\subset\{1,2,..,n\},\ j\in\{j1,j2,..,jk\}\subset\{1,2,..,n\}} \quad (7)$$

a sub-matrix of M. We denote with $\alpha=\det(U)$. For this chosen matrix U, we denote the complementary matrix:

$$\overline{U}=(m_{ij})_{i\in\{1,2,..,n\}\setminus\{i1,..,ik\},\ j\in\{1,2,..,n\}\setminus\{j1,..,jk\}} \quad (8)$$

We call an algebraic complement of the minor α (the cofactor of α) the determinant with sign $\overline{\alpha}=(-1)^{i1+..+ik+j1+..+jk}\det(\overline{U})$.

If we fix the lines $i1,..,ik$ we have:

$$\det(M)=\sum_{1\le j1<..<jk\le n}\alpha\overline{\alpha}. \quad (9)$$

The formula generalizes the Laplace expansion formula according to a line of the matrix M determinant. $\overline{U}$ is a matrix of dimension (n-k)x(n-k) and considering a matrix V being the square sub-matrix of indices $\{m_1,..,m_p\}$, $\{l_1,..,l_p\}$ of determinant β and correspondingly matrix $\overline{V}$ we can write:

$$\det(\overline{U})=\sum_{1\le l1;..;lp\le n-k}\beta\cdot(-1)^{m1+..+mp+l1+..lp}\det(\overline{V}) \quad (10)$$

Therefore we have:

$$\det(M)=\sum_{1\le j1<..<jk\le n}\ \sum_{1\le l1<..<lp\le n-k}$$
$$\alpha\beta\left(-1\right)^{i1+..+ik+j1+..+jk+m1+..+mp+l1+..+lp}\det(\overline{V})=$$
$$=\sum_{j}\sum_{l}\alpha\beta\gamma. \quad (11)$$

Considering the previous notations, the following result it exits:

Proposition

Consider the square polynomial matrices with complex coefficients, of size *n*, noted *A, B, C, L*, Z=O$_n$ and matrix $M=\begin{pmatrix} A & Z & B \\ Z & A & B \\ L & L & C \end{pmatrix}$.

Then det(M) is dividable by det(A). The proof is presented in [22]. Immediate results are:

P1. The eigenvalues of free at one end and fixed at the other end symmetrical beams are also eigenvalues of the whole mechanical system.

P2. For the eigenvalues common to the system in Fig. 2 and to the entire system in Fig.1, the eigenvectors are of the form:

$$\Phi=\begin{Bmatrix} \Phi_1 \\ -\Phi_1 \\ 0 \end{Bmatrix} \quad (12)$$

(the components of the eigenmodes corresponding to the two identical beams are skew symmetric).

P3. For the other eigenvalues, which are not obtained from P1, the eigenvectors are of the form:

$$\Phi = \begin{Bmatrix} \Phi_1 \\ \Phi_1 \\ \Phi_2 \end{Bmatrix} \tag{13}$$

(the components of the eigenmodes corresponding to the two identical beams are symmetric).

Table 1. Eigenfrequencies

Entire structure		Substructure	
Mode No.	Frequency (Hz)	Mode No.	Frequency (Hz)
1	3,72		
2	6,39	1	9,02
3	7,04		
4	11,95		
5	12,52		
6	15,37		
7	15,51		
8	18,47		
9	19,74	2	20,74
10	21,92	5	57,10
11	22,63	6	60,13
12	24,90		
13	26,98	3	30,49
14	35,82		
15	55,35	4	52,89
16	55,51		
17	93,04		
18	93,60		
19	159,84	7	158,38
20	160,04	8	158,58

In Table 1 are presented the eigenfrequencies for the structures from Fig.1 and Fig.2. Inspection of table 1 confirms the results obtained in the papers [9],[17]-[22], with to exceptions. The cause of these differences is to be studied and can be caused by the calculation algorithm.

In Fig.6 is presented the 2nd eigenmode of vibration for the entire structure and in Fig.7 the 1st eigenmode for the half structure. In an analogous way are presented, by comparison, the skew eigenmodes corresponding to the entire structure and for half of the structure (Fig.8 and Fig.9, Fig.10 and Fig.11, Fig.12 and Fig.13, Fig.14 and Fig.15, Fig.16 and Fig.17).

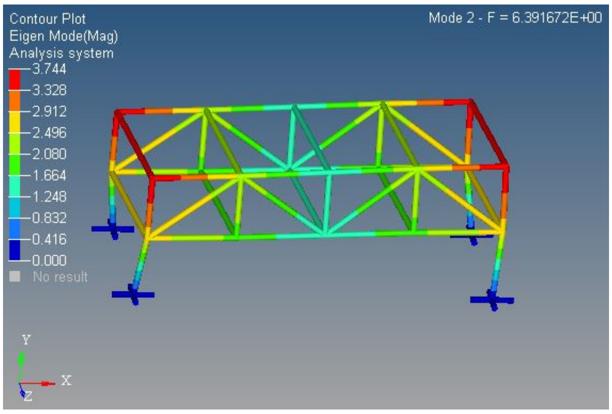

Figure 6. Mode 2

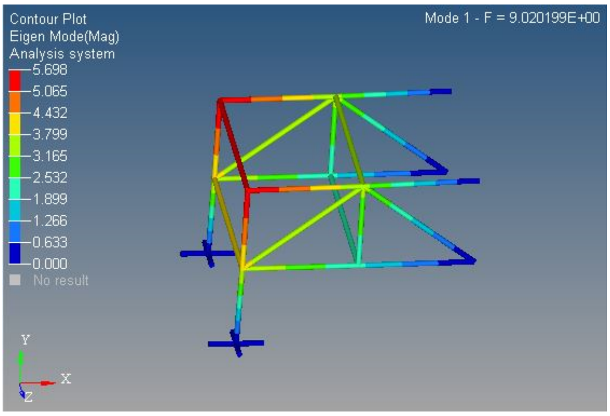

Figure 7. Mode 1

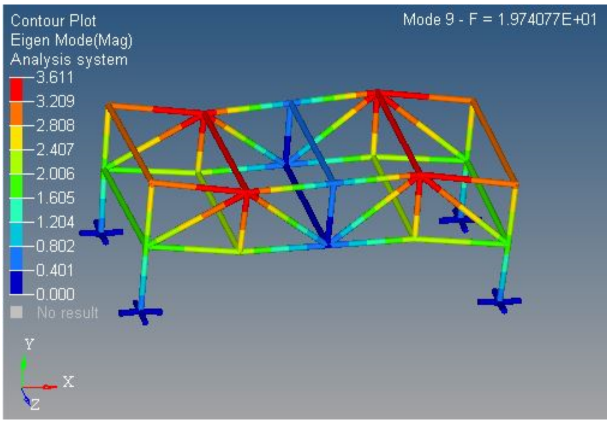

Figure 8. Mode 9

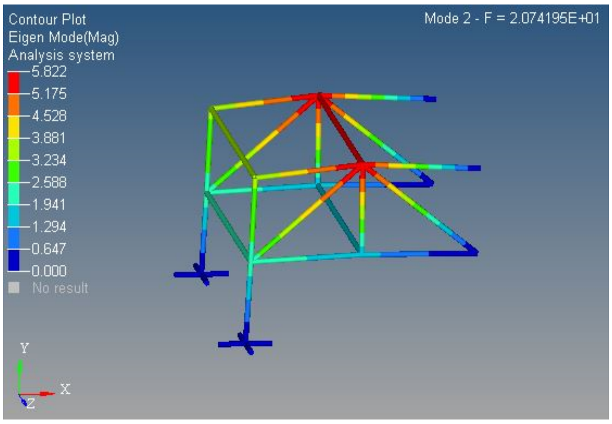

Figure 9. Mode 2

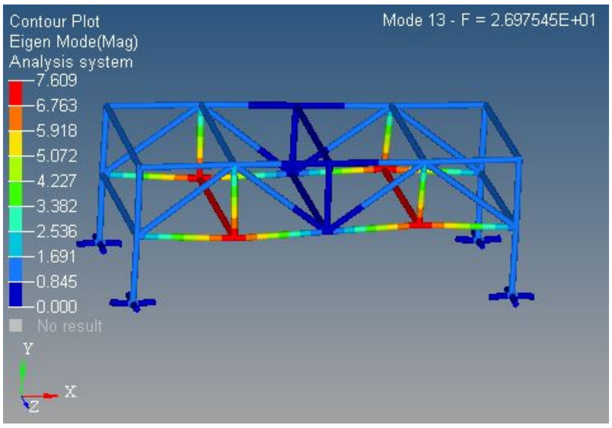

Figure 10. Mode 13

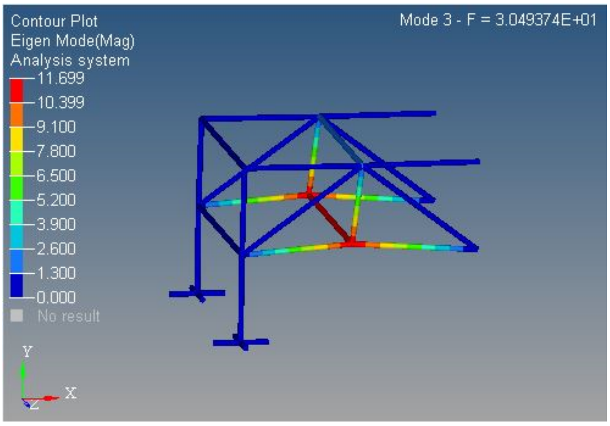

Figure 11. Mode 3

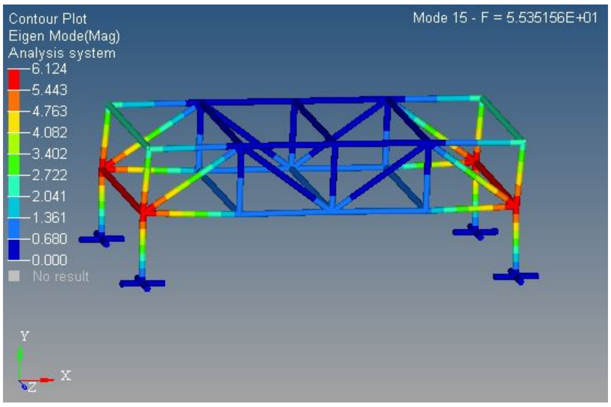

Figure 12. Mode 15

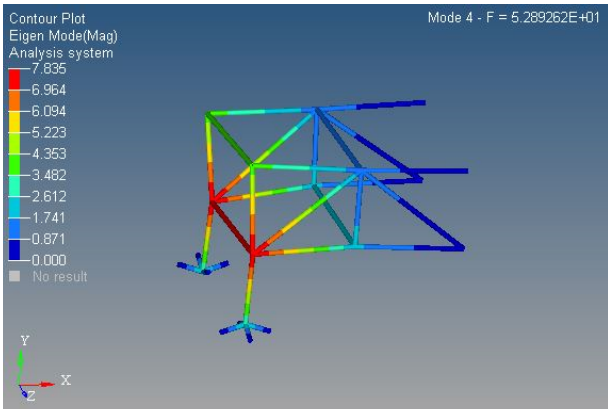

Figure 13. Mode 4

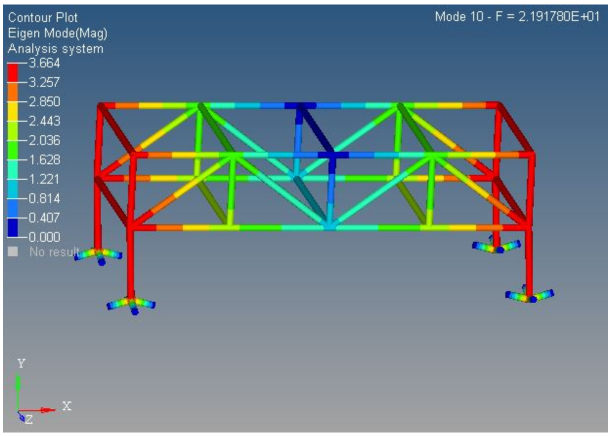

Figure 14. Mode 10

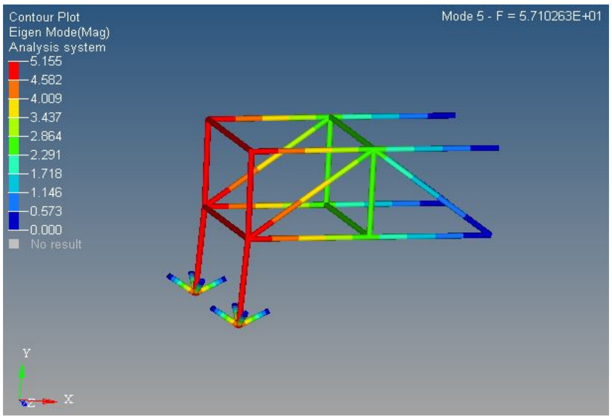

Figure 15. Mode 5

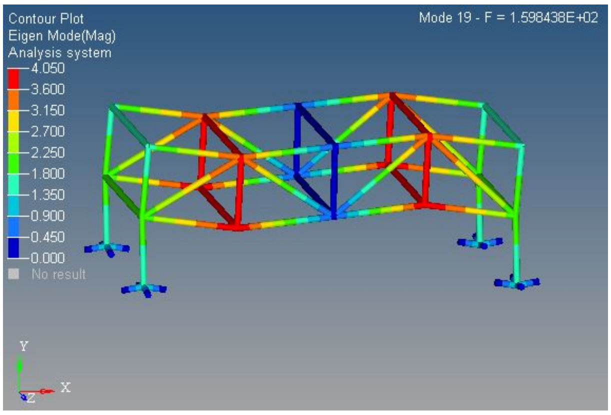

Figure 16. Mode19

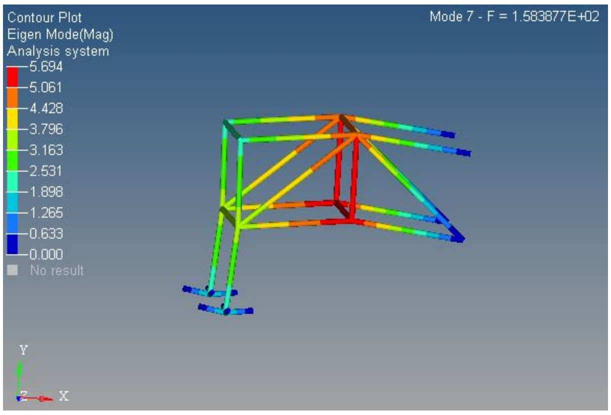

Figure 17. Mode 7

It is observed that for the eigenvalues of the system, which are the same with those of a half structure, the eigenmodes of vibration are skewsymmetric. For the other eigenvalues, the eigenmodes of vibration are symmetric. The properties presented in [17]-[22] can be shown in the Fig. 6-17. We have presented in parallel the eigenmodes for the entire structure and for a half structure.

4. CONCLUSIONS

The mechanical systems having some identical parts present some interesting properties considering the vibrations. Some types of structures were studied in the literature presented in Introduction. In this paper is made a study of such a system built by two identical parts. The study is based on a laboratory-scale structure in order to verify the properties determined by symmetry. The results are presented the section 3.

The properties presented in other papers for some particular systems are verified in the case of a three-dimensional mechanical system composed of bars. The main advantage using these properties is lowering the time and the costs of simulation.

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