The Dynamic Model of Tubular Vibratory Mill with Rotary Chamber and Double Lateral Drive

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Abstract: - Tubular vibratory mill with rotary chamber and double lateral drive was developed following research by specialists to improve the performance of vibratory mills. This paper presents the dynamic model of a tubular vibratory mill that is driven by two adjustable speed electrovibrators, operating independently of each other. The authors of this article propose - for half the tubular vibratory mill - a dynamic model which is designed as a rigid body with three degrees of freedom. Using the kinetostatic formulation, the differential motion equations of the vibratory mill are obtained. Finally, the calculation model is used to determine the vibration parameters of the PALLA-U 20 vibratory mill (Germany), whose construction is identical to the type specified in the title of paper.

Keywords: - dynamic model, vibratory mill, vibration parameters

1. INTRODUCTION

It is known to exist a proportional dependence between the increase of the solid materials surface area by grinding and the increase of the efficiency of physico-chemical processes which uses these materials. [7]

A classification of the vibratory mills can be made according based on the vibration generator type used to obtain the excitation required for grinding (Figure 1):

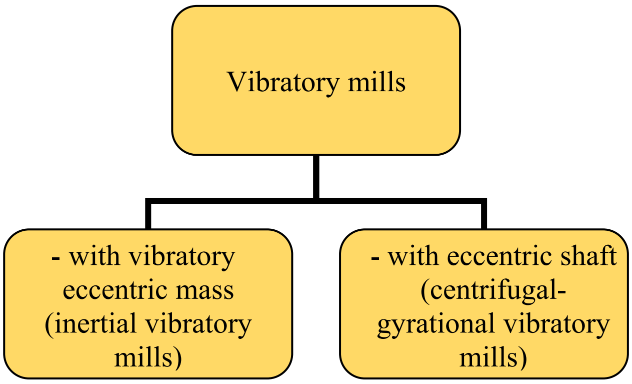

Figure 1. Vibratory mills classification by type of vibration generator

Compared to other milling equipments, whether they are inertial or girational, the vibratory mills are characterized by some very important features. Thus, in relation to the rotary drum mills, the vibratory mills are characterized by: simplicity of connection to the feed device and the collecting hopper of the ground product, up to 3 to 4 times lower mounting surface (ground footprint), weighing 4 to 5 times less, 6 to 8 times lower cost of milling bodies, 4 to 10 times lower energy consumption, but also by a productivity of 4 to 20 times lower than that of. [7, 15, 18, 21]

Compared to stirred media mills and jet mills, the vibratory mills are simpler, easier to operate and have an energy consumption of 6 to 10 times lower. [7, 15, 18, 21]

As disadvantages of the vibratory mills, we note: the negative effects of vibrations on the environment (especially on the foundation) requirements of expensive anti-vibration isolation measures, high noise during operation (which can reach 90 to 120 dB) [12, 17 , 21].

Until now, research has pursued the development of vibration grinding technologies in many research centers, such as: the Technical University of Clausthal [9, 10, 21], the Russian company Vibrocom [22] and the University of Science and Technology from Cracow – AGH [18, 19, 20, 21].

Also, research on the dynamics of vibratory mills or antivibration isolation was also carried out at the Polytechnic University of Bucharest, as well as at the Technical University of Civil Engineering in Bucharest [6, 7].

Dynamic models such as one of a vibratory mill with two eccentric masses have been developed, by writing equations with six dynamic degrees of freedom based on Lagrange's equations [13].

The new imputs presented by the authors in this paper refer to the developement of the dynamic model which describes most accurately the tubular vibratory mill with rotary chamber and double lateral drive (the constructive version of PALLA (Germany). [23]

The differential equations solutions for the characteristics presented in the technical book of the mill in question it is also presented.

2. TUBULAR VIBRATORY MILL WITH ROTARY CHAMBER AND DOUBLE LATERAL DRIVE

The studies conducted by specialists in order to improve the performance of the vibratory mills led - among other things - to the constructive version of a rotary chamber with double lateral drive mill (Figure 2).

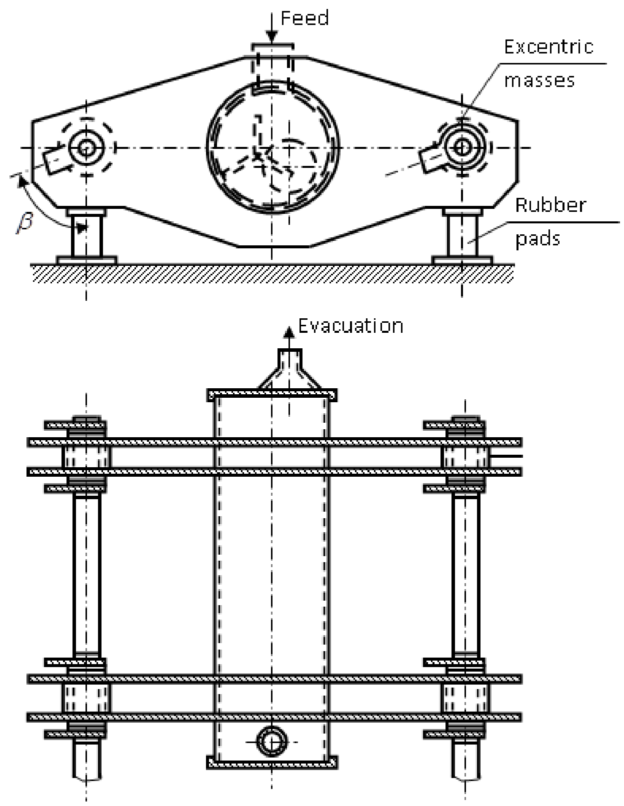

Figure 2. The constructive scheme of them tubular vibratory mill with rotary chamber and double lateral drive

This vibratory mill type it's made of a tubular grinding chamber having an inside diameter *D* (typically *D = 1000 mm*) and a length *L* (usually *L = 2500 mm*). The thickness of the wall *t* of the grinding chamber, including its armor, is usually *35 mm*. In order to minimize the influence of torsional vibrations relative to the axis of the mill, the grinding tube is supported with by two pairs of elastic supports. The vibratory mill is powered by two electro-vibrators, which work independently of each other, with adjustable speeds. To generate the vibrations of the mill, both electro-vibrators have eight pairs of unbalanced masses. The elastic support system consists of four rubber pads. For the evacuation of the product to be milled, the mill tube is eqquipped with an interchangeable sieve up to *500 mm* in diameter, located in the quadrant IV of its cover. The ground material is discharged as a finished product through the outlet pipe or - as the case may be - is reintroduced into the next stage of the milling process.

3. THE DYNAMIC MODEL OF TUBULAR VIBRATORY MILL WITH ROTARY CHAMBER AND DOUBLE LATERAL DRIVE

The dynamic model proposed for half of the tubular vibratory mill (*L/2 = 1250 mm*) is designed as a rigid solid body with three degrees of freedom: x, y and ξ (Figure 3).

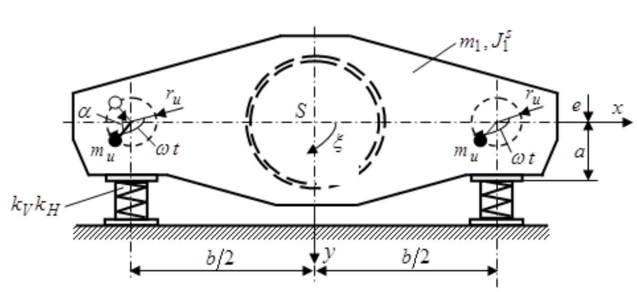

Figure 3. The dynamic model of tubular vibratory mill with rotary chamber and double lateral drive

During the operation of the mill it's tube produces linear force vibrations along horizontal x, vertical y and rotation directions ξ (around the longitudinal axis) x. The origin of axis system coincides with the center of gravity of the empty mill, which is motionless. In stationary operating mode, which for vibratory mills (as for most other vibratory machines) takes place in the resonance range, the damping effect in the elastic support system can be neglected.

The following notations are used to write the dynamic model (see figure 3):

m_1 - the mass of the empty mill;

J_1^S - moment of inertia of the mill body in relation to the center of mass S;

r_u - the distance from the eccentric mass of the vibro generator to it's axis of rotation;

m_u - the value of the eccentric mass;

k_V, k_H - the rigidity constants of the elastic bearing elements along the vertical (y) and horizontal (x) directions;

a - the vertical distance from the center of the empty mill to the level of the elastic support elements;
$b/2$ - the horizontal distance from the center of the empty mill to the symmetry axis of an elastic support element;
e – the eccentricity, measured on the vertical direction, of the axis of the vibro generator from the center of mass of the empty mill.

From the geometric data of this tubular vibratory mill (see Figures 2 and 3), can be determined the mass of the empty mill, m_1, the coordinates of the center of mass of the empty mill S, and the moment of inertia of the mill's body to the center of weight S, J_1^S.

The eccentricity r_u is determined with the relation:

$$r_u = y_u \cos\frac{\beta}{2} \tag{1}$$

where:
y_u is the vertical distance from point S to one of the eccentric masses;
β - the angle between the symmetry axes of the two unbalanced masses (all eight pairs of unbalanced masses can be set off from 15^0 to 15^0).

Using the principle of d'Alembert, we obtain the differential equations of movement of the vibratory mill, obtaining a linear system of non-homogeneous differential equations of the second order:

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & J_1^S & 0 \\ 0 & 0 & m_1 \end{bmatrix}\begin{bmatrix} \ddot{x} \\ \ddot{\xi} \\ \ddot{y} \end{bmatrix} + \begin{bmatrix} k_{xx} & k_{x\xi} & 0 \\ k_{\xi x} & k_{\xi\xi} & 0 \\ 0 & 0 & k_{yy} \end{bmatrix}\begin{bmatrix} x \\ \xi \\ y \end{bmatrix} = m_u r_u \omega^2 \begin{bmatrix} |e|[\cos\omega t + \cos(\omega t + \alpha)] + \\ + \frac{b}{2}[\sin\omega t - \sin(\omega t + \alpha)] \end{bmatrix} \tag{2}$$

In relations (2), the mass distribution matrix is:

$$[M] = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & J_1^S & 0 \\ 0 & 0 & m_1 \end{bmatrix}, \tag{3}$$

and the stiffness matrix is

$$[K] = \begin{bmatrix} k_{xx} & k_{x\xi} & 0 \\ k_{\xi x} & k_{\xi\xi} & 0 \\ 0 & 0 & k_{yy} \end{bmatrix}. \tag{4}$$

Using the principle of virtual work we can determine the elements of the stiffness matrix [K], as follows:

$$k_{xx} = 2k_H; k_{yy} = 2k_V;$$

$$k_{\xi\xi} = 2(k_V \cdot \frac{b^2}{4} + k_H \cdot a^2);$$

$$k_{\xi x} = k_{x\xi} = 2 \cdot k_H \cdot a.$$

The system of differential equations (2) decomposes into:

$$\begin{bmatrix} m_1 & 0 \\ 0 & J_1^S \end{bmatrix}\begin{bmatrix} \ddot{x} \\ \ddot{\xi} \end{bmatrix} + \begin{bmatrix} k_{xx} & k_{x\xi} \\ k_{\xi x} & k_{\xi\xi} \end{bmatrix}\begin{bmatrix} x \\ \xi \end{bmatrix} = \begin{bmatrix} 1 \\ |e| \end{bmatrix} m_u r_u \omega^2 [\cos\omega t + \cos(\omega t + \alpha)]; \tag{5}$$

$$\begin{bmatrix} m_1 & 0 \\ 0 & J_1^S \end{bmatrix}\begin{bmatrix} \ddot{x} \\ \ddot{\xi} \end{bmatrix} + \begin{bmatrix} k_{xx} & k_{x\xi} \\ k_{\xi x} & k_{\xi\xi} \end{bmatrix}\begin{bmatrix} x \\ \xi \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{b}{2} \end{bmatrix} m_u r_u \omega^2 [\sin\omega t - \sin(\omega t + \alpha)]; \tag{6}$$

$$m_1 \ddot{y} + k_{yy} y = m_u r_u \omega^2 [\sin\omega t + \sin(\omega t + \alpha)] \tag{7}$$

The proposed solutions are:

$$x_1(t) = U_1[\cos\omega t + \cos(\omega t + \alpha)]; \tag{8}$$

$$\xi_1(t) = -\varepsilon_1[\cos\omega t + \cos(\omega t + \alpha)]; \tag{9}$$

$$x_2(t) = U_2[\sin\omega t - \sin(\omega t + \alpha)]; \tag{10}$$

$$\xi_2(t) = \varepsilon_2[\sin\omega t - \sin(\omega t + \alpha)]; \tag{11}$$

$$y(t) = A[\sin\omega t + \sin(\omega t + \alpha)] \tag{12}$$

By replacing in the equations of the system (5), (6) and (7) we obtain:

$$\begin{bmatrix} m_1 & 0 \\ 0 & J_1^S \end{bmatrix}\begin{bmatrix} -U_1 \\ -\varepsilon_1 \end{bmatrix}\omega^2 + \begin{bmatrix} k_{xx} & k_{x\xi} \\ k_{\xi x} & k_{\xi\xi} \end{bmatrix}\begin{bmatrix} U_1 \\ \varepsilon_1 \end{bmatrix} = \begin{bmatrix} 1 \\ |e| \end{bmatrix} m_u r_u \omega^2; \tag{13}$$

$$\begin{bmatrix} m_1 & 0 \\ 0 & J_1^S \end{bmatrix}\begin{bmatrix} -U_1 \\ -\varepsilon_1 \end{bmatrix}\omega^2 + \begin{bmatrix} k_{xx} & k_{x\xi} \\ k_{\xi x} & k_{\xi\xi} \end{bmatrix}\begin{bmatrix} U_2 \\ \varepsilon_2 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{b}{2} \end{bmatrix} m_u r_u \omega^2; \tag{14}$$

$$m_1(-A)\omega^2 + k_{yy} A = m_u r_u \omega^2. \tag{15}$$

Referring to the notations:

$$\omega_x^2 = \frac{k_{xx}}{m_1}; \tag{16}$$

$$\omega_\xi^2 = \frac{k_{\xi\xi}}{J_1^S}; \quad (17)$$

$$\rho = \frac{k_{x\xi}}{m_1}; \quad (18)$$

$$\gamma = \frac{k_{\xi x}}{J_1^S}; \quad (19)$$

$$\omega_y^2 = \frac{k_{yy}}{m_1}, \quad (20)$$

the homogeneous equations attached to the system made up of (13), (14) and (15) become:

$$\begin{bmatrix} \omega_x^2 - \omega^2 & \rho \\ \gamma & \omega_\xi^2 - \omega^2 \end{bmatrix} \begin{bmatrix} U_1 \\ \varepsilon_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (21)$$

$$\begin{bmatrix} \omega_x^2 - \omega^2 & \rho \\ \gamma & \omega_\xi^2 - \omega^2 \end{bmatrix} \begin{bmatrix} U_2 \\ \varepsilon_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (22)$$

$$\left(\omega_y^2 - \omega^2\right)A = 0 \quad (23)$$

The three proper pulsations are:

$$\omega_1 = \sqrt{\frac{1}{2}\left(\omega_x^2 + \omega_\xi^2\right) + \sqrt{\frac{1}{4}\left(\omega_x^2 - \omega_\xi^2\right) + \rho\gamma}}\ ; \quad (24)$$

$$\omega_2 = \sqrt{\frac{1}{2}\left(\omega_x^2 + \omega_\xi^2\right) - \sqrt{\frac{1}{4}\left(\omega_x^2 - \omega_\xi^2\right) + \rho\gamma}}\ ; \quad (25)$$

$$\omega_3 = \sqrt{\frac{k_{yy}}{m_1}}\ . \quad (26)$$

The system of non-homogeneous differential equations (13) has the solution:

$$U_1(\omega) = m_u r_u \omega^2 \frac{\frac{1}{m_1}\left(\omega_\xi^2 - \omega^2\right) - \frac{|e|}{J_1^S}\rho}{\left(\omega_x^2 - \omega^2\right)\left(\omega_\xi^2 - \omega^2\right) - \rho\gamma}; \quad (27)$$

$$\varepsilon_1(\omega) = m_u r_u \omega^2 \frac{\frac{|e|}{J_1^S}\left(\omega_x^2 - \omega^2\right) - \frac{\gamma}{m_1}}{\left(\omega_x^2 - \omega^2\right)\left(\omega_\xi^2 - \omega^2\right) - \rho\gamma}. \quad (28)$$

The solutions of the system (14) are:

$$U_2(\omega) = -m_u r_u \omega^2 \frac{\frac{b}{2}\cdot\frac{\rho}{J_1^S}}{\left(\omega_x^2 - \omega^2\right)\left(\omega_\xi^2 - \omega^2\right) - \rho\gamma}; \quad (29)$$

$$\varepsilon_2(\omega) = m_u r_u \omega^2 \frac{\frac{b}{2}\cdot\frac{1}{J_1^S}\left(\omega_x^2 - \omega^2\right)}{\left(\omega_x^2 - \omega^2\right)\left(\omega_\xi^2 - \omega^2\right) - \rho\gamma}. \quad (30)$$

For equation (15) can be given as a solution:

$$A(\omega) = m_u r_u \omega^2 \frac{1}{m_1\left(\omega_y^2 - \omega^2\right)}. \quad (31)$$

4. EXAMPLE OF CALCULATION

For the vibratory mill of type PALLA-U 20 (Germany) [23], same as the analyzed mill, the technical data are [23]: *m_1=2736 kg; J_1^S=834,72 kgm^2; m_u=49 kg; a=0,512 m; e=0,018 m; b/2=1,025 m; k_v=583 N/mm; y_u=291mm.*

The mass distribution matrix will be:

$$[M] = \begin{bmatrix} 2736 & 0 & 0 \\ 0 & 834{,}72 & 0 \\ 0 & 0 & 2736 \end{bmatrix}$$

and stiffness matrix coefficients will have values:

$$k_{xx} = 2\cdot 149440 = 298880\ \text{N/m}$$

$$k_{yy} = 2\cdot 583000 = 1166000\ \text{N/m}$$

$$k_{\xi\xi} = 2(583000\cdot\frac{2{,}05}{4} + 149440\cdot 0{,}512)^2 = 1303378$$

N/m

$$k_{\xi x} = k_{x\xi} = 2\cdot 149440\cdot 0{,}512 = 153027\ \text{N}$$

Will then result $\omega_1 = 41{,}3\ \text{s}^{-1}$; $\omega_2 = 10{,}1\ \text{s}^{-1}$; $\omega_3 = 20{,}6\ \text{s}^{-1}$.

In the following figures [8, 14] are graphically plotted amplitude dependencies calculated by frequency ω ($U_1 = U_1(\omega)$ in Figure 4a; $U_2 = U_2(\omega)$ in Figure 4b); $\varepsilon_1 = \varepsilon_1(\omega)$ in Figure 5*a*, $\varepsilon_2 = \varepsilon_2(\omega)$ in Figure 5*b* and $A = A(\omega)$ in Figure 6). In representation of the curves, was used a special calculation program, which is not the subject of the present paper. For normal rotation speed (*n=900* rot/min), $U_1(\omega)$ tends towards value $\frac{m_u r_u}{m_1} = \frac{49\cdot 172}{2736} = 3{,}08\ \text{mm}$.

The curve converges to the $\frac{m_u r_u e}{J_1^S} = \frac{49\cdot 0{,}172\cdot 0{,}018}{834{,}7} = 0{,}00018$,

which corresponds to an angle of *$0{,}01^0$* (rotational vibrations).

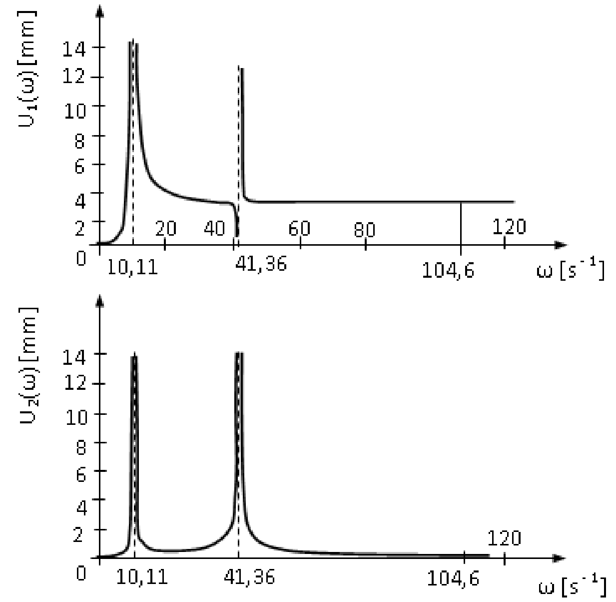

Figure 4. The curves variation, $U_1=U_1(\omega)$ (figure 4a) and $U_2=U_2(\omega)$ (figure 4b), for tubular vibratory mill with rotary chamber and double lateral drive

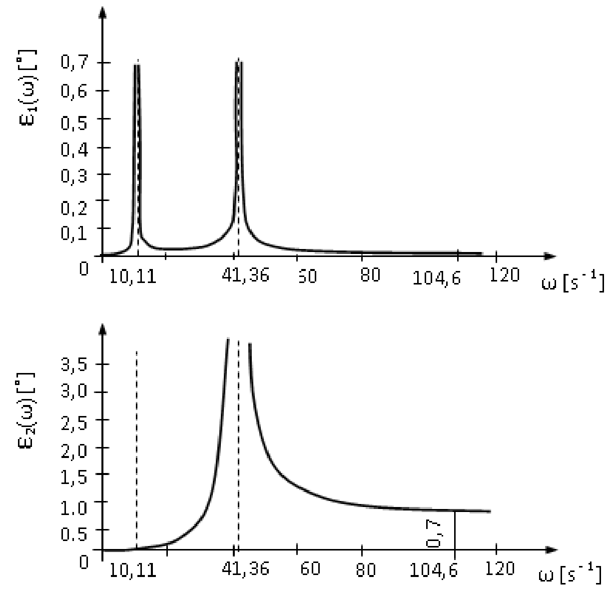

Figure 5. The curves variation, $\varepsilon_1=\varepsilon_1(\omega)$ (figure 5a) and $\varepsilon_2=\varepsilon_2(\omega)$ (figure 5b), for tubular vibratory mill with rotary chamber and double lateral drive

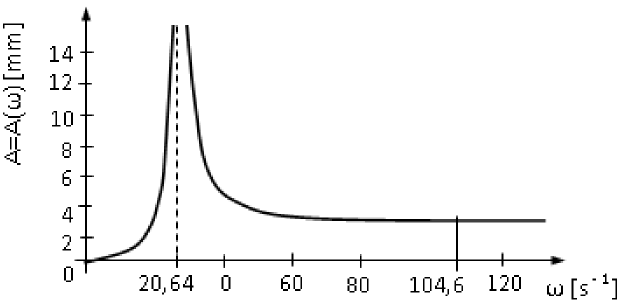

Figure 6. The curve variation $A=A(\omega)$, for tubular vibratory mill with rotary chamber and double lateral drive

The curve $\varepsilon_2(\omega)$ becomes $\frac{m_u r_u b}{2J_1^S}=0,01$, value corresponding to an angle of *0,57*^{0}.

The amplitude of the vertical vibration $A(\omega)$ tends towards the same value as the amplitude of the horizontal vibration $U_1(\omega)$.

The most important conclusion is the following: if the values mentioned on the graphs are compared, $U_1=3,2$ mm and $A=3,25$ mm for $\omega=104,6s^{-1}$ (i.e. *n=1000* rot/min), it can bee seen that, in the corresponding operating speed range, the tubular vibratory mill moves on a circular trajectory.

The parameters that characterize the elastic support system of the mill

The elastic support system consists of four rubber elements with a ring cross-section. These four rubber elements are mounted in parallel and work on compression in a vertical plane and on shear in the horizontal plane.

The amplitude of the dynamic force F_{Tx} in the *x* direction, which the mill transmits to the foundation on which it is placed through the four elastic elements, is given by the relation [5]:

$$F_{Tx}=4k_x\sqrt{A_x^2+a_z^2A_\varphi^2}\ . \qquad (32)$$

The vibrations isolation factor is [10, 11, 13]:

$$I_x=1-\frac{F_{Tx}}{P_0}=1-\frac{F_{Tx}}{m_0r\omega^2} \qquad (33)$$

which must have values: $I_x = 0,70\ldots0,90$.

The amplitude of the dynamic force F_{Tz} in the *z* direction, which the mill sends to the foundation on which it is placed through the four elastic elements, is given by the relation [1]:

$$F_{Tx}=4k_z\sqrt{A_z^2+a_z^2A_\varphi^2}\ . \qquad (34)$$

The vibrations isolation factor is [10, 11, 13]:

$$I_z=1-\frac{F_{Tz}}{P_0}=1-\frac{F_{Tz}}{m_0r\omega^2} \qquad (35)$$

which must have values: $I_x = 0,80\ldots0,95$.

4. CONCLUSIONS

Comparing the parameters used (horizontal, vertical and rotational vibration amplitudes and frequencies) resulting from the proposed calculation model, with those shown in the technical book of the

PALLA-U 20 vibratory mill [23], it has been found that the proposed dynamic model is perfectly suitable for establishing the constructive and functional characteristics of the vibratory mills of the type that was the subject of the present study.

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[23] * * * *Prospekte der Firma Klöckner-Humboldt-Deutz AG Koln (Humboldt-Schwingmühle) PALLA 20 U.*