Analysis of the Support Vibrations For the ELI-NP System with Impact on the Human Body and Optoelectronic Equipment

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Abstract: - The article presents research results on the support system for the basic structure plate of the ELI-NP Romania equipment. Thus, a flat platform with two rectangular symmetry planes was studied on which the entire laser installation is mounted and equipped. Modal analysis is essential for identifying self-motions associated with the vibration modes specific to the elastic support system. This requirement constitutes the fundamental basis for assessing the system's response to external excitations generated by environmental sources of vibration or zonal seismic movements.

Keywords: - modal vibrations, forced vibrations, optoelectronic platform, effects on the human body, effects on the optoelectronic system.

1. INTRODUCTION

The approach in this study involves the movement of a rigid body with elastic supports, allowing for the identification of the six degrees of freedom.

Because the material system is made of a plate with geometric symmetry in relation to the two median planes, longitudinally and transversely, it follows that the single axis of symmetry stands as the characteristic and determined element for the geometric and mass configuration.

Based on the modal analysis, the eigen pulses and eigenvectors of the support plate were determined from the ELI–Măgurele platform in Bucharest, Romania [1, 2, 3, 4].

2. ANALYSIS OF DECOUPLED VIBRATIONS OF RIGID ELASTIC BONDS

Based on the Lagrange equations of the second order, a system of differential equations of motion was obtained, with six equations statically coupled (by the existence of non-diagonal coefficients of the stiffness matrix) and dissipative coupled (by the existence of non-diagonal coefficients of the damping matrix). This system is difficult to solve analytically or using matrix formalism because it requires a large volume of computation, and the sixth-degree polynomial equation of the eigenequations generates difficulties in solving and analysis. In addition, the expressions of dynamic parameters are very complicated, and it is not possible to directly highlight the influence of mass, dimensional, elastic, and viscous characteristics, as well as disturbing factors (amplitude, pulsation), on these dynamic parameters.

The solution involves the automatic numerical calculation of the system of second-order differential equations of motion, resulting in a system of 12 first-order differential equations that can be solved without difficulty. However, when using numerical analysis, it appears to be a disadvantage to highlight the influence of the physical characteristics of the dynamical system. Thus, the analysis is performed punctually through repeated tests with sets of different values for the input data (masses, lengths, stiffness coefficients, viscous damping coefficients, etc.).

For this purpose, both for eliminating certain couplings of motions and for obtaining the analytical solution of the dynamical system model, specific dimensional and structural conditions can be established to decouple the system of equations into simpler and more easily integrated subsystems. In addition, if the rigid bonds are considered elastic or have low damping, the equations of motion can be simplified by canceling the damping term. This results in a simplified system of dynamic equations for the elastic case [5, 6, 7, 8].

2.1. Vibrations of the rigid support plate with a vertical axis of symmetry

Let us consider a rigid body with a four-point lower base elastic support, as seen in Figure 1.

The rigid has two vertical planes of symmetry, longitudinal (yCz) and transversal (xCz) ones, meaning it has a vertical axis of symmetry (Cz).

The symmetry of the rigid body is characterized by its geometric configuration, mass distribution, positioning of the supports, and the geometric and physical identity of these supports, all of which have equal elastic constants.

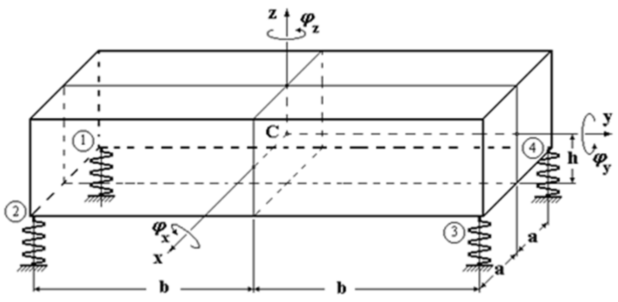


Figure 1. Rigid with four-point lower base elastic support

Owing to the aforementioned symmetries, part of the coupling terms in the stiffness matrix cancel out, so we have:

$$\sum k_{ix} y_i = 0 \qquad \sum k_{iy} x_i = 0$$
$$\sum k_{iz} y_i = 0 \qquad \sum k_{iz} x_i = 0 \tag{1}$$

$$\sum k_{iy} z_i x_i = 0 \qquad \sum k_{ix} y_i z_i = 0$$
$$\sum k_{iz} x_i y_i = 0 \tag{2}$$

In this case, the stiffness matrix is:

$$\underline{C} = \begin{bmatrix} 4k_x & 0 & 0 & 0 & -4hk_x & 0 \\ 0 & 4k_y & 0 & 4hk_y & 0 & 0 \\ 0 & 0 & 4k_z & 0 & 0 & 0 \\ 0 & 4hk_y & 0 & 4\left(b^2k_z + h^2k_y\right) & 0 & 0 \\ -4hk_x & 0 & 0 & 0 & 4\left(h^2k_x + a^2k_z\right) & 0 \\ 0 & 0 & 0 & 0 & 0 & 4\left(a^2k_y + b^2k_x\right) \end{bmatrix} \tag{3}$$

or, in tabular form:

$\underline{C}$ =

X	Y	Z	φ_x	φ_y	φ_z	
$4k_x$	0	0	0	$-4hk_x$	0	X
0	$4k_y$	0	$-4hk_y$	0	0	Y
0	0	$4k_z$	0	0	0	Z
0	$-4hk_y$	0	$4\left(b^2k_z + h^2k_y\right)$	0	0	φ_x
$-4hk_x$	0	0	0	$4\left(h^2k_x + a^2k_z\right)$	0	φ_y
0	0	0	0	0	$4\left(a^2k_y + b^2k_x\right)$	φ_z

From the tabular form of the stiffness matrix, by the disappearance of the coupling terms, the system decouples into four subsystems described by the coordinates $(X, \varphi_y), (Y, \varphi_x)$, Z and φ_z.

The matrices of these subsystems are structured as follows:

a) the subsystem (X, φ_y)

• inertia matrix

$$\underline{A}_1 = \begin{bmatrix} m & 0 \\ 0 & J_y \end{bmatrix}; \quad (4)$$

•• stiffness matrix

$$\underline{C}_1 = \begin{bmatrix} 4k_x & -4hk_x \\ -4hk_x & 4\left(h^2k_x + a^2k_z\right) \end{bmatrix}; \quad (5)$$

b) the subsystem (Y, φ_x)

• inertia matrix

$$\underline{A}_2 = \begin{bmatrix} m & 0 \\ 0 & J_x \end{bmatrix}; \quad (6)$$

•• stiffness matrix

$$\underline{C}_2 = \begin{bmatrix} 4k_y & 4hk_y \\ 4hk_y & 4\left(b^2k_z + h^2k_y\right) \end{bmatrix}; \quad (7)$$

c) the vertical vibration, Z, is determined by m and $4k_z$;

d) yaw (gyratory) vibration φ_z is determined by J_z and $4\left(a^2k_y + b^2k_x\right)$.

According to the decoupling obtained, the systems of the differential equations of free vibrations are as follows:

– for the coupled lateral vibrations and roll :

$$\begin{cases} m\ddot{X} + 4k_xX - 4hk_x\varphi_y = 0 \\ J_y\ddot{\varphi}_y - 4hk_xX + 4\left(h^2k_x + a^2k_z\right)\varphi_y = 0; \end{cases} \quad (8)$$

– for the coupled longitudinal vibrations and pitch

$$\begin{cases} m\ddot{Y} + 4k_yY + 4hk_y\varphi_x = 0 \\ J_x\ddot{\varphi}_x + 4hk_yY + 4\left(b^2k_z + h^2k_y\right)\varphi_x = 0; \end{cases} \quad (9)$$

– for the vertical vibrations, it is the equation:

$$m\ddot{Z} + 4k_zZ = 0; \quad (10)$$

– for the yaw (gyratory) vibration, the equation is :

$$J_z\ddot{\varphi}_z + 4\left(a^2k_y + b^2k_x\right)\varphi_z = 0. \quad (11)$$

2.2. Analysis of the eigenmodes of vibration

To calculate the eigen pulsations and eigenvalues (eigenmodes of vibration), consider the systems of differential equations of motion given by relations (8) ... (11). By dividing the relations with the mass elements characteristic of the rigid, the canonical forms of the equations of motion and dynamic matrices are obtained.

• Lateral motion coupled with roll motion (swaying):

– The canonical form of the of motion equations system i

$$\begin{cases} \ddot{X} + 4\dfrac{k_x}{m}X - 4h\dfrac{k_x}{m}\varphi_y = 0 \\ \ddot{\varphi}_y - 4\dfrac{hk_x}{J_y}X + 4\dfrac{\left(h^2k_x + a^2k_z\right)}{J_y}\varphi_y = 0. \end{cases} \quad (12)$$

– The dynamic matrix of the system is:

$$\underline{D}_1 = \underline{A}_1^{-1}\underline{C}_1 = \begin{bmatrix} 4\dfrac{k_x}{m} & -4h\dfrac{k_x}{m} \\ -4\dfrac{hk_x}{J_y} & 4\dfrac{\left(h^2k_x + a^2k_z\right)}{J_y} \end{bmatrix}. \quad (13)$$

– The equation of eigenpulsations is:

$$p^4 - 4\left(\frac{k_x}{m} + \frac{h^2k_x + a^2k_z}{J_y}\right)p^2 + \frac{16}{mJ_y}\left[k_x\left(h^2k_x + a^2k_z\right) - h^2k_x^2\right] = 0, \quad (14)$$

or

$$p^4 - \frac{4}{m}\left(k_x + \frac{h^2k_x + a^2k_z}{i_y^2}\right)p^2 + \frac{16}{m^2}\frac{a^2k_zk_x}{i_y^2} = 0, \quad (15)$$

where: $J_x = mi_x^2$; $J_y = mi_y^2$; $J_z = mi_z^2$ are the moments of inertia;

i_x , i_y , i_z – the radii of inertia (gyration) of the rigid according along the axes Cx, Cy, respectively Cz.

The solutions of equation (16) represent the eigen pulsations of the rigid body for the coupled lateral and rolling motions (swaying), as follows:

$$p_{1,2}^2 = \frac{2}{m}\left[k_x + \frac{h^2k_x + a^2k_z}{i_y^2} \pm \sqrt{k_x^2 + 2k_x\frac{h^2k_x - a^2k_z}{i_y^2} + \left(\frac{h^2k_x + a^2k_z}{i_y^2}\right)^2}\right] \quad (16)$$

By using the notations:

$$\alpha_1 = \frac{1}{m}\sum k_{ix} z_i = -\frac{4}{m} h k_x$$

$$\alpha_2 = \frac{1}{J_y}\sum k_{ix} z_i = -\frac{4}{J_y} h k_x$$

$$p_X = \sqrt{\frac{\sum k_{ix}}{m}} = 2\sqrt{\frac{k_x}{m}}$$ - eigenpulsation of the uncoupled lateral vibration;

$$p_{\varphi_y} = \sqrt{\frac{\sum z_i^2 k_{ix} + \sum x_i^2 k_{iz}}{J_y}} = 2\sqrt{\frac{h^2 k_x + a^2 k_z}{J_y}}$$

- eigen pulsation of the uncoupled roll vibration (swaying),

the equation (16) turns into:

$$p^4 - \left(p_X^2 + p_{\varphi_y}^2\right) p^2 + \left(p_X^2 p_{\varphi_y}^2 - \alpha_1 \alpha_2\right) = 0, \quad (17)$$

therefore, the squares of the eigen pulsation are:

$$p_{1,2}^2 = \frac{1}{2}\left[p_X^2 + p_{\varphi_y}^2 \pm \sqrt{\left(p_X^2 - p_{\varphi_y}^2\right)^2 + 4\alpha_1\alpha_2}\right]. \quad (18)$$

Assuming that there is an ordering relation $p_1 < p_2$ between the eigen pulsation values, their expressions are:

$$p_1 = \sqrt{\frac{1}{2}\left[p_X^2 + p_{\varphi_y}^2 - \sqrt{\left(p_X^2 - p_{\varphi_y}^2\right)^2 + 4\alpha_1\alpha_2}\right]}; \quad (19)$$

$$p_2 = \sqrt{\frac{1}{2}\left[p_X^2 + p_{\varphi_y}^2 + \sqrt{\left(p_X^2 - p_{\varphi_y}^2\right)^2 + 4\alpha_1\alpha_2}\right]}. \quad (20)$$

The eigen vectors $\underline{\underline{v}}_1 = \left[1 \ \mu_1\right]^{\mathrm{T}}$ and $\underline{\underline{v}}_2 = \left[1 \ \mu_2\right]^{\mathrm{T}}$ are obtained by calculating the eigenpulsation values:

$$\mu_1 = \frac{p_1^2 - d_{11}}{d_{12}} = -\frac{1}{2\alpha_1}\left[p_X^2 + p_{\varphi_y}^2 + \sqrt{\left(p_X^2 - p_{\varphi_y}^2\right)^2 + 4\alpha_1\alpha_2}\right]; \quad (21)$$

$$\mu_2 = \frac{p_2^2 - d_{11}}{d_{12}} = -\frac{1}{2\alpha_1}\left[p_X^2 + p_{\varphi_y}^2 - \sqrt{\left(p_X^2 - p_{\varphi_y}^2\right)^2 + 4\alpha_1\alpha_2}\right], \quad (22)$$

where d_{11} and d_{12} are the corresponding elements of the dynamic matrix, $\underline{D}_1$.

•• Longitudinal motion coupled with pitch motion (gallops):

– The canonical form of the motion equation system is:

$$\begin{cases} \ddot{Y} + 4\dfrac{k_y}{m} Y + 4h\dfrac{k_y}{m}\varphi_x = 0 \\ \ddot{\varphi}_x + 4\dfrac{h k_y}{J_x} Y + 4\dfrac{\left(b^2 k_z + h^2 k_y\right)}{J_x}\varphi_x = 0. \end{cases} \quad (23)$$

– The dynamic matrix of the system is:

$$\underline{D}_2 = \underline{A}_2^{-1} \underline{C}_2 = \begin{bmatrix} 4\dfrac{k_y}{m} & -4h\dfrac{k_y}{m} \\ -4\dfrac{h k_y}{J_x} & 4\dfrac{\left(b^2 k_z + h^2 k_y\right)}{J_x} \end{bmatrix}. \quad (24)$$

– The equation of eigenpulsations is:

$$p^4 - 4\left(\frac{k_y}{m} + \frac{b^2 k_z + h^2 k_y}{J_x}\right) p^2 + \frac{16}{m J_x}\left[k_y\left(b^2 k_z + h^2 k_y\right) - h^2 k_y^2\right] = 0, \quad (25)$$

or

$$p^4 - \frac{4}{m}\left(k_y + \frac{b^2 k_z + h^2 k_y}{i_x^2}\right) p^2 + \frac{16}{m^2}\frac{b^2 k_y k_z}{i_x^2} = 0. \quad (26)$$

Based on the above, the expression of eigenpulsations for the coupled vibrations, longitudinal, and pitch (gallops) is:

$$p_{3,4}^2 = \frac{2}{m}\left[k_y + \frac{b^2 k_z + h^2 k_y}{i_x^2} \pm \sqrt{k_y^2 + 2k_y\frac{h^2 k_y - b^2 k_z}{i_x^2} + \left(\frac{b^2 k_z + h^2 k_y}{i_x^2}\right)^2}\right]. \quad (27)$$

By using the notations:

$$\beta_1 = \frac{1}{m}\sum k_{iy} z_i = -\frac{4}{m} h k_y;$$

$$\beta_2 = \frac{1}{J_x}\sum k_{iy} z_i = -\frac{4}{J_x} h k_y;$$

$$p_Y = \sqrt{\frac{\sum k_{iy}}{m}} = 2\sqrt{\frac{k_y}{m}}$$ – eigenpulsation of uncoupled longitudinal vibration;

$$p_{\varphi_x} = \sqrt{\frac{\sum y_i^2 k_{iz} + \sum z_i^2 k_{iy}}{J_x}} = 2\sqrt{\frac{b^2 k_z + h^2 k_y}{J_x}}$$ - eigenpulsation of the uncoupled pitch vibration (gallops), and equation (27) becomes:

$$p^4 - \left(p_Y^2 + p_{\varphi_x}^2\right)p^2 + \left(p_Y^2 p_{\varphi_x}^2 - \beta_1\beta_2\right) = 0, \quad (28)$$

so that results from the squares of the eigen pulsation:

$$p_{3,4}^2 = \frac{1}{2}\left[p_Y^2 + p_{\varphi_x}^2 \pm \sqrt{\left(p_Y^2 - p_{\varphi_x}^2\right)^2 + 4\beta_1\beta_2}\right]. \quad (29)$$

If $p_3 < p_4$, the eigenpulsation expressions become:

$$p_3 = \sqrt{\frac{1}{2}\left[p_Y^2 + p_{\varphi_x}^2 - \sqrt{\left(p_Y^2 - p_{\varphi_x}^2\right)^2 + 4\beta_1\beta_2}\right]}; \quad (30)$$

$$p_4 = \sqrt{\frac{1}{2}\left[p_Y^2 + p_{\varphi_x}^2 + \sqrt{\left(p_Y^2 - p_{\varphi_x}^2\right)^2 + 4\beta_1\beta_2}\right]}. \quad (31)$$

The eigenvectors $\underline{\underline{v}}_3 = \left[1 \;\; \mu_3\right]^{\mathrm{T}}$ and $\underline{\underline{v}}_4 = \left[1 \;\; \mu_4\right]^{\mathrm{T}}$ are obtained by calculating the eigenpulsation values:

$$\mu_3 = \frac{p_3^2 - d_{11}}{d_{12}} = -\frac{1}{2\beta_1}\left[p_Y^2 + p_{\varphi_x}^2 + \sqrt{\left(p_Y^2 - p_{\varphi_x}^2\right)^2 + 4\beta_1\beta_2}\right]; \quad (32)$$

$$\mu_4 = \frac{p_4^2 - d_{11}}{d_{12}} = -\frac{1}{2\beta_1}\left[p_Y^2 + p_{\varphi_x}^2 - \sqrt{\left(p_Y^2 - p_{\varphi_x}^2\right)^2 + 4\beta_1\beta_2}\right], \quad (33)$$

where d_{11} and d_{12} are the corresponding elements of the dynamic matrix, $\underline{\underline{D}}_2$.

••• Uncoupled vertical motion

The equation of motion along vertical axis is:

$$\ddot{Z} + 4\frac{k_z}{m}Z = 0 \quad (34)$$

so that the fifth eigen pulsation of the rigid, elastically supported can be obtained with a vertical axis of symmetry in the form of:

$$p_5 = p_Z = 2\sqrt{\frac{k_z}{m}}. \quad (35)$$

•••• The yaw uncoupled motion (gyratory)

The equation gives the free yaw vibration:

$$\ddot{\varphi}_z + 4\frac{a^2k_y + b^2k_x}{J_z}\varphi_z = 0, \quad (36)$$

so that the sixth eigenpulsation can be obtained:

$$p_6 = p_{\varphi_z} = 2\sqrt{\frac{a^2k_y + b^2k_x}{J_z}}. \quad (37)$$

Knowing the calculated eigenpulsations p_1, p_2, p_3, p_4, p_5, p_6 and the eigenvalues μ_1, μ_2, μ_3, μ_4, the laws of motion for the free vibrations rigid elastically supported, with two planes of symmetry (transverse and longitudinal) are:

$$\begin{cases} X(t) = C_1\sin\left(p_1t + \theta_1\right) + C_2\sin\left(p_2t + \theta_2\right) \\ \varphi_y(t) = \mu_1C_1\sin\left(p_1t + \theta_1\right) + \mu_2C_2\sin\left(p_2t + \theta_2\right) \end{cases} \quad (38)$$

$$\begin{cases} Y(t) = C_3\sin\left(p_3t + \theta_3\right) + C_4\sin\left(p_4t + \theta_4\right) \\ \varphi_x(t) = \mu_3C_3\sin\left(p_3t + \theta_3\right) + \mu_4C_4\sin\left(p_4t + \theta_4\right) \end{cases} \quad (39)$$

$$Z(t) = C_5\sin p_5t = C_5\sin\left(2\sqrt{\frac{k_z}{m}}t + \theta_5\right) \quad (40)$$

$$\varphi_z(t) = C_6\sin p_6t = C_6\sin\left(2\sqrt{\frac{a^2k_y + b^2k_x}{J_z}}t + \theta_6\right), \quad (41)$$

where C_1, C_2, C_3, C_4, C_5, C_6, θ_1, θ_2, θ_3, θ_4, θ_5, θ_6 are constants with values determined from the initial conditions of motion.

The calculation model, differential equations of motion, eigenpulsations, pulsations of the uncoupled motions, and expressions of the coupling terms of the free vibrations of the rigid with a vertical axis of symmetry (Figure 2) are summarized in Table 1 [9, 10, 11, 12, 13].

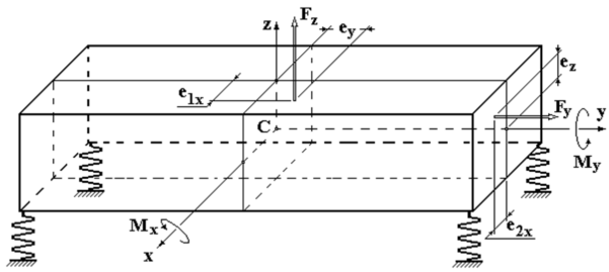
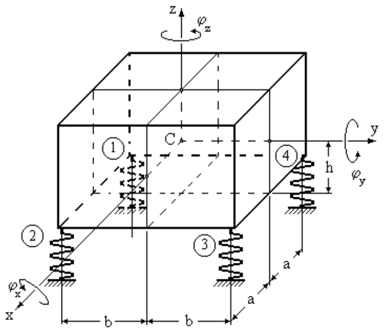


Figure 2. The rigid body with a vertical axis of symmetry

Table 1. Rigid with two planes of symmetry – free vibrations

Model for calculation	**Equations of free vibrations**
	$\begin{cases} m\ddot{X}+4k_xX-4hk_x\varphi_y=0 \\ J_y\ddot{\varphi}_y-4hk_xX+4\left(h^2k_x+a^2k_z\right)\varphi_y=0 \end{cases}$ $\begin{cases} m\ddot{Y}+4k_yY+4hk_y\varphi_x=0 \\ J_x\ddot{\varphi}_x+4hk_yY+4\left(b^2k_z+h^2k_y\right)\varphi_x=0 \end{cases}$ $m\ddot{Z}+4k_zZ=0 \qquad J_z\ddot{\varphi}_z+4\left(a^2k_y+b^2k_x\right)\varphi_z=0$
Eigen pulsations	$p_1=\sqrt{\frac{1}{2}\left[p_X^2+p_{\varphi_y}^2-\sqrt{\left(p_X^2-p_{\varphi_y}^2\right)^2+4\alpha_1\alpha_2}\right]}$ $p_2=\sqrt{\frac{1}{2}\left[p_X^2+p_{\varphi_y}^2+\sqrt{\left(p_X^2-p_{\varphi_y}^2\right)^2+4\alpha_1\alpha_2}\right]}$ $p_3=\sqrt{\frac{1}{2}\left[p_Y^2+p_{\varphi_x}^2-\sqrt{\left(p_Y^2-p_{\varphi_x}^2\right)^2+4\beta_1\beta_2}\right]}$ $p_4=\sqrt{\frac{1}{2}\left[p_Y^2+p_{\varphi_x}^2+\sqrt{\left(p_Y^2-p_{\varphi_x}^2\right)^2+4\beta_1\beta_2}\right]}$ $p_5=p_Z=2\sqrt{\frac{k_z}{m}} \qquad p_6=p_{\varphi_z}=2\sqrt{\frac{a^2k_y+b^2k_x}{J_z}}$
	where: – pulsations of the uncoupled vibrations are: $p_X=\sqrt{\frac{\sum k_{ix}}{m}}=2\sqrt{\frac{k_x}{m}}$ $p_Y=\sqrt{\frac{\sum k_{iy}}{m}}=2\sqrt{\frac{k_y}{m}} \qquad p_Z=\sqrt{\frac{\sum k_{iz}}{m}}=2\sqrt{\frac{k_z}{m}}$ $p_{\varphi_x}=\sqrt{\frac{\sum y_i^2k_{iz}+\sum z_i^2k_{iy}}{J_x}}=2\sqrt{\frac{b^2k_z+h^2k_y}{J_x}}$ $p_{\varphi_y}=\sqrt{\frac{\sum z_i^2k_{ix}+\sum x_i^2k_{iz}}{J_y}}=2\sqrt{\frac{h^2k_x+a^2k_z}{J_y}}$ $p_{\varphi_z}=\sqrt{\frac{\sum x_i^2k_{iy}+\sum y_i^2k_{ix}}{J_z}}=2\sqrt{\frac{a^2k_y+b^2k_x}{J_z}}$ – terms of coupling are $\alpha_1=\frac{1}{m}\sum k_{ix}z_i=-\frac{4}{m}hk_x \qquad \alpha_2=\frac{1}{J_y}\sum k_{ix}z_i=-\frac{4}{J_y}hk_x$ $\beta_1=\frac{1}{m}\sum k_{iy}z_i=-\frac{4}{m}hk_y \qquad \beta_2=\frac{1}{J_x}\sum k_{iy}z_i=-\frac{4}{J_x}hk_y$

3. ANALYSIS OF THE STATIONARY FORCED VIBRATIONS OF THE RIGID ELASTICALLY SUPPORTED

To analyze the dynamic parameters of the forced vibrations in the harmonic regime, the physical model shown in Figure 2 is considered. The support of the rigid is elastic, and the dynamic response is synchronous with the perturbation factors (forces, torques), in phase or phase opposition with them [14,15,16,17,18]..

Harmonic excitation by perturbation torque

a. Harmonic forced vibrations in pitch motion (gallops) coupled with the longitudinal motion (along the axis of propagation). The perturbation torque is given by the moment vector

$$\overline{M}=M_x\overline{i}=\left(M_{0x}\sin\omega t\right)\overline{i}, \qquad (42)$$

and the vector of the generalized perturbing forces is of the form:

$$\underline{\underline{f}} = \left[0,0,0,M_{0x}\sin\omega t,0,0\right]^{\mathrm{T}}, \qquad (43)$$

where ω is the pulsation of the perturbation torque.

Due to the decoupling in subsystems of the whole system, the perturbation excites only the subsystem $\left(Y,\varphi_x\right)$, so that the differential equations of forced vibrations are:

$$\begin{cases} m\ddot{Y} + 4k_y Y + 4hk_y\varphi_x = 0 \\ J_x\ddot{\varphi}_x + 4hk_y Y + 4\left(b^2k_z + h^2k_y\right)\varphi_x = M_{0x}\sin\omega t. \end{cases} \qquad (44)$$

The forced longitudinal vibrations and the pitch ones are expressed by the particular solution of the system (44). This solution is:

$$\begin{cases} Y_f\left(t\right) = A_Y\sin\omega t \\ \varphi_{xf}\left(t\right) = A_{\varphi_x}\sin\omega t. \end{cases} \qquad (45)$$

By substituting the second-order derivatives in system (44), we obtain an algebraic system with the amplitudes of the forced vibrations as solutions:

$$\begin{cases} A_Y = \dfrac{-4hk_yM_{0x}}{\left(4k_y - m\omega^2\right)\left[4\left(b^2k_z + h^2k_y\right) - J_x\omega^2\right] - 16h^2k_y^2} \\ A_{\varphi_x} = \dfrac{\left(4k_y - m\omega^2\right)M_{0x}}{\left(4k_y - m\omega^2\right)\left[4\left(b^2k_z + h^2k_y\right) - J_x\omega^2\right] - 16h^2k_y^2}. \end{cases} \qquad (46)$$

Considering the expression of eigen pulsations of uncoupled longitudinal and pitch vibrations, as well as the expressions of the coupling coefficients, the restricted expressions of the amplitudes are:

$$\begin{cases} A_Y = \dfrac{\beta_2M_{0x}}{m\left[\left(p_Y^2 - \omega^2\right)\left(p_{\varphi_x}^2 - \omega^2\right) - \beta_1\beta_2\right]} \\ A_{\varphi_x} = \dfrac{\left(p_Y^2 - \omega^2\right)M_{0x}}{mi_x^2\left[\left(p_Y^2 - \omega^2\right)\left(p_{\varphi_x}^2 - \omega^2\right) - \beta_1\beta_2\right]}. \end{cases} \qquad (47)$$

Depending on the eigenpulsations of the coupled vibrations, there are the following:

$$\begin{cases} A_Y = \dfrac{\beta_2M_{0x}}{m\left(p_3^2 - \omega^2\right)\left(p_4^2 - \omega^2\right)} \\ A_{\varphi_x} = \dfrac{\left(p_Y^2 - \omega^2\right)M_{0x}}{mi_x^2\left(p_3^2 - \omega^2\right)\left(p_4^2 - \omega^2\right)}, \end{cases} \qquad (48)$$

where p_3 and p_4 are the eigenpulsations of the subsystem $\left(Y,\varphi_x\right)$.

The calculation model, differential equations of motion, and expressions of the amplitudes of the forced vibrations of the rigid body with an axis of vertical symmetry under perturbation with a moment of harmonic pitch are listed in Table 2.

b. Harmonic forced vibrations with roll motion (swaying) coupled with lateral motion (sliding). The moment vector of the perturbation torque is [19,20,21]:

$$\bar{M} = M_y\bar{j} = \left(M_{0y}\sin\omega t\right)\bar{j}, \qquad (49)$$

and the vector of the generalized perturbing forces is of the form:

$$\underline{\underline{f}} = \left[0,0,0,0,M_{0y}\sin\omega t,0\right]^{\mathrm{T}}. \qquad (50)$$

4. CONCLUSIONS

The model of the rigid with elastic supports, with geometric and mass symmetries, can be adopted for any dynamic analysis at free and forced vibrations.

This study presents the analytical relations of the calculation in coupled and decoupled dynamic regimes, offering the possibility of engineering evaluations of the system capability. The research conducted served as the basis for evaluating the ELI-NP optoelectronic platform in Măgurele, Romania.

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