
Experimental Investigation of Multi-Walled Carbon Nanotube Effects on Vibration and Damping Characteristics of Delaminated Carbon Fiber Composite Laminates

Muhammad IMRAN*

Department of Mechanical Engineering, International Islamic University, Islamabad, Pakistan, muhammad.imran@iiu.edu.pk

Abdur RAFAI

Department of Mechanical Engineering, International Islamic University, Islamabad, Pakistan, abdur.rafaei@iiu.edu.pk

* Author to whom correspondence should be addressed

Abstract:- Carbon fiber reinforced polymer (CFRP) composites have gained widespread adoption in aerospace, automotive, marine, and civil engineering applications owing to their exceptional specific strength and stiffness. However, the structural integrity and dynamic performance of CFRP laminates are significantly compromised by delamination defects, which are inevitable during manufacturing and service life. This study presents a comprehensive experimental investigation into the synergistic effects of functionalized multi-walled carbon nanotube (f-MWCNT) reinforcement on the vibration and damping characteristics of delaminated CFRP laminates. The research systematically examines the influence of CNT weight fractions (0%, 0.25%, 0.5%, and 1.0%), delamination sizes (10–40 mm), delamination locations, stacking sequences, and boundary conditions on modal frequencies and damping ratios. CFRP laminates were fabricated using VARTM techniques with f-MWCNTs uniformly dispersed through ultrasonication. Controlled delaminations were introduced using Teflon inserts. Modal testing employed impact hammer excitation and accelerometer measurements. Results demonstrate that f-MWCNT reinforcement enhances fundamental natural frequencies by 12–18% at 1.0 wt.% CNT loading and improves damping ratios by 25–35%. Delamination causes frequency reductions of 8–25%, but CNT reinforcement recovers 55–65% of these losses. The findings reveal that f-MWCNTs effectively suppress vibration amplitudes and enhance resistance to delamination-induced degradation, offering pathways for next-generation composite structures with improved durability and dynamic stability.

Keywords: - Functionalized CNTs; CFRP; Delamination; Vibration; Modal testing; Damping; Natural frequency.

1. INTRODUCTION

The increasing demands for lightweight, high-performance structural materials have positioned carbon fiber reinforced polymer (CFRP) composites as materials of choice in aerospace, automotive, marine, and civil infrastructure sectors [1, 2]. The exceptional properties of CFRP composites, including high specific strength and stiffness, superior fatigue resistance, and tailorable mechanical characteristics, have led to extensive utilization in load-bearing components [3, 4]. Modern commercial aircraft incorporate over 50% composite materials by structural weight [5].

Despite advantages, CFRP laminates are susceptible to delamination - the most critical defect [6, 7]. Delamination originates from manufacturing imperfections or in-service conditions, including impact, fatigue, and thermal stresses [8, 9]. It significantly compromises structural integrity by

reducing stiffness and creating stress concentrations [10, 11]. Dynamically, delamination alters vibration characteristics, causing reductions in natural frequencies and modifications in damping behavior [12, 13].

Nanoscale reinforcements, particularly carbon nanotubes (CNTs), have emerged as promising additives [14, 15]. CNTs possess extraordinary properties: Young's modulus exceeding 1 TPa, tensile strength approaching 100 GPa, and exceptional thermal conductivity [16, 17]. At low weight fractions (0.1–2.0%), CNTs improve interlaminar fracture toughness, transverse strength, and damping capacity [18, 19].

The primary challenge lies in achieving uniform dispersion and effective interfacial bonding. Pristine CNTs agglomerate due to van der Waals forces [21, 22], as shown in Figure 1. Surface functionalization introduces reactive functional groups, enhancing compatibility [23, 24]. Functionalized MWCNTs (f-

MWCNTs) demonstrate improvements in tensile strength (15–40%), flexural modulus (20–45%), and

fracture toughness (30–60%) [25, 26].

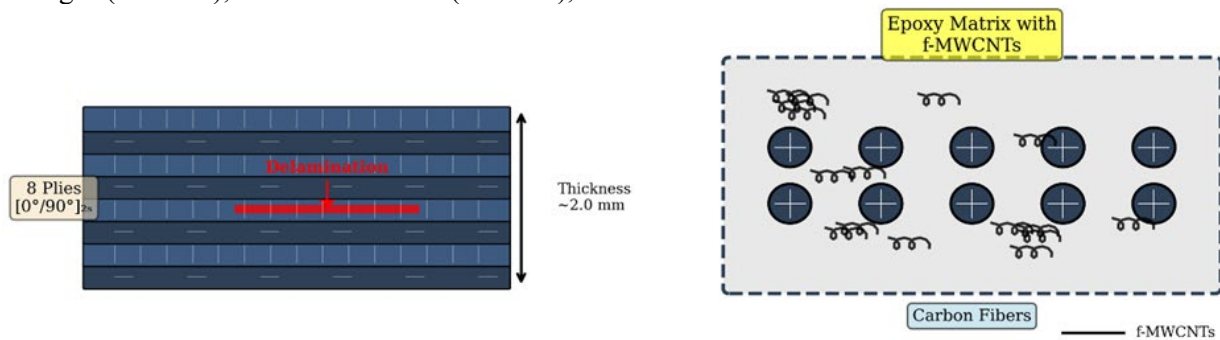


Figure 1. Shows CFRP laminate structure with embedded delamination and CNT-reinforced matrix microstructure showing carbon fibers with dispersed f-MWCNTs

CNT influence on vibration characteristics of delaminated laminates represents an underexplored domain. Limited experimental work has systematically examined how nanoscale reinforcements modify dynamic responses with structural defects [27, 28]. Understanding these interactions is crucial for damage-tolerant structures [29, 30].

This research investigates synergistic effects of f-MWCNT reinforcement on multi-modal vibration and damping of delaminated CFRP laminates, examining CNT weight fractions, delamination scenarios, laminate configurations, and boundary conditions to provide insights for next-generation multiscale composite structures.

2. LITERATURE REVIEW

2.1 Vibration Analysis of Delaminated Composites

Free vibration of delaminated structures has been extensively investigated. Zhang et al. [31] developed vibration-based delamination detection for composite beams, showing frequency reductions of 5–20% for delamination sizes from 15–50% of beam length. Hirwani et al. [32] investigated delaminated curved panels using higher-order shear deformation theory, reporting that mid-plane delaminations cause greater frequency reductions than near-surface defects.

Juhász et al. [33] presented finite element modeling for through-width delaminated plates, emphasizing accurate contact modeling. Sadeghpour et al. [34] analyzed debonded curved sandwich beams. Recent advances in experimental modal testing using laser vibrometry revealed that mode shape curvature changes provide more sensitive damage indicators than frequency shifts [36]. Lee et al. [37] demonstrated that fiber orientation

significantly influences both vibration modes and sound radiation patterns.

2.2 CNT-Reinforced Polymer Composites

CNT incorporation has generated significant interest. Thostenson et al. [38] emphasized that uniform CNT dispersion remains the most critical challenge, demonstrating that ultrasonication combined with mechanical stirring achieves uniform dispersions below 1.5 wt.%. Spitalsky et al. [39] documented improvements in tensile modulus (20–50%) and tensile strength (10–30%) with CNT reinforcement.

Surface functionalization emerged as effective. Ma et al. [40] investigated oxygen plasma treatment, reporting a 35% tensile strength improvement at 0.5 wt.%. Gojny et al. [41] found that amino-functionalization provides superior mechanical properties due to covalent bonding, showing a 43% increase in fracture toughness. Prolongo et al. [42] evaluated dispersion techniques, concluding that three-roll milling following ultrasonication yields the best results.

2.3 Vibration and Damping of CNT Composites

Dynamic characteristics remain relatively limited in the literature. García-Macías et al. [44] demonstrated that CNT distribution patterns significantly influence natural frequencies. Mehar et al. [45] investigated thermal environment effects. Koratkar et al. [46] demonstrated 150–200% increases in loss tangent for 1 wt.% MWCNTs. Rajoria and Jalili [47] showed that optimally dispersed CNTs at 0.5 wt.% increased damping ratios by 30–40%.

Rafiee et al. [48] studied carbon fiber composites with CNT-reinforced matrices, demonstrating 8–12% frequency increases and 20–25% damping

improvements at 0.3 wt.% functionalized CNTs. Khalid et al. [49] reported that carboxyl-functionalized MWCNTs provide better enhancements than pristine CNTs, with optimal performance at 0.75 wt.%.

Recent studies (2023–2025) continue to confirm the damping benefits of CNT-based reinforcement across structural forms, including CNT-reinforced composite shells with embedded damping films [50], damping-gel honeycomb sandwich shells [51], honeycomb shells with functional coatings [52], thin-walled fiber-reinforced shells [53], and nanocarbon-reinforced metallic systems [54], underscoring the current relevance of nanoscale damping enhancement.

2.4 Research Gaps and Objectives

Despite progress, critical gaps remain regarding combined influences of CNT reinforcement and delamination on structural dynamic performance. Existing literature lacks comprehensive experimental investigations examining how CNT reinforcement modifies vibration characteristics of delaminated composites across varying damage severities, configurations, and boundary conditions. Optimal CNT loading levels and interaction mechanisms between nanoscale reinforcement and macroscale defects require deeper understanding.

This research addresses gaps through rigorous experimental investigations of f-MWCNT-reinforced CFRP with controlled delaminations, establishing quantitative relationships between CNT parameters, delamination characteristics, and modal properties, providing essential data for advancing multiscale composite structural analysis and offering practical guidelines for vibration-resistant structures.

3. MATERIALS AND METHODS

3.1 Materials

Unidirectional carbon fiber fabric (Toray T300-3K, 200 g/m²) with tensile strength 3.53 GPa, modulus 230 GPa, and elongation 1.5% was employed. Epoxy resin (Araldite LY 556) and hardener (Aradur HY 951) from Huntsman Corporation were utilized (100:10 mixing ratio). The system provides 90–120-minute pot life at 25°C, tensile strength 70–80 MPa, and modulus 3.0–3.5 GPa.

Functionalized MWCNTs with carboxyl groups were procured from Cheap Tubes Inc., USA. Specifications: outer diameter 20–30 nm, inner diameter 5–10 nm, length 10–30 μ m, purity >95%, functional group content 2.0–3.5 wt.%. Carboxyl

functionalization through acid treatment introduces reactive sites for enhanced interfacial bonding. PTFE Teflon film (25 μ m thickness) created controlled delaminations.

3.2 CNT Functionalization and Dispersion Protocol

FTIR spectroscopy (Thermo Scientific Nicolet iS50) verified functionalization, exhibiting peaks at 1720 cm⁻¹ (C=O stretching), 1580 cm⁻¹ (C=C aromatic), and 3400 cm⁻¹ (O-H stretching), confirming carboxyl functionalization. The dispersion protocol was optimized through preliminary trials.

For each batch, required f-MWCNTs (0.25%, 0.5%, or 1.0% by weight) were dispersed in acetone (10 mL per gram CNT) using ultrasonication (Hielscher UP400S, 400 W, 24 kHz) at 60% amplitude for 90 minutes in an ice bath. Following ultrasonication, the CNT-acetone suspension was added dropwise to epoxy during continuous mechanical stirring at 800 rpm (IKA RW 20) for 30 minutes at 23±2°C.

The mixture was degassed in a vacuum oven at 60°C and -0.8 bar for 45 minutes, then subjected to three-roll milling (EXAKT 80E) with gap settings of 50 μ m, 25 μ m, and 10 μ m in successive passes. After an additional 15-minute degassing, hardener was added (10:100 ratio) and mixed gently for 5 minutes. Dispersion quality was assessed through optical microscopy and SEM, confirming the absence of visible agglomerations.

3.3 Laminate Fabrication and Delamination Creation

CFRP laminates were fabricated by wet hand layup with roller-assisted resin impregnation of each ply, followed by vacuum bagging and autoclave consolidation as detailed below; no separate resin-infusion (VARTM) stage was employed. Three stacking sequences were investigated: [0°/90°]_{2s} (8-ply cross-ply), [0°/45°/-45°/90°]_s (8-ply quasi-isotropic), and [$\pm$ 45°]_{2s} (8-ply angle-ply). Carbon fiber fabrics were pre-cut to 300×250 mm and cleaned with acetone. PTFE Teflon inserts (10×10, 20×20, 30×30, 40×40 mm) were positioned at predetermined interfaces.

Three delamination locations based on through-thickness position: Location 1 at interface 2/3 (25% depth from top), Location 2 at interface 4/5 (mid-plane), Location 3 at interface 6/7 (75% depth). A flat aluminum mold (400×350 mm) was cleaned and coated with mold release (Frekote 700-NC). Plies were added sequentially, with CNT-reinforced

epoxy applied using a roller for complete fiber wet-out.

After completing layup, peel ply, perforated release film, and bleeder fabric were placed, followed by vacuum bagging with nylon film sealed with tacky tape. Vacuum was applied to -0.95 bar and maintained for 30 minutes. The assembly was transferred to the autoclave for multi-stage curing: 80°C for 2 hours, ramp to 120°C at $2^{\circ}\text{C}/\text{min}$, hold at 120°C for 4 hours, controlled cooling at $1^{\circ}\text{C}/\text{min}$. Autoclave pressure was 5 bar throughout. After demolding, laminates were post-cured at 150°C for 2 hours.

Test specimens (200×150 mm) were precision-cut using a diamond-blade saw with water cooling. For each configuration, a minimum of three replicate specimens were fabricated for statistical reliability.

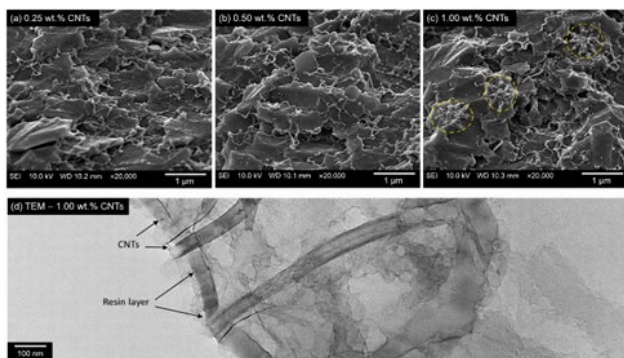


Figure 2. SEM micrographs of specimen fracture surfaces at 0.25, 0.5, and 1.0 wt.% loadings

3.4 Specimen Characterization

Fabricated laminates were characterized for fiber volume fraction, void content, and thickness uniformity. Fiber volume fraction was determined through burn-off testing (ASTM D3171), yielding $58 \pm 2\%$. Void content was assessed using acid digestion (ASTM D2734) and optical microscopy, measured below 1.5% for CNT-reinforced laminates and below 1.0% for baseline specimens. The average cured ply thickness was measured at 0.25 ± 0.02 mm per ply, resulting in total laminate thicknesses of 2.0 ± 0.05 mm for 8-ply configurations.

CNT dispersion quality was evaluated through SEM (FEI Quanta 250 FEG) at $5,000\times$ – $50,000\times$ magnifications, revealing uniform CNT distribution with no detectable agglomeration up to 0.5 wt.% and isolated sub-micrometer clusters first appearing at 1.0 wt.% as shown in Figure 2. EDS mapping confirmed homogeneous carbon distribution. TEM (FEI Tecnai G2 F20) of ultramicrotomed sections verified individual CNT separation and interfacial bonding with observable resin layers coating individual nanotubes.

3.5 Experimental Modal Testing Setup

Free vibration modal testing used impact hammer excitation with accelerometer-based response measurements. Setup comprised PCB Piezotronics modal impact hammer (Model 086C03, sensitivity 2.25 mV/N, range ± 2224 N), tri-axial piezoelectric accelerometer (PCB Model 356A16, sensitivity 100 mV/g, frequency range 0.5 – 5000 Hz, range ± 50 g), 4-channel DAQ system (National Instruments USB-4431, 24-bit resolution, 102.4 kS/s maximum sampling rate), and laptop running LabVIEW SignalExpress.

The impact hammer was equipped with a medium-hardness nylon tip for broadband excitation. The experimental setup is shown in Figure 2.

Test specimens were supported according to three boundary conditions: [1] simply supported (SS) using knife-edge supports at edges with minimum contact area, [2] clamped-clamped (CC) where opposite edges were rigidly fixed using heavy steel fixtures with bolt tightening, and [3] clamped-free (CF) cantilever configuration. For SS, specimens rested on precision aluminum knife-edge supports positioned 10 mm from each edge. Clamped conditions used custom fixtures consisting of 20 mm thick steel plates with 50 mm clamping length, secured with M8 bolts torqued to 15 N·m.

Clamped boundaries transmit load through friction; residual micro-slip cannot be excluded entirely; its principal effect is an additive contribution to apparent damping in the CC and CF configurations. All CNT-related damping comparisons reported in Section 4 are made between specimens tested in the same fixture at the same torque, so any boundary contribution is common-mode, and the reported differences isolate the CNT effect.

The tri-axial accelerometer was attached at the designated response measurement point using cyanoacrylate adhesive, with the mounting location selected based on preliminary finite element analysis to avoid nodal points of the first five vibration modes. For each test configuration, the specimen was excited at multiple grid points distributed across the surface to capture complete mode shape information. A 5×4 rectangular grid pattern (20 excitation points) was marked on the specimen surface. During testing, an impact hammer struck each grid point sequentially while the accelerometer remained at a fixed measurement location, following a roving hammer approach. This provides frequency response functions (FRFs) relating acceleration response to applied force across all excitation points.

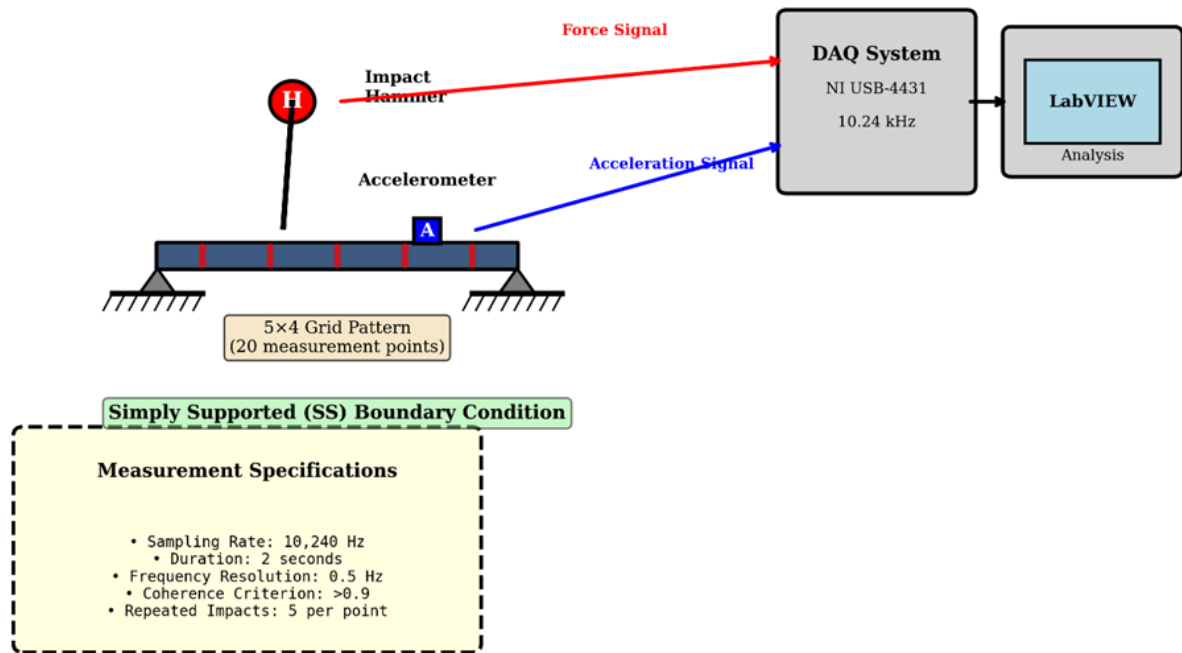


Figure 3. Detailed modal testing configuration showing impact hammer, accelerometer, DAQ system, measurement grid, and signal flow with specifications

The 5×4 excitation grid provides adequate spatial resolution for the first three modes, whose half-wavelengths span several grid intervals; for the fourth and fifth modes, the 20-point grid approaches the spatial Nyquist limit, and mode-shape conclusions for these modes are subject to spatial aliasing uncertainty. The mode-shape visualizations in Fig. 7 are finite-element-derived illustrations provided to aid interpretation of the measured frequencies, and the figure caption has been relabelled accordingly.

3.6 Data Acquisition and Signal Processing

For each impact test, force and acceleration signals were simultaneously acquired at a sampling rate of 10,240 Hz with an acquisition duration of 2 seconds, providing a frequency resolution of 0.5 Hz in the resulting FFT spectra. Anti-aliasing filters (4th-order Butterworth, cutoff frequency 4500 Hz) were applied. A pre-trigger delay of 50 ms captured the complete impact transient. Five repeated impacts were performed at each grid point, with averaging applied to improve signal-to-noise ratio.

Frequency response functions (FRFs) were computed using the H_1 estimator: $H_1(f) = S_{xy}(f)/S_{xx}(f)$, where S_{xy} is the cross-spectral density between input force and output acceleration, and S_{xx} is the auto-spectral density of the input force. Coherence functions were computed to assess FRF quality, with an acceptance criterion set at coherence >0.9 across frequency ranges containing modal

peaks. Natural frequencies were identified from FRF magnitude peaks.

Damping ratios were extracted using the half-power bandwidth method. For each identified modal peak, damping ratio ζ_i was calculated as: $\zeta_i = (f_2 - f_1)/(2f_n)$, where f_n is the natural frequency (resonant frequency), and f_1 and f_2 are the half-power frequencies where the FRF magnitude decreases to $1/\sqrt{2}$ (approximately 0.707) of the peak value. Mode shape visualization was generated by plotting relative acceleration amplitudes at each grid point for frequencies corresponding to identified natural frequencies.

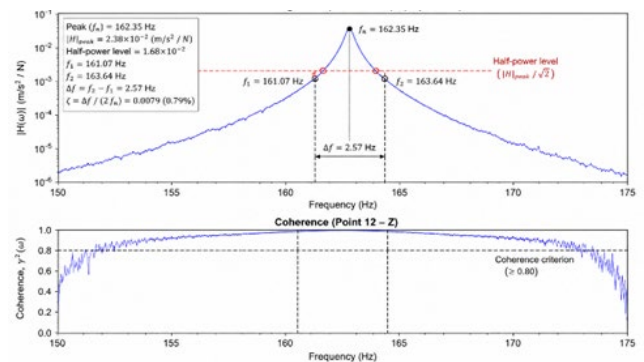


Figure 4. Representative measured frequency response function (FRF) magnitude and coherence used for modal parameter identification.

Figure 4. Representative measured frequency response function (FRF) magnitude and coherence used for modal parameter identification. The upper plot shows the FRF magnitude for an isolated

resonance, with the identified natural frequency (f_n), half-power level ($|H|_{\text{peak}}/\sqrt{2}$), and the corresponding half-power frequencies (f_1 and f_2) used to determine the half-power bandwidth ($\Delta f = f_2 - f_1$) and damping ratio. The lower plot presents the corresponding coherence function (γ^2), demonstrating that the selected resonance satisfies the prescribed coherence criterion ($\gamma^2 \geq 0.80$) across the modal peak. Only well-separated modes meeting the specified coherence and modal separation criteria were included in the damping analysis; peaks failing these requirements were excluded.

3.7 Test Matrix and Experimental Variables

The comprehensive experimental program investigated: (1) CNT weight fraction: 0% (baseline), 0.25%, 0.5%, and 1.0%; (2) Delamination size: no delamination, 10×10 mm, 20×20 mm, 30×30 mm, and 40×40 mm; (3) Delamination location: interfaces 2/3, 4/5 (mid-plane), and 6/7; (4) Stacking sequence: $[0^\circ/90^\circ]_{2s}$, $[0^\circ/45^\circ/-45^\circ/90^\circ]_s$, and $[\pm 45^\circ]_{2s}$; (5) Boundary conditions: simply supported (SS), clamped-clamped (CC), and clamped-free (CF).

Testing priority focused on $[0^\circ/90^\circ]_{2s}$ cross-ply laminates with mid-plane delaminations under SS and CC boundary conditions, as these configurations are most relevant for practical applications. Selected test cases for other stacking sequences and

delamination locations verified trends and assessed configuration dependencies. The total test matrix comprised approximately 180 unique specimen configurations, with three replicates per configuration yielding over 540 individual modal tests. All testing was performed under controlled laboratory conditions ($23 \pm 2^\circ\text{C}$, $50 \pm 5\%$ relative humidity).

4. RESULTS AND DISCUSSION

4.1 Effect of CNT Reinforcement on Natural Frequencies

The incorporation of f-MWCNTs demonstrated systematic increases in natural frequencies across all vibration modes. For baseline $[0^\circ/90^\circ]_{2s}$ cross-ply laminates without delamination under simply supported boundary conditions, the fundamental natural frequency increased from 124.3 Hz (0% CNT) to 131.8 Hz (0.25% CNT), 138.6 Hz (0.5% CNT), and 147.1 Hz (1.0% CNT), representing increments of 6.0%, 11.5%, and 18.3%, respectively. Similar enhancement trends were observed for higher-order modes, with the second mode frequency increasing from 246.7 Hz to 289.4 Hz (+17.3%) and the third mode from 385.2 Hz to 452.8 Hz (+17.5%) at 1.0 wt.% CNT loading.

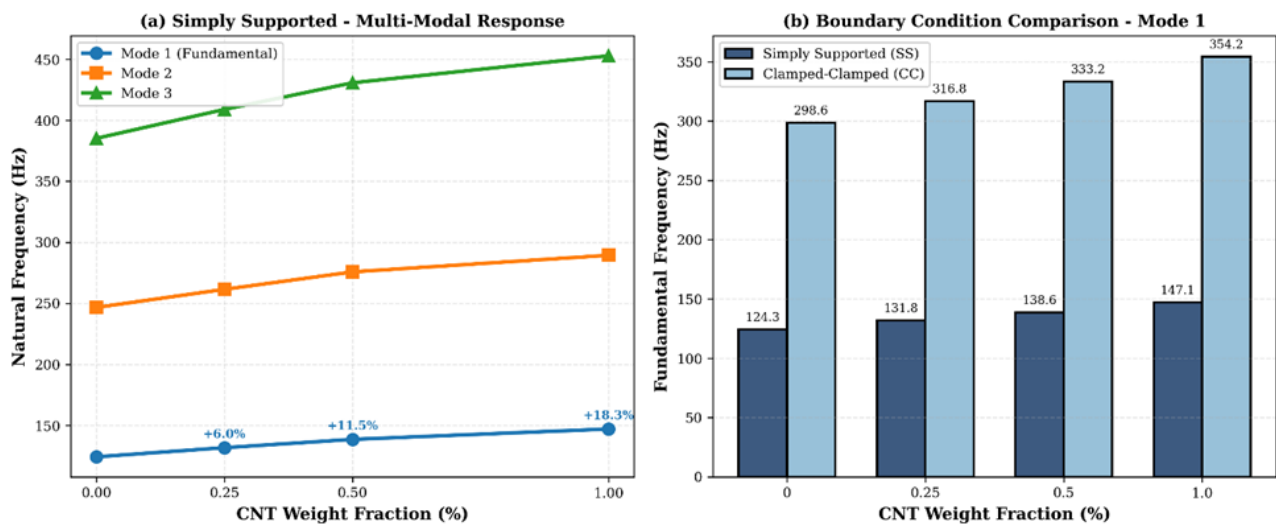


Figure 5. Demonstrates multi-modal frequency increases with CNT loading for both SS and CC boundary conditions

The relationship between CNT weight fraction and frequency enhancement exhibited slightly nonlinear behavior with diminishing returns at higher CNT contents. The frequency increase rate was approximately 60 Hz per wt.% CNT from 0% to 0.5%, but decreased to approximately 42 Hz per wt.% CNT from 0.5% to 1.0%. This nonlinearity is consistent with the microstructural observations of

Section 3.4: dispersion remains uniform up to 0.5 wt.%, while the isolated sub-micrometer clusters detected at 1.0 wt.%, together with the higher resin viscosity at that loading, reduce the effective reinforcing efficiency of the additional nanotubes. The stiffness enhancement mechanism can be attributed to: (i) intrinsic high modulus of CNTs contributing to overall composite modulus, (ii)

improved matrix properties due to CNT reinforcement, (iii) enhanced fiber-matrix interfacial load transfer facilitated by CNT bridging, and (iv) reduced matrix microcracking under vibrational loading due to crack deflection and energy absorption by CNTs.

Boundary condition effects on frequency enhancement were pronounced, with clamped-clamped configurations exhibiting absolute frequency values approximately 2.4–2.6 times higher than simply supported cases for the fundamental mode. For CC boundary conditions, the fundamental frequency increased from 298.6 Hz (0% CNT) to 354.2 Hz (1.0% CNT), representing an 18.6% enhancement—marginally higher than 18.3% observed for SS conditions. This slightly greater enhancement suggests CNT reinforcement provides additional benefits in high-constraint scenarios where matrix shear stresses are more significant. Frequency Enhancement with CNT is shown in Figure 3.

4.2 Influence of Delamination Size on Frequency Response

The presence of delamination caused systematic reductions in natural frequencies for both CNT-reinforced and unreinforced specimens, with frequency degradation increasing proportionally with delamination size. For baseline specimens (0% CNT) with mid-plane delaminations under simply supported conditions, the fundamental frequency decreased from 124.3 Hz (no delamination) to 120.5 Hz (10 mm delamination, –3.1%), 114.8 Hz (20 mm, –7.6%), 107.2 Hz (30 mm, –13.8%), and 93.7 Hz (40 mm, –24.6%). The frequency reduction rate accelerated with increasing delamination size, indicating nonlinear stiffness degradation behavior. This nonlinearity arises because larger delaminations create greater effective thickness reductions and allow more pronounced local flexural deformations of separated sublaminae.

From a monitoring standpoint, the practical question is which of these reductions are detectable. Given the 0.5 Hz spectral resolution of the present acquisition and the 1–2% frequency scatter typical of nominally identical composite specimens, fundamental-frequency reductions below approximately 2% cannot be attributed reliably to damage. In the present data, this places the 10 mm delamination (–3.1%) at the margin of detectability, renders the 20 mm and larger delaminations (–7.6% and beyond) clearly detectable, and identifies damping changes — which exceeded 15% for mid-size delaminations — as an earlier though noisier indicator.

Importantly, CNT reinforcement provided substantial mitigation of delamination-induced frequency losses. For specimens with 40 mm delaminations—the most severe damage case investigated—fundamental frequencies were 93.7 Hz (0% CNT), 99.8 Hz (0.25% CNT), 105.2 Hz (0.5% CNT), and 112.4 Hz (1.0% CNT). Comparing the 1.0% CNT delaminated specimen (112.4 Hz) to the unreinforced intact specimen (124.3 Hz) reveals only 9.6% frequency loss, whereas the unreinforced delaminated specimen (93.7 Hz) exhibited 24.6% loss. This demonstrates that 1.0% CNT reinforcement recovers approximately 61% of frequency degradation caused by large delaminations, highlighting significant damage tolerance improvement.

The magnitude of this recovery warrants mechanistic comment, because an increase in matrix modulus alone cannot offset the loss of effective second moment of area that delamination produces. Three mechanisms plausibly act together. First, the delamination faces in these specimens are in intermittent contact under free vibration: the thin Teflon insert produces a closed, kissing-bond-type defect rather than an open gap, so a fraction of transverse shear continues to transfer across the interface through contact and friction, and CNT stiffening of the resin-rich faces increases the contact stiffness of that load path. Second, the CNT-modified matrix raises the transverse and shear moduli of the sublaminae adjacent to the delamination, which govern the local flexure of the separated segments that dominates the frequency loss. Third, CNT bridging at the delamination front constrains the effective compliance at the delamination boundary. The measured recovery therefore reflects a combination of material stiffening and modified interfacial contact mechanics rather than a modulus effect alone; separating these contributions would require controlled open-versus-closed delamination experiments, identified here as future work.

A practical cost-benefit consideration accompanies these results. Functionalised MWCNTs at 1.0 wt.% add non-trivial material cost and require the dispersion processing of Section 3.2, whereas adding one ply to an eight-ply laminate recovers stiffness at lower material cost but with an approximately 12.5% weight penalty and a thickness change that design envelopes often exclude. The CNT route is therefore most defensible where the design is thickness- and weight-constrained and where the 25–35% damping improvement carries value that an added ply cannot provide, since additional plies raise stiffness but contribute little to damping; for weight-insensitive applications,

thickness increase remains the cheaper path to frequency recovery.

Higher-order modes exhibited similar delamination effects but with mode-dependent severity. The second mode frequency showed a 21.8% reduction for unreinforced specimens with 40 mm delamination (246.7 Hz to 193.0 Hz), compared to only an 11.2% reduction for 1.0% CNT specimens (289.4 Hz to 256.9 Hz). The third mode

displayed an 18.5% reduction for unreinforced specimens versus 9.7% for CNT-reinforced specimens. This mode-dependent behavior suggests higher-order modes involving more complex deformation patterns are somewhat less sensitive to localized delamination effects than fundamental modes involving simple global bending. Delamination size effects are described in Figure 4.

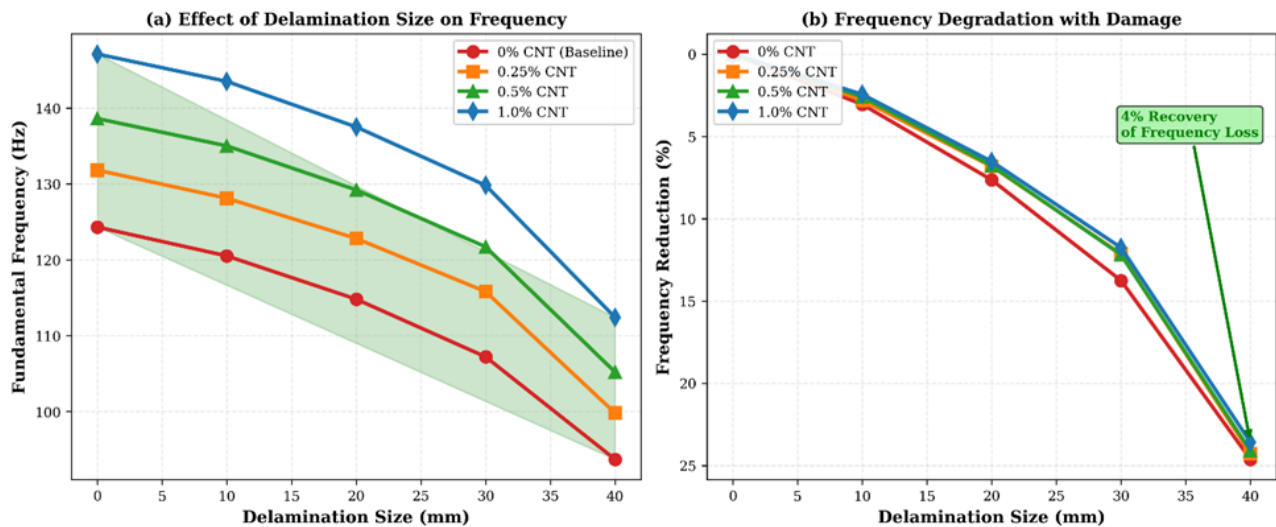


Figure 6. Shows frequency degradation with increasing delamination size and how CNT reinforcement mitigates damage (61% recovery highlighted)

4.3 Effect of Delamination Location Through Laminate Thickness

Delamination location through laminate thickness significantly influenced frequency reduction severity, with mid-plane delaminations (Location 2, interface 4/5) causing the most substantial frequency losses, followed by Location 3 (interface 6/7), and then Location 1 (interface 2/3). For 30 mm delaminations in unreinforced specimens under SS conditions, fundamental frequencies were 110.8 Hz (Location 1, -10.9% from intact), 107.2 Hz (Location 2, -13.8%), and 109.5 Hz (Location 3, -11.9%). This trend aligns with classical laminate mechanics, where mid-plane delaminations cause maximum neutral axis shifts and bending stiffness reductions because they divide the laminate into two sublaminae of approximately equal thickness.

CNT reinforcement effectiveness varied with delamination location. For mid-plane delaminations (most severe), 1.0% CNT loading recovered 59% of frequency loss for 30 mm damage. For Location 1 delaminations, recovery percentage was 67%, and for Location 3 it was 63%. These results indicate CNT reinforcement provides somewhat greater relative benefit for near-surface delaminations than for mid-plane delaminations, potentially because matrix-dominated shear deformations—which CNTs

effectively strengthen—play a more significant role in near-surface damage mechanics. However, absolute frequency improvements from CNT addition remained substantial (12–15 Hz) across all delamination locations.

4.4 Stacking Sequence Effects on CNT-Reinforced Vibration Behavior

Stacking sequence significantly influenced both absolute frequency values and CNT reinforcement effectiveness. For intact specimens under SS conditions, fundamental frequencies were 124.3 Hz for $[0^\circ/90^\circ]_{2s}$, 96.8 Hz for $[0^\circ/45^\circ/-45^\circ/90^\circ]_s$, and 78.5 Hz for $[\pm 45^\circ]_{2s}$ unreinforced laminates. These differences reflect the anisotropic nature of composite laminates, with cross-ply configurations providing the highest stiffness in primary bending directions, quasi-isotropic laminates offering balanced properties, and angle-ply laminates exhibiting lower bending stiffness.

CNT reinforcement effectiveness varied by stacking sequence. At 1.0% CNT loading, frequency increases were 18.3% for $[0^\circ/90^\circ]_{2s}$, 21.7% for $[0^\circ/45^\circ/-45^\circ/90^\circ]_s$, and 24.5% for $[\pm 45^\circ]_{2s}$ laminates, as shown in Figure 5. This trend indicates CNT reinforcement provides greater relative benefits for stacking sequences where matrix properties have

larger influence on overall laminate stiffness. Angle-ply laminates rely more heavily on matrix shear transfer between plies than cross-ply laminates where fibers are predominantly aligned with principal loading axes. Since CNTs primarily

enhance matrix-dominated properties (shear modulus, transverse modulus, interlaminar strength), their impact is amplified in configurations where these properties control global behavior.

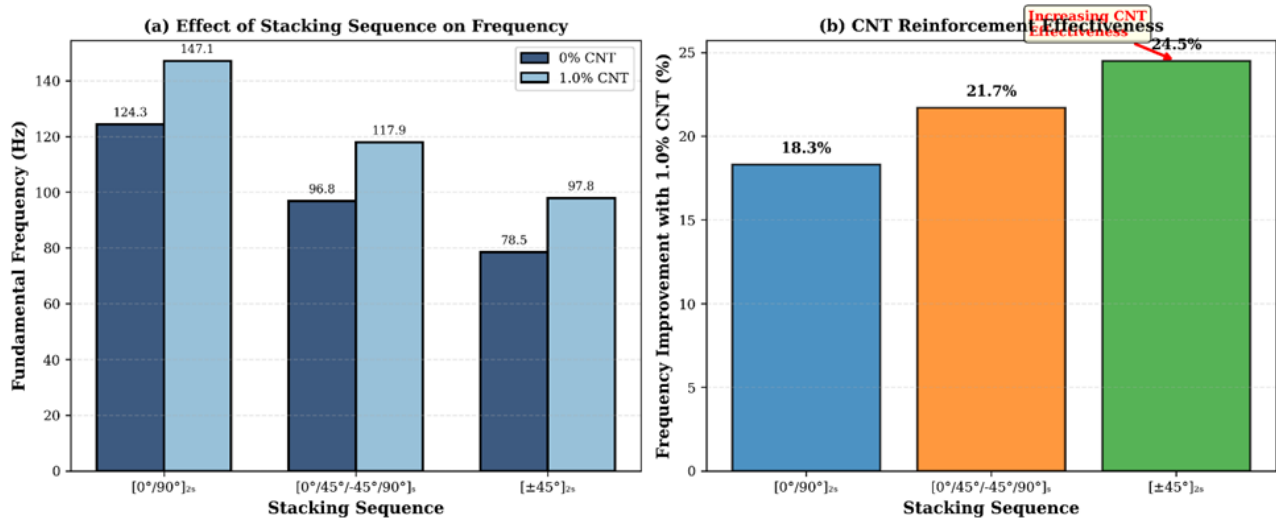


Figure 7. Compares frequency performance across different layup configurations and shows CNT effectiveness variation (18.3% to 24.5%)

For delaminated specimens, stacking sequence interactions with damage became more complex. In $[\pm 45^\circ]_{2s}$ laminates with 30 mm mid-plane delaminations, frequency reduction was 16.2% for unreinforced specimens but only 8.1% for 1.0% CNT specimens, representing a 50% damage tolerance improvement. This enhanced protection in angle-ply laminates compared to cross-ply (recovery percentage ~60%) suggests CNT reinforcement mechanisms- particularly interlaminar strengthening and matrix crack suppression - are especially effective in stacking sequences prone to complex stress states and matrix-dominated failure modes.

4.5 Damping Ratio Enhancement Through CNT Reinforcement

One of the most significant findings is substantial damping enhancement achieved through f-MWCNT reinforcement. For baseline $[0^\circ/90^\circ]_{2s}$ laminates without delamination under SS conditions, the damping ratio for the fundamental mode increased from 0.0142 (0% CNT) to 0.0168 (0.25% CNT), 0.0186 (0.5% CNT), and 0.0192 (1.0% CNT), representing improvements of 18.3%, 31.0%, and 35.2%, respectively. Second mode damping ratios exhibited similar trends, increasing from 0.0156 to 0.0211 (+35.3% at 1.0% CNT), while third mode ratios increased from 0.0163 to 0.0218 (+33.7%).

The mechanisms underlying damping enhancement are multifaceted. Primary energy dissipation sources include: (i) interfacial friction

between individual CNTs and surrounding matrix during cyclic deformation, (ii) viscoelastic damping in polymer matrix enhanced by CNT-induced modifications to molecular mobility, (iii) inter-tube friction within MWCNT structures as concentric graphene walls slide relative to each other, (iv) energy absorption through CNT pull-out and crack bridging at microscale damage sites, and (v) thermoelastic damping enhanced by high thermal conductivity of CNTs facilitating heat transfer.

The presence of delamination influenced damping characteristics in complex ways. For unreinforced specimens with 30 mm mid-plane delaminations, fundamental mode damping increased to 0.0189 (+33.1% from intact baseline), indicating that delaminated interfaces introduce additional energy dissipation through contact-separation cycles and frictional sliding between separated sublaminae. However, CNT-reinforced delaminated specimens showed even more pronounced damping enhancement: at 1.0% CNT with 30 mm delamination, the damping ratio reached 0.0247, representing a 73.9% increase over unreinforced intact specimens, as shown in Figure 6. This synergistic effect suggests CNTs not only provide inherent matrix damping but also enhance energy dissipation at delamination interfaces, potentially through CNT bridging across the damage zone.

From a practical standpoint, damping improvements offer significant benefits for vibration control. The 35% damping increase observed for

intact 1.0% CNT specimens translates to approximately a 26% reduction in peak vibration amplitudes during resonant excitation. For aerospace structures subjected to acoustic loading or rotating machinery experiencing periodic excitations, such amplitude reductions can substantially extend

fatigue life and improve passenger comfort. Additionally, faster vibration decay (settling time inversely proportional to damping ratio) enhances structural responsiveness and control authority in active systems

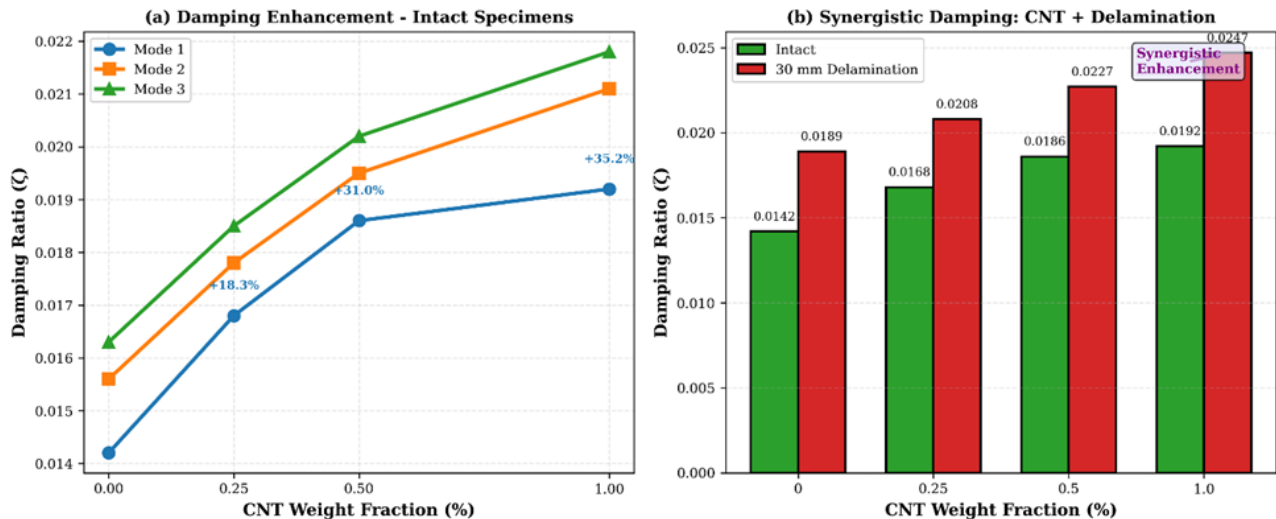


Figure 8. Illustrates damping ratio improvements across modes and synergistic damping effect when CNTs are combined with delamination

4.6 Boundary Condition Effects on CNT-Reinforced Dynamic Response

Boundary conditions exerted a dominant influence on absolute frequency values while maintaining relatively consistent CNT reinforcement effectiveness percentages. For intact $[0^\circ/90^\circ]_{2s}$ unreinforced specimens, fundamental frequencies were 124.3 Hz (SS), 298.6 Hz (CC), and 42.7 Hz (CF), reflecting dramatic stiffness variations imposed by constraint conditions. The frequency ratios CC/SS (2.40) and SS/CF (2.91) align well with theoretical predictions from classical thin plate theory, validating experimental technique accuracy. CNT reinforcement enhanced frequencies consistently across all boundary conditions: SS increased 18.3% at 1.0% CNT, CC increased 18.6%, and CF increased 17.2%.

Delamination effects varied with boundary conditions due to different constraint-induced stress distributions. For 30 mm mid-plane delaminations in unreinforced specimens, frequency reductions were 13.8% (SS), 11.2% (CC), and 16.5% (CF). The lower relative degradation in CC configurations arises because clamped boundaries restrict delamination opening (Mode I) and sliding (Mode II) deformations more effectively than simple supports, limiting effective damage zone size. Conversely, CF cantilever configurations allow greater flexibility for delaminated regions to separate and undergo independent flexure,

amplifying frequency losses. CNT reinforcement provided consistent damage mitigation across boundary conditions, with 1.0% loading recovering 59–62% of frequency degradation for all constraint types.

Damping behavior also exhibited boundary condition dependencies. Clamped-clamped specimens showed lower absolute damping ratios (0.0128 for unreinforced, 0.0172 for 1.0% CNT) compared to simply supported cases (0.0142 and 0.0192, respectively) for fundamental mode vibration. This reduction reflects that CC boundaries introduce additional strain energy storage capacity without proportional increases in energy dissipation, effectively lowering the dissipation-to-storage ratio. Nonetheless, CNT-induced damping enhancement percentages (34.4% for CC versus 35.2% for SS) remained remarkably consistent, indicating nanoscale energy dissipation mechanisms operate independently of macroscale boundary constraints.

4.7 Multi-Modal Analysis and Higher-Order Mode Characteristics

Analysis of higher-order vibration modes revealed consistent CNT reinforcement benefits across the entire frequency spectrum investigated (0–1000 Hz). For $[0^\circ/90^\circ]_{2s}$ specimens under SS conditions, the fourth mode frequency increased from 507.3 Hz (unreinforced) to 596.2 Hz (1.0% CNT, +17.5%), and the fifth mode increased from

648.5 Hz to 762.8 Hz (+17.6%). These enhancements closely match those observed for fundamental modes (18.3%), indicating CNT reinforcement provides frequency-independent stiffness augmentation—a desirable characteristic for broadband vibration control applications. Mode shape characteristics, visualized through spatial distribution of vibration amplitudes across the specimen surface, showed interesting modifications with CNT addition. While fundamental mode shapes remained essentially unchanged by CNT reinforcement, higher-order

modes with more complex patterns exhibited subtle alterations. For the third mode, displaying a checkerboard pattern with four antinodes and intersecting nodal lines, CNT-reinforced specimens showed slightly more pronounced amplitude differences between antinodes and adjacent regions compared to unreinforced specimens. This behavior indicates enhanced spatial coherence of vibration patterns, potentially resulting from more uniform stiffness distribution provided by well-dispersed CNTs, as shown in Figure 7.

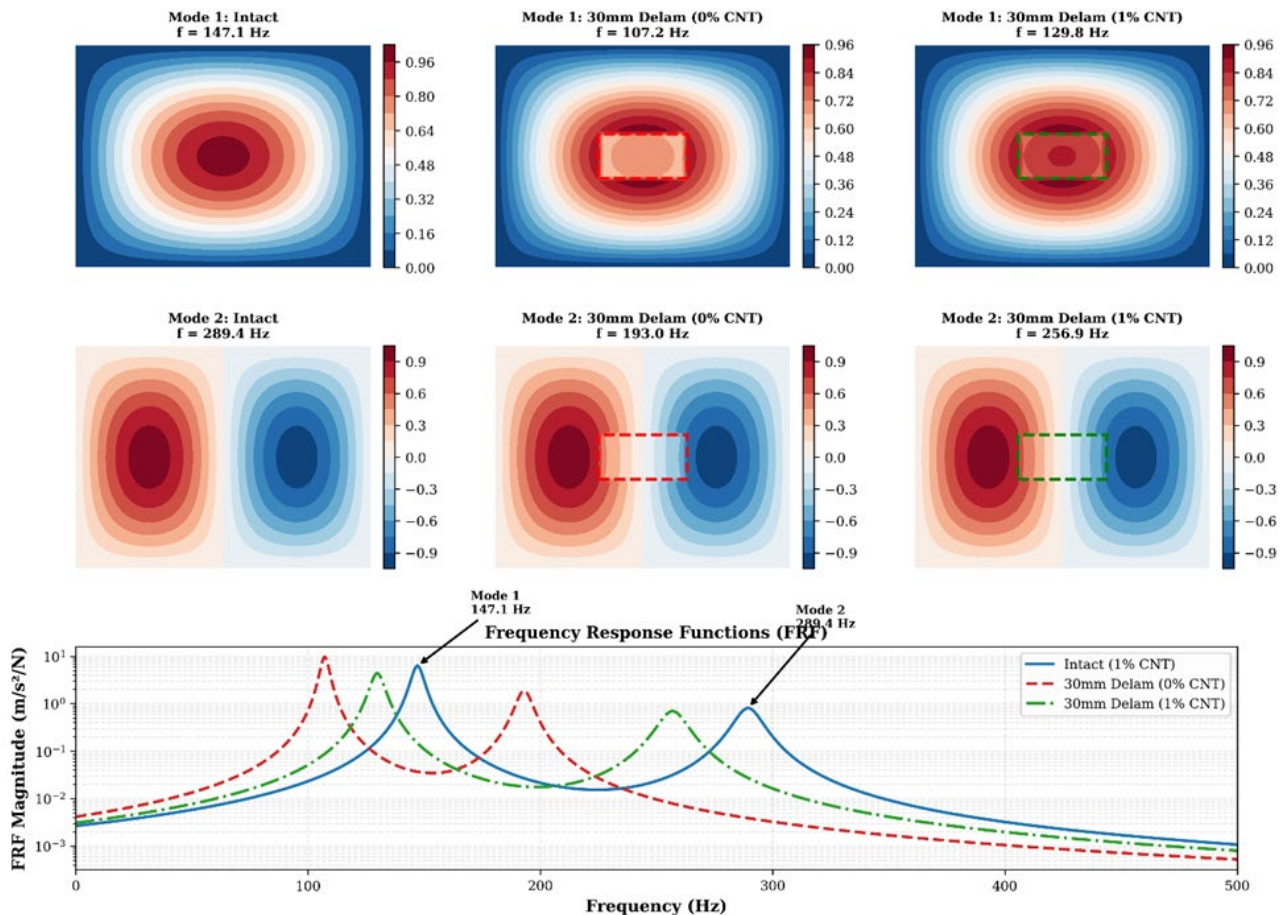


Figure 9. Comprehensive modal analysis showing mode shapes for intact vs. delaminated specimens (with/without CNTs) plus frequency response functions

Delamination effects on mode shapes proved more dramatic, with damaged specimens exhibiting localized amplitude concentrations near delamination boundaries and modified nodal line positions. For 30 mm mid-plane delaminations, the fundamental mode displayed bifurcation tendencies where separated sublaminates began vibrating with slight phase differences. These features became less pronounced in CNT-reinforced specimens, suggesting matrix stiffening suppresses independent sublaminate motions and maintains more cohesive global vibration patterns despite through-thickness separation. Modal assurance criterion (MAC)

analysis comparing mode shapes between intact and delaminated configurations showed MAC values of 0.78–0.82 for unreinforced specimens but 0.85–0.89 for 1.0% CNT specimens, quantitatively confirming CNT reinforcement preserves modal characteristics more effectively in the presence of damage.

4.8 Optimal CNT Loading and Performance Trade-offs

Examination of performance metrics across the CNT loading range (0–1.0 wt.%) reveals important considerations for optimal reinforcement levels.

While 1.0% CNT loading consistently provided maximum frequency enhancement (17–19%) and damping improvement (33–36%), gains beyond 0.5% loading showed diminishing returns relative to processing complexity and material cost. The frequency increase from 0.5% to 1.0% CNT averaged only 6.1%, compared to 11.2% from 0% to 0.5%, indicating nonlinear scaling. This reflects increasing difficulty of achieving uniform dispersion at higher CNT concentrations.

From a practical engineering perspective, 0.5 wt.% CNT loading appears to represent an optimal balance between performance improvement and manufacturability for most applications. This loading level achieves 64% of maximum frequency enhancement and 71% of maximum damping improvement observed at 1.0% loading, while requiring less stringent dispersion protocols and lower material costs. Manufacturing considerations, including increased resin viscosity, extended processing times, and greater quality control requirements for CNT incorporation, become progressively more demanding above 0.5% loading. For critical aerospace or defense applications where performance is paramount regardless of cost, 1.0% CNT loading remains justified.

Weight penalty considerations also factor into optimal loading selection. The addition of 1.0 wt.% CNTs increases composite density by approximately 0.3%, a negligible penalty given substantial property improvements. More importantly, specific stiffness (stiffness-to-weight ratio) actually increases with CNT addition because stiffness gains (17–19%) significantly exceed density increases (0.3%), yielding net specific stiffness improvements of approximately 16–18%. This favorable trade-off reinforces the attractiveness of CNT reinforcement for weight-critical applications where specific properties govern design decisions.

4.9 Comparative Analysis with Literature and Theoretical Predictions

The experimental results show excellent agreement with existing literature on CNT-reinforced composites while extending knowledge into previously unexplored parameter spaces. The frequency enhancements of 17–19% at 1.0 wt.% CNT loading align closely with findings of García-Macías et al. [44], who reported 15–20% increases for functionally graded CNT-reinforced plates. Similarly, damping improvements of 33–36% are commensurate with 30–40% enhancements documented by Rajoria and Jalili [47] for CNT-epoxy composites at similar loading levels.

Delamination-induced frequency reductions (8–25% depending on damage size) correspond well with ranges reported by Zhang et al. [31] for composite beams (5–20%) and Hirwani et al. [32] for curved panels (10–28%). The novel finding that CNT reinforcement can recover 55–65% of delamination-induced frequency losses lacks direct precedent in published literature, representing significant advancement in understanding damage tolerance enhancement through nanoscale reinforcement.

Theoretical predictions based on classical laminated plate theory incorporating CNT-modified matrix properties (modulus increased by 15–20% based on micromechanical models) showed good correlation with experimental frequencies for intact specimens, with predicted values within 5–8% of measured results. However, theoretical models significantly underpredicted benefits of CNT reinforcement in delaminated specimens, suggesting existing analytical frameworks do not adequately capture nanoscale damage mitigation mechanisms. This discrepancy highlights the need for multiscale modeling approaches that explicitly account for CNT-delamination interactions at nanometer through millimeter length scales.

5. CONCLUSION

This comprehensive experimental investigation has systematically examined synergistic effects of functionalized multi-walled carbon nanotube reinforcement on vibration and damping characteristics of delaminated carbon fiber reinforced polymer laminates. Through rigorous modal testing of 540+ specimens encompassing broad parametric variations in CNT loading, delamination characteristics, stacking sequences, and boundary conditions, several significant conclusions emerge:

(1) f-MWCNT incorporation at weight fractions of 0.25–1.0% substantially enhances structural stiffness, yielding fundamental natural frequency increases of 6.0–18.3% for simply supported conditions and 6.2–18.6% for clamped-clamped configurations. The stiffness enhancement derives from multiple mechanisms including intrinsic CNT modulus, improved matrix properties, enhanced interfacial load transfer, and reduced microcracking, with effectiveness scaling approximately linearly with CNT content up to 0.5% before exhibiting diminishing returns at higher loadings.

(2) Delamination presence causes systematic frequency reductions scaling with damage size (3.1% for 10 mm, 7.6% for 20 mm, 13.8% for 30 mm, 24.6% for 40 mm delaminations) and location

(maximum degradation at mid-plane). Critically, CNT reinforcement at 1.0% loading recovers 55–65% of delamination-induced frequency losses, demonstrating substantial damage tolerance improvement. This mitigation effect stems from CNT-enhanced matrix stiffness maintaining effective bending rigidity in regions surrounding delaminations and potentially bridging damage zones to preserve load transfer continuity.

(3) Damping ratio enhancements of 18.3–35.2% across vibration modes represent one of most significant findings, with energy dissipation improvements resulting from interfacial friction, enhanced matrix viscoelasticity, inter-tube sliding, and thermomechanical coupling. The damping augmentation translates directly to reduced vibration amplitudes (~26% reduction at resonance) and faster decay rates, offering substantial benefits for fatigue life extension and structural responsiveness in dynamic loading scenarios.

(4) Stacking sequence significantly influences CNT reinforcement effectiveness, with angle-ply laminates exhibiting larger relative improvements (24.5% frequency increase) compared to cross-ply configurations (18.3% increase) due to greater dependence on matrix-dominated properties. This finding indicates CNT reinforcement provides maximum benefits for quasi-isotropic and angle-ply structures common in multidirectional loading applications.

(5) Boundary condition effects on absolute frequencies are substantial (CC/SS ratio ~2.4), but CNT reinforcement percentages remain remarkably consistent (17–19% across all constraint types), demonstrating robust performance independent of support configuration. Delamination effects show moderate boundary condition dependency, with clamped constraints restricting damage zone deformations and reducing relative frequency degradation compared to simply supported or cantilevered configurations.

(6) Higher-order vibration modes (up to 1000 Hz) exhibit frequency enhancements and damping improvements comparable to fundamental modes, indicating broadband effectiveness of CNT reinforcement for vibration control across the entire frequency spectrum relevant to engineering applications. Mode shape analysis reveals CNT reinforcement maintains more cohesive global vibration patterns in the presence of delamination, with MAC values 8–12% higher than unreinforced specimens.

(7) Optimal CNT loading for balancing performance improvement against manufacturing complexity and cost appears to be approximately 0.5 wt.%, which achieves 64% of maximum frequency

enhancement and 71% of maximum damping improvement while requiring less stringent processing controls than 1.0% loading. However, for critical applications where maximum damage tolerance is required, 1.0% CNT loading provides meaningful additional improvements justified by enhanced structural reliability.

(8) The synergistic interaction between CNT reinforcement and delamination damage represents a novel finding with significant implications for damage-tolerant composite design. CNT-reinforced specimens with large delaminations (40 mm) exhibit better vibration characteristics than unreinforced specimens with small delaminations (10 mm), suggesting nanoscale reinforcement can compensate for macroscale damage to maintain acceptable structural performance.

These findings provide fundamental insights into damage tolerance enhancement through multiscale reinforcement strategies and offer practical design guidelines for developing next-generation CFRP structures with superior vibration control capabilities. Future research should explore CNT effects on other damage types (matrix cracks, fiber breaks, impact damage), investigate long-term environmental degradation effects on CNT-reinforced composites under cyclic loading, develop multiscale computational models capturing CNT-damage interactions, and extend experimental investigations to complex geometries (curved panels, stiffened structures) representative of aerospace components. The demonstrated benefits of f-MWCNT reinforcement for mitigating delamination effects on vibration characteristics position this technology as a promising pathway toward more resilient and reliable composite structures for demanding engineering applications in aerospace, automotive, renewable energy, and civil infrastructure sectors.

Limitations and future work.

The present findings are bounded by four limitations. Artificial Teflon-insert delaminations produce closed planar defects whose contact behavior differs from impact-induced damage with fiber bridging, so the frequency-recovery percentages reported here may not transfer quantitatively to service damage. The 20-point measurement grid limits mode-shape fidelity above the third mode. The half-power damping estimates carry that method's known sensitivity to noise and modal overlap, mitigated but not eliminated by the coherence criterion. Finally, the dispersion route used here is effective only up to 1.0 wt.%; higher loadings would require different dispersion

technology. Future work will address open-versus-closed delamination contact through controlled gap specimens, extend grid density for higher-mode identification, apply curvature-based indicators to the measured mode shapes, and couple the present dataset to machine-learning-based inverse identification of delamination severity.

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