
Selection of the Mathematical Parametric Model of a Device or Process Based on Experimental Data

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Abstract: - To study the response of a dynamic system to various inputs, it is necessary to establish a mathematical model of the system. Such a model can be developed either from the design of the respective dynamic system (a method that may introduce significant deviations from real behavior) or from experimental results obtained during the operation of the system or its components under laboratory and/or real operating conditions. This article presents a methodology for selecting an appropriate mathematical model based on performance indices calculated for phenomenological models using the same inputs as the modeled dynamic systems or processes. The method is illustrated using two parametric mathematical models (phenomenological models) of a magnetorheological (MR) fluid damper.

Keywords: - magnetorheological fluids, performance indices, genetic algorithm, parametric model

1. INTRODUCTION

The characterization of Magnetorheological (MR) fluids represents a quintessential challenge in nonlinear system identification. Due to their rapid and reversible transition from a Newtonian fluid to a semi-solid state under a magnetic field, MR fluids exhibit complex hysteretic and non-symmetric behaviors that are difficult to capture through purely mechanistic laws [1-3].

Consequently, researchers often rely on phenomenological models—such as the Bingham, Bouc-Wen, or Sigmoid models—to describe these observed effects [3-6].

In order to exemplify the methodology for selecting parametric mathematical models based on experimental observations, this research employs data collected from testing a mono-tube shock absorber equipped with a gas accumulator. [3].

The system operated using the MRF-132 AD type magnetorheological fluid (MR), commercially developed by LORD Corporation [5, 6].

As part of the controllable fluids family, MR fluids exhibit the unique ability to change their flow characteristics in response to electromagnetic stimuli. This allows for real-time control of the damping forces by simply altering the intensity of the electric current that generates the magnetic field.

Over the past 30 years, controllable fluids have captured the attention of scientists due to their remarkable applications in developing dynamic behaviour control devices of a wide variety of mechanical or physiological systems: conveyor systems (couplers, microactuators, microvalves, muscular prostheses, etc.), braking systems, vibration damping systems, sealing systems, etc [7, 8].

Controllable fluids are bi-phase fluids including a mix of solid particles in an electrically and/or magnetically neutral carrier liquid.

Controllable fluids include ferrofluids [3, 9], electro-rheological (ER) fluids, and magnetorheological (MR) fluids [1- 3, 5, 7, 10–15].

Ferrofluids are colloidal suspensions of nanometer-sized particles (5–10 nm) of magnetite or manganese-zinc type ferrite [9]. Due to the dimensions of the particles within the ferrofluid, under the action of a non-uniform magnetic field, the internal pressure of the fluid is altered proportionally to the local intensity of the applied field (the colloidal suspension is attracted toward areas of maximum field intensity). Unlike ferrofluids, electro-rheological (ER) and magnetorheological (MR) fluids [1-3, 5, 7, 10–15] are dispersions of solid particles (polarizable for ER and magnetizable for MR) with dimensions of 3–5 μm — approximately 1000 times larger than ferrofluid particles — in electrically and/or magnetically neutral carrier liquids (generally mineral oil, silicone oil, or water). However, the most efficient ER fluids developed to date have a yield strength of only 3–3.6 kPa and do not tolerate impurities, which can be introduced during production or operation, as these impurities alter the insulating properties of the carrier liquid.

Recently developed MR fluids can operate within a broader temperature range (-40°C to 150°C for MR, compared to 10°C to 90°C for ER) without significantly changing their yield strength due to thermal fluctuations and are, in fact, unaffected by impurities. From a control perspective, MR fluids require an external magnetic field generated by a voltage of 5–15 V with a maximum current of 2 A [3].

For magnetorheological (MR) fluids [1-3, 5, 6, 15], when a magnetic field is applied, the dispersed particles form oriented chains parallel to the magnetic field lines (Figure 1).

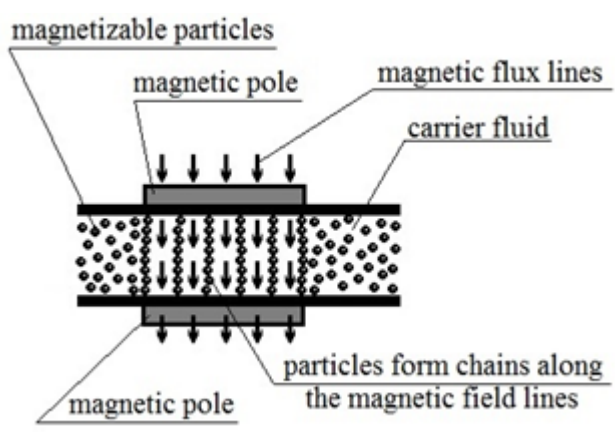


Figure 1. Formation of particle chains in controllable MR fluids upon applying a magnetic field

This structuring results in an increase in shear stress in a direction perpendicular to the magnetic field lines.

The force required to break these particle chains opposes the movement of the inertial

mass, thereby providing damping. This behavior makes magnetorheological (MR) fluids ideal for use in vibration dampers.

Under the application of a magnetic field, MR fluids respond within millisecond timescales. Consequently, prototype MR dampers have been developed that reach rheological equilibrium in less than 6.5 ms following field application. From this perspective, semi-active vibration control systems using MR dampers are superior to those utilizing variable-valve hydraulic dampers or controllable dry friction.

The efficient use of MR dampers in vibration control systems requires the prior determination of an analytical model to estimate their behavior under imposed conditions or to analyze how specific control rules apply to the dynamic systems equipped with these dampers. This model must be valid over a broad range of operational parameters—including piston displacement, velocity, and power dissipation—as well as for the continuous variation of the electric control current typical of technical operating conditions. Due to the complex interactions within a dynamic system and its environment, analytical models describing such behavior – regardless of the system's simplicity—serve as approximations across different levels of the system's operation.

2. ESTABLISHMENT OF THE PHENOMENOLOGICAL MODEL OF MR FLUID DAMPER

The development of a mathematical model [7, 15, 16] for a dynamic system can be achieved either based on the design of the respective system—a method that often introduces major deviations from reality—or based on experimental results obtained during its operation (under laboratory and/or field conditions).

The development of a model based on experimental results primarily consists of three phases [3, 15-17]:

- **Obtaining and processing** experimental data [5, 6, 16-19];
- **Selecting** phenomenological models that are compatible with the dynamic system [20-27];
- **Identifying** the specific model (from those selected above) that best describes the behavior of the dynamic system based on the experimental data [3, 16–27].

The first phase involves the establishment of an experimental program [19, 23] dedicated to obtaining data necessary for the identification of the dynamic system. The program has to contain those dynamic tests of the analyzed system based on

which a maximum amount of information on the system behavior (work domain, response time, operation limits, etc.) could be obtained. While carrying out the tests included in the program, both input signals (excitation) and output signals (system response) are recorded simultaneously. These recordings form the experimental data set based on which the mathematical model of the dynamic system is developed.

If dynamic tests cannot be performed experimentally, data records of input and output signals obtained during the operation of the dynamic system under field conditions may be utilized. The second phase relies on employing established mathematical structures (initial, known models) to describe the dynamic system [4-6, 21-25], albeit with unknown parameters and characteristics. These unknowns are determined using specialized algorithms that process experimental results obtained during testing or operational use. This phase is critical and often the most challenging, as it depends on the intuition, experience, and engineering judgment of the designer to select, combine, or construct models with unknown physical parameters based on physical laws and predetermined relationships. In certain cases, primary standard models (e.g., elastic, viscous friction or dry friction elements) can be used without direct reference to the physical context.

The third phase involves determining the unknown parameters of the selected model [26–28]. The objective is to ensure that, for the input signals recorded in the first phase, the model response reproduces the actual system response as faithfully as possible. This process of establishing an analytical model based on experimental data is part of the field known as system identification.

In the case of parametric models, the identification process (adapting the MR damper model to experimental data) involves the following steps [3]:

- **Parameter Estimation:** The values of the unknown model parameters are established based on experimental data for various constant control currents, while maintaining a consistent periodic relative displacement of the damper fixing points.
- **Functional Correlation:** The laws of variation for these parameters—as a function of the control electric current—are established based on the data obtained in the previous step.
- **Model Validation:** The resulting model is then validated using independent experimental data sets—distinct from those used for initial identification. These validation sets

correspond to constant periodic displacements under varying electric control current profiles.

During the first phase of the identification process, the numerical values of the unknown model parameters are determined. This estimation is treated as an optimization problem, where the objective function aims to minimize the discrepancy between the measured damping force and the force calculated by the model. This typically involves a multi-parameter optimization to achieve the best fit.

To demonstrate the development of a phenomenological parametric model for an MR fluid shock absorber, the present study utilizes the experimental data originally obtained by Sireteanu T. and Stancioiu D.

Figure 2 presents the experimental setup [3] used to record the damping force (output) as a function of time, considering various types and values of displacement and velocity (inputs), under different levels and profiles of the control current.

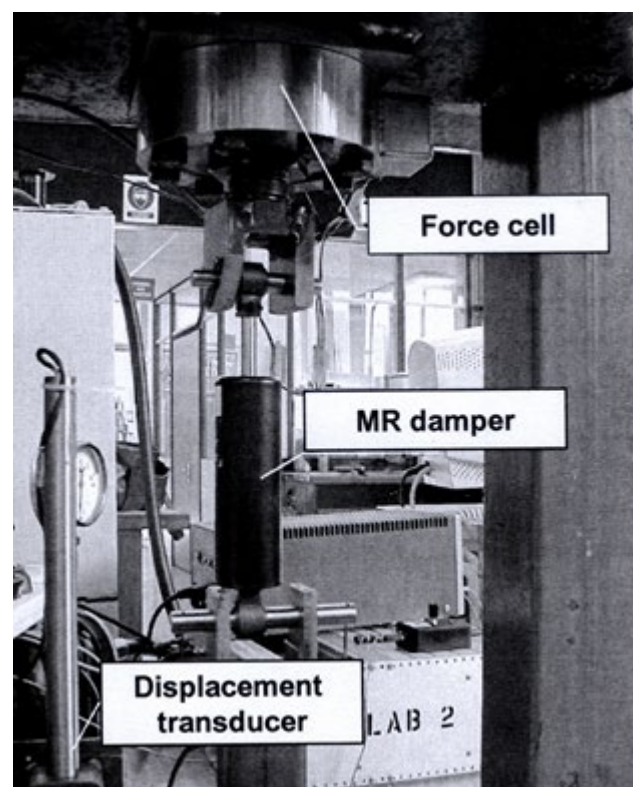


Figure 2 Experimental setup used for assessing the parametric model of the tested MR damper

A critical requirement for the test stand is the ability to acquire and record the displacement and velocity signals (input signals) and the force signal (output signal) without introducing additional phase shifts.

The sampling frequency used was 100 Hz per channel.

The characteristics of the tested monotube MR fluid damper [2] are presented in Table 1

Table 1. The characteristics of the shock absorber

Compressed/expanded limit length	155 mm/208 mm
Outer diameter of the cylinder	41.4 mm
Rod diameter	10 mm
Electrical characteristics	
Electrical supply current	2 A max.
Electrical supply voltage	12 V DC
Response time	The time it takes to reach 90% of maximum force at a step-type control current variation from 0 to 1 A and a piston speed of 51 mm/s
Damping force (peak-to-peak)	
At a piston speed of 58 mm/s at 1 A	> 2224 N
At a piston speed of 200 mm/s at 0 A	> 667 N
Electromagnetic circuit resistance measured at 29 °C	5.2 Ω
Maximum working temperature	71 °C

For the aforementioned MR fluid, the carrier liquid is synthetic hydrocarbon-based oil (incompatible with natural or hydrocarbon-based rubbers) [1, 5, 11].

The fluid characteristics are presented in Table 2.

Table 2 MR fluid characteristics

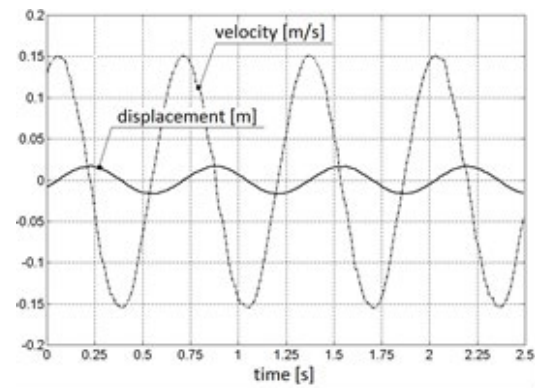
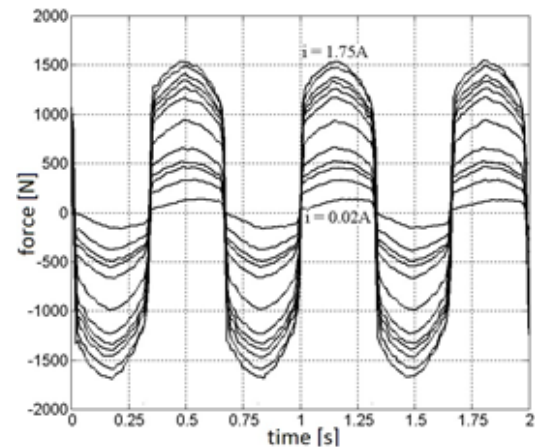
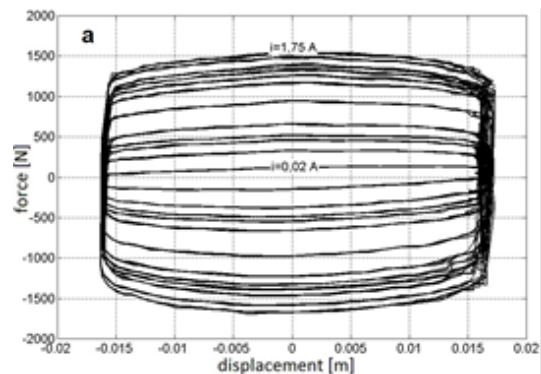
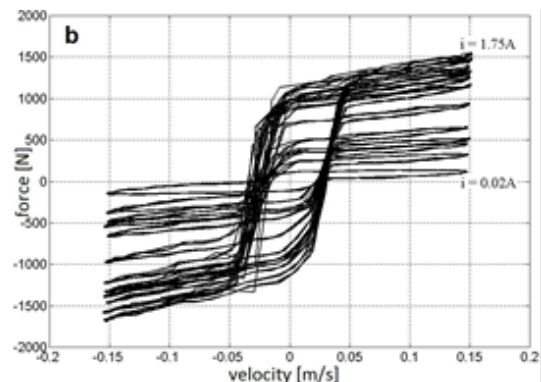
MRF density [gr/cc]	3.09 (bulk density)
Solid particles density [gr/cc]	7.87 (Fe particles)
Volume percent solid [V%]	32.1 (81.5% wt)
Shear viscosity [Pa×s]	0.23 at 600 [1/s] 0.19 at 1090 [1/s]

For the identification of the model parameters, the experimental results obtained for a sinusoidal relative displacement with a frequency of $f = 1.5$ Hz and an amplitude of $X = 16$ mm (Figure 3) were used as input data.

These results correspond to the following constant values of the control current intensity: $I = 0.02$ A, 0.1 A, 0.2 A, 0.4 A, 0.6 A, 0.8 A, 1.05 A, 1.45 A and 1.75 A.

Figure 4 illustrates the damping force response to the excitation shown in Figure 3 under the aforementioned constant control current values:

Figures 5a and 5b show the functional characteristics of the MR shock absorber

**Figure 3.** The input signal (excitation) - relative displacement with $f = 1.5$ Hz and $X = 16$ mm**Figure 4.** The experimental force response for excitation presented in Figure 3**Figure 5a.** Force vs. displacement diagrams (work characteristics)**Figure 5b.** Force vs. velocity diagrams (hydraulic characteristics)

These characteristics were obtained experimentally using the input signal shown in Figure 3, the constant control current intensities listed above, and the corresponding experimental force response (Figure 4). To validate the identified model, a test load with periodic excitation was applied, consisting of a triangular relative displacement of the shock absorber fixing points with a frequency of $f = 1.0$ Hz and an amplitude of $X = 16$ mm (Figure 6a).

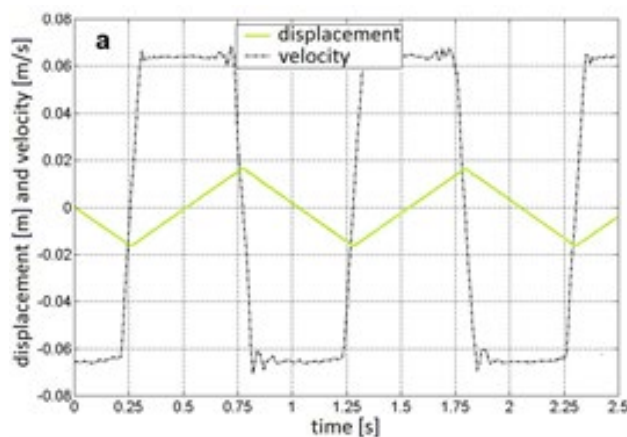


Figure 6a Periodic excitation with relative triangular-type displacement of the damper fixing points

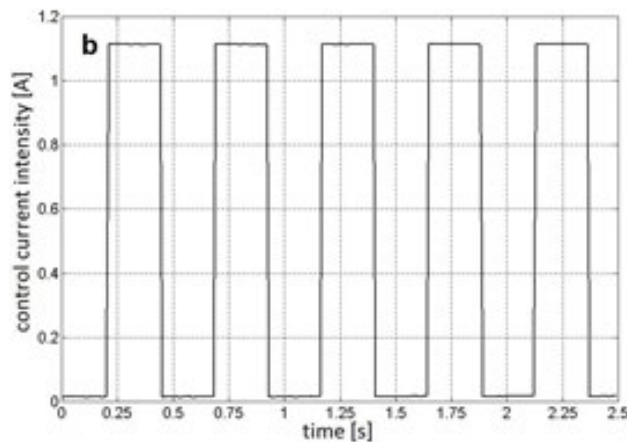


Figure 6b. Control current variation

The intensity of the electric control current varies cyclically, following a square wave (rectangular) profile with a frequency of $f = 2.0$ Hz and an amplitude of $X = 1.12$ A (Figure 6b).

The damping force obtained experimentally under periodic triangular excitation (Figure 6a) and a square-wave (rectangular) control current profile (Figure 6b) is shown in Figure 7.

The functional characteristics of the MR damper, derived from the experimental test inputs (Figures 6a and 6b) and the recorded output force (Figure 7), are illustrated in Figures 8a and 8b:

The phenomenological models used to validate the methodology were originally presented by Butz T. and von Stryk O [24].

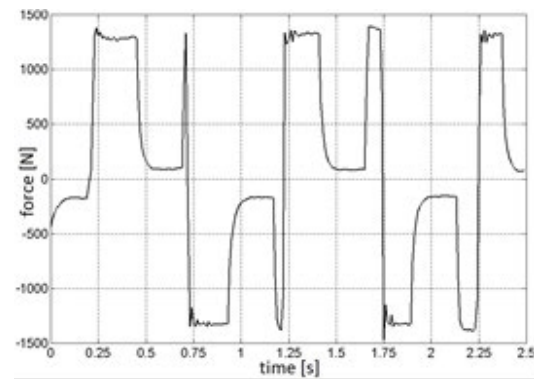


Figure 7. Experimental force of the test input

Given the inherent nonlinearities and the large number of parameters associated with these phenomenological descriptions, traditional gradient-based optimization methods often struggle with local minima.

To overcome these limitations, this research integrates Genetic Algorithms (GA) as a robust global search strategy. By leveraging evolutionary principles such as selection and crossover, GA facilitates the identification of the optimal parametric set that minimizes the error between the experimental data of the MRF-132AD fluid and the simulated response of the chosen mathematical model.

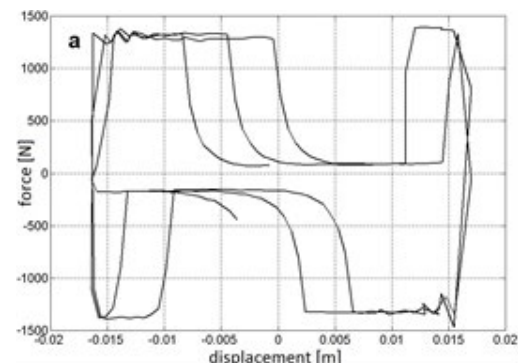


Figure 8a. Work characteristic

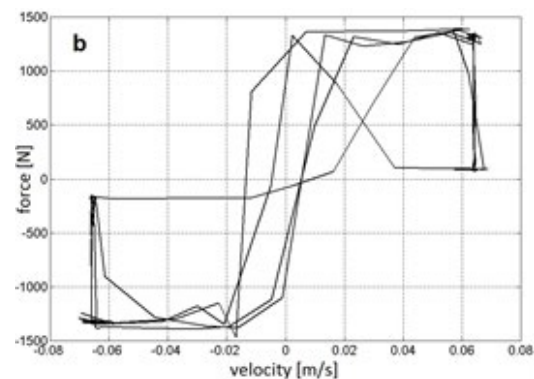


Figure 8b. Hydraulic characteristic

Consequently, GA was selected as a highly efficient method for solving 'black box' optimization problems involving multiple parameters. The

foundation of this approach lies in the mechanisms of genetics and natural selection [29-32]. In this framework, each parameter to be optimized represents a gene, while the vector formed by the complete set of parameters is termed a chromosome.

The number of genes within a chromosome varies depending on the specific parametric model, corresponding directly to the number of parameters requiring optimization. The GA process begins with an initial group of chromosomes, known as population zero, consisting of N_{pop} individuals. Each gene in this initial population is randomly generated (using a white noise distribution) within its specific predefined range, forming the starting matrix of the algorithm.

$$M_{GA} = [N_{pop}, N_{gen}] \quad (1)$$

In the above relation, N_{gen} is the number of genes.

The process begins with an initial (zero) population of candidate solutions that evolves over successive generations to obtain increasingly optimal results through various selection mechanisms.

Since the parameters under optimization are continuous variables, a floating-point representation was employed, specifically utilizing a Real-Coded Genetic Algorithm (RCGA).

In this study, a population of 60 individuals (chromosomes) was maintained over a maximum of 300 generations using an elitist strategy [3]. This approach ensures that high-performing individuals have a greater probability of being selected as parents for the next generation, preventing the loss of the best solutions found so far [29, 30].

The core components of the implemented genetic algorithm include:

- **Selection:** This phase utilized the roulette wheel method combined with an elitist strategy to choose individuals for reproduction based on their fitness scores.
- **Crossover (Recombination):** This operator combines genetic material from selected parents to produce offspring, introducing new candidate solutions into the population. The primary objective of crossover is to broaden the search domain and facilitate progress by exploring new regions of the state space.

The specific crossover operator employed is the Type 2.2, which generates two descendants from two parents to populate the subsequent generation.

The crossover probability was 0.7.

- **Mutation:** This operator involves changing the value of a single gene within a chromosome (individual), thereby introducing new genetic material that could not be produced through other mechanisms. Mutation is essential for maintaining and

enhancing population diversity. The implemented operator is a non-uniform mutation (generation-dependent), which utilizes a non-uniform probability distribution to select the parent for mutation.

The mutation rate was set to 0.05 for both models.

This study evaluates two rheological models for the MR fluid damper, ranging from the simplest to the most complex.

To evaluate the performance of each chromosome, a fitness function (fitness index) was defined.

The optimization objective is to minimize the cost function (objective function). The MSE (Mean Squared Error) function, defined according to Equation (2), was chosen as the objective function [36].

$$MSE = \frac{1}{N} \sum_{i=1}^N (F^{exp} - F^{sim})^2 \quad (2)$$

$N = 250$ is the total number of elements in the vector.

As the algorithm convergence rate was not predetermined, the optimization was terminated once the maximum number of generations was reached.

The objective function (2) was used to calculate the fitness index for every individual (chromosome) in the population. The fitness index is defined as: $fit = MSE^{-1}$

2.1. The Modified Bingham Model

According to this model (Figure 9), in the form proposed by Stanway et al. in 1985 [8], the damping force acting on the piston is defined such that the displacement occurs at the velocity $\dot{x}$:

$$F = f_c \text{sign } \dot{x} + c_0 \cdot \dot{x} \quad (3)$$

By convention, $F > 0$, $\dot{x} > 0$ is considered at the expansion stroke. For the Modified Bingham model, the chromosome used in the genetic algorithm consists of two real genes.

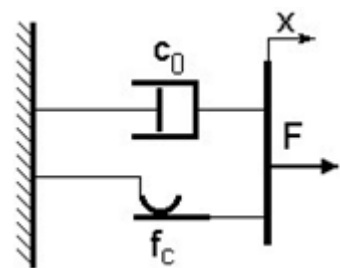


Figure 9. Modified Bingham Model

These genes represent the control-current-dependent variable parameters: $f_c(i)$ and $c_0(i)$ (the Coulomb friction force and the viscous friction coefficient).

The chromosome is:

$$Cr^{Bingham} = [f_c \ c_0] \quad (4)$$

For both genes, the zero population selection intervals were:

$$I^{Bingham} = (0.00, 5000.00]$$

Figures 10 and 11 present the variation of the parameters f_c and c_0 as a function of the control current, as determined using the genetic algorithm.

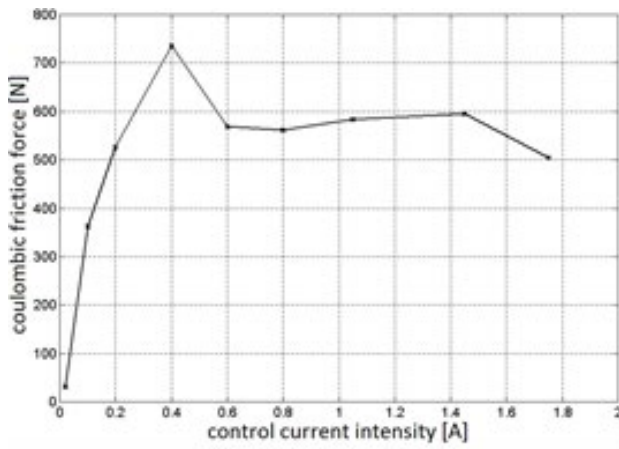


Figure 10. Coulombic friction force f_0 vs. i

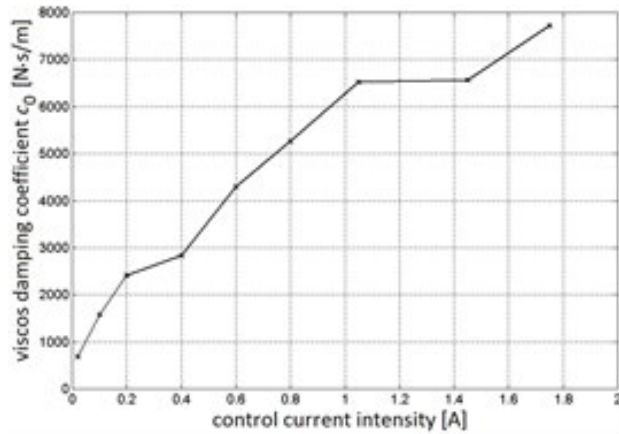


Figure 11. Viscous damping coefficient c_0 vs. i

Based on the global objective function (damping force F – Equation 3), applying the test load yields the model force response and its mechanical characteristics (F vs. t , F vs. d , and F vs. v), as shown in Figures 12, 13, and 14.

For comparison, the red lines represent the experimental force and experimental characteristics previously shown in Figures 6 and 7.

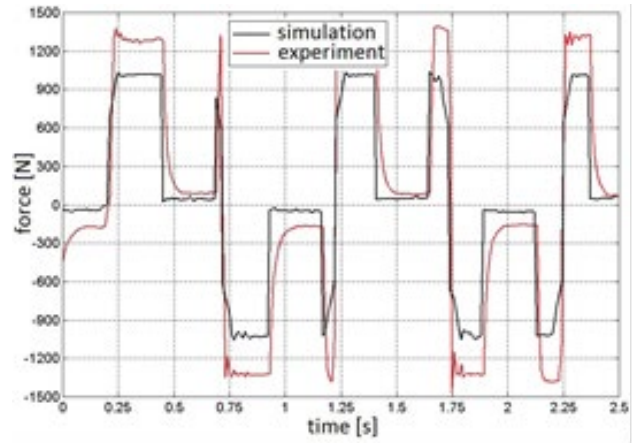


Figure 12. Force vs. time for the test load

Despite its computational efficiency and simplicity, the Modified Bingham model (Stanway model) exhibits several significant structural deficiencies in characterizing magnetorheological (MR) dampers.

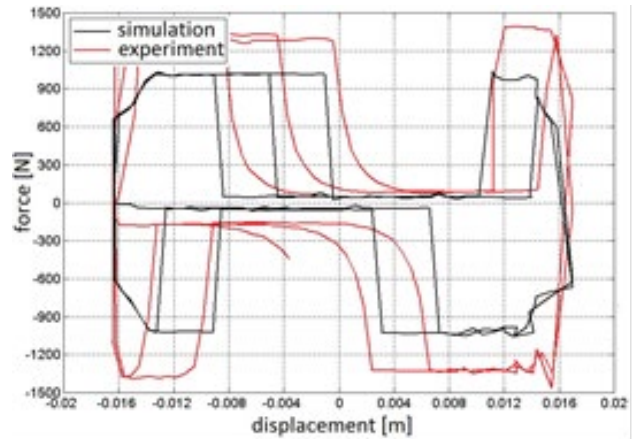


Figure 13. Work characteristic (F vs. d) for the test load

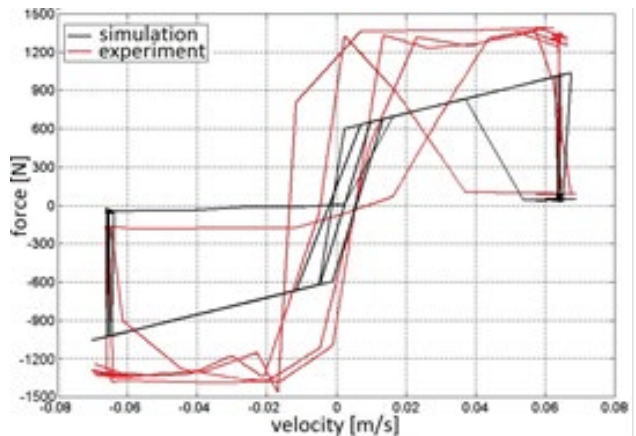


Figure 14. Hydraulic characteristic (F vs. v) for the test load

The three most critical among these are:

- due to its reliance on constant viscosity, the model is unable to capture the damper pre- and post-yield behavior, missing the critical transition

between solid and fluid phases, as shown in Figure 14 [5, 6, 8, 25, 26];

- as shown in Figure 12, the model is unable to account for MR fluid elasticity during rapid transitions of the control current, leading to a discontinuous response in the simulated force; the basic modified Bingham model lacks saturation awareness due to its linear assumption of Coulomb force (f_0). Defining this force (f_0) as a nonlinear function of the control current – via a GA (Figure 10) – eliminates this deficiency.

2.2. The Modified Bouc – Wen Model

To improve the simulation of the MR damper response at the yield point (yield initiation), Spencer et al. proposed a Modified Bouc–Wen model in 1997 [3, 21, 22, 24–28, 34]. Subsequently, in 2010, Sireteanu et al. proposed an extension of this Modified Bouc–Wen model [33]. Figure 15 presents the phenomenological mechanical structure of the proposed model.

The damping force F produced by the model for an x displacement is:

$$F = \alpha \cdot z + c_0 \cdot (\dot{x} - \dot{y}) + k_0 \cdot (x - y) + k_1 \cdot (x - x_0) \quad (5)$$

$$= c_1 \cdot \dot{y} + k_1 \cdot (x - x_0)$$

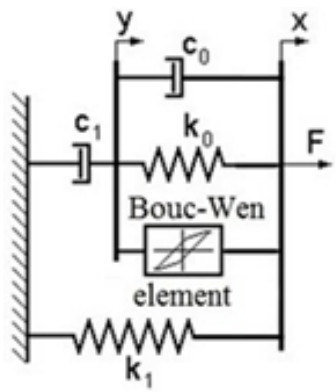


Figure 15. Modified Bouc – Wen Model

where the hysteretic component (Bouc-Wen element – evolutionary variable) is:

$$\dot{z} = -\gamma \cdot |\dot{x} - \dot{y}| \cdot z \cdot |z|^{n-1} - \beta \cdot (\dot{x} - \dot{y}) \cdot |z|^n + \delta \cdot (\dot{x} - \dot{y}) \quad (6)$$

and:

$$\dot{y} = \frac{1}{c_0 + c_1} \cdot [\alpha \cdot z + c_0 \cdot \dot{x} + k_0 \cdot (x - y)] \quad (7)$$

In the above relations:

- β, γ, δ – parameters controlling the linearity during unloading and the smoothness of the transition from the pre-yield to the post-yield region;

- α – parameter representing the stiffness component associated with the evolutionary variable z ;
- k_0 – stiffness parameter at high velocities;
- k_1 – stiffness parameter associated with the gas accumulator;
- c_0 – viscous damping coefficient;
- c_1 – damping coefficient of the dashpot included to produce the roll-off effect at low velocities;
- x_0 – initial displacement of the spring with stiffness k_0 .

The parameter identification and testing of the Modified Bouc–Wen model were carried out under the hypothesis of zero initial motion ($x_0 = 0$) [35].

For this model, the genetic algorithm used in the identification process contains 9 real genes represented by the model parameters:

$$\alpha, \beta, \gamma, \delta, c_0, k_0, c_1, k_1 \text{ and } n.$$

The chromosome is:

$$Cr^{Bouc-Wen} = [\alpha \ \beta \ \gamma \ \delta \ c_0 \ k_0 \ c_1 \ k_1 \ n] \quad (8)$$

For the genes within the model chromosome, selection intervals for population zero were set as follows:

- for the parameters $\alpha, \beta, \gamma, \delta, c_0, k_0$ and k_1 selection interval is:

$$I_1 = (0.00, 5000.00]$$

- for the parameter c_1 selection interval is:

$$I_2 = (0.00, 25000.00]$$

- for the parameter n selection interval is:

$$I_3 = (0.00, 10.00]$$

Figures 16 to 24 present the variation of the chromosome genes as a function of the control current, as determined using the genetic algorithm.

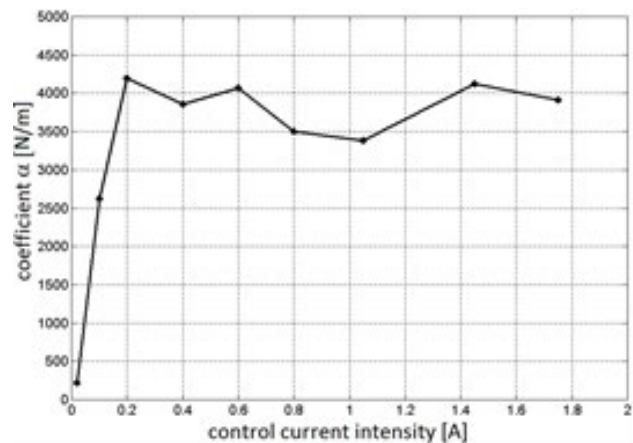


Figure 16. Parameter α vs. t

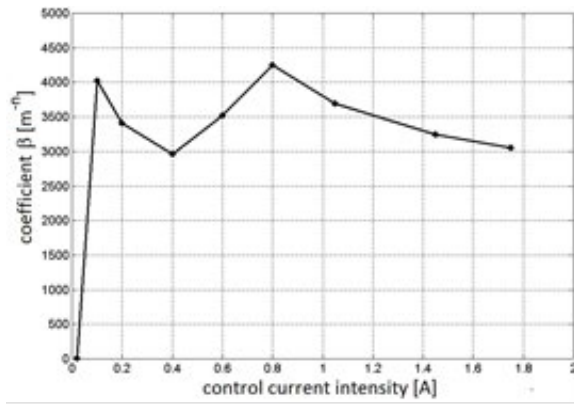


Figure 17. Parameter β vs. i

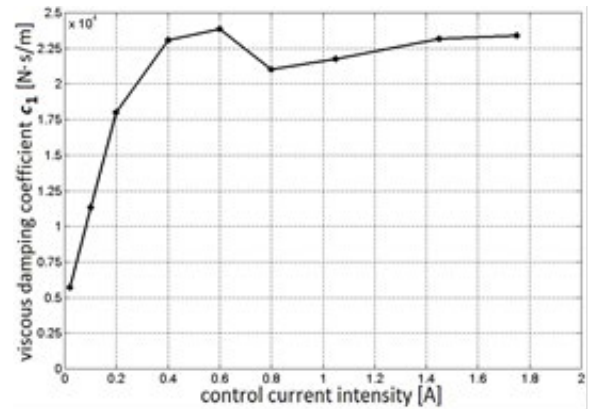


Figure 21. Viscous damping coefficient c_1 vs. i

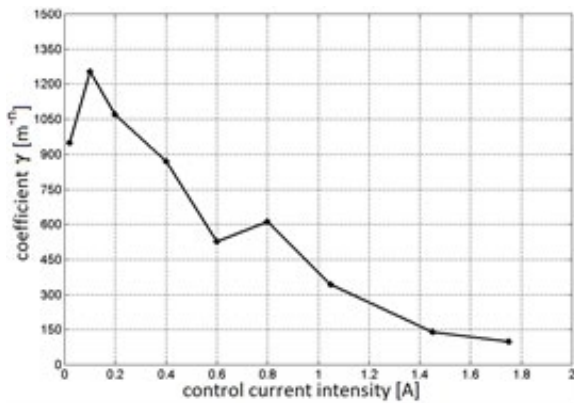


Figure 18. Parameter γ vs. i

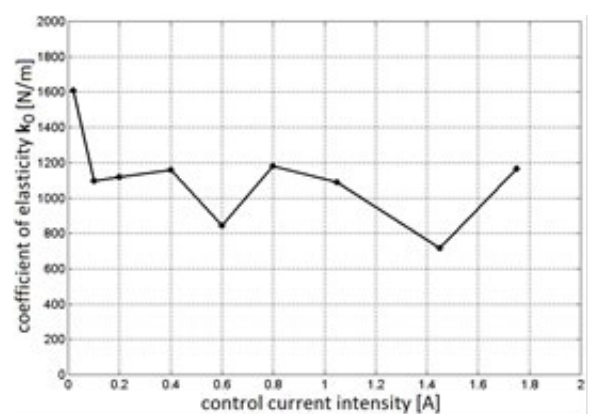


Figure 22. Coefficient of elasticity k_0 vs. i

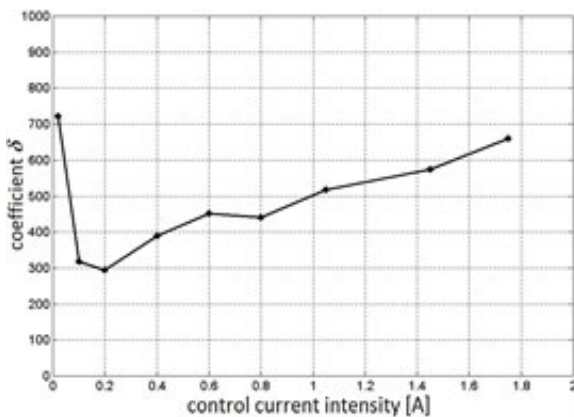


Figure 19. Parameter δ vs. i

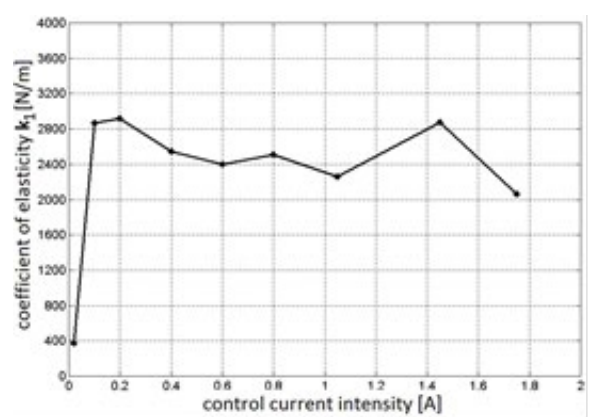


Figure 23. Coefficient of elasticity k_1 vs. i

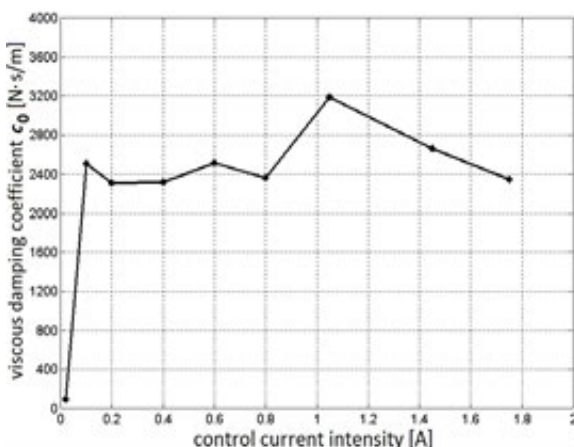


Figure 20. Viscous damping coefficient c_0 vs. i

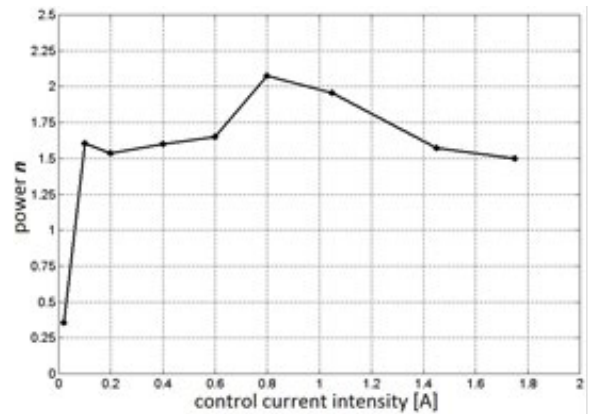


Figure 24. Power n vs. i

The model response to the test excitation is presented in Figures 25 to 27.

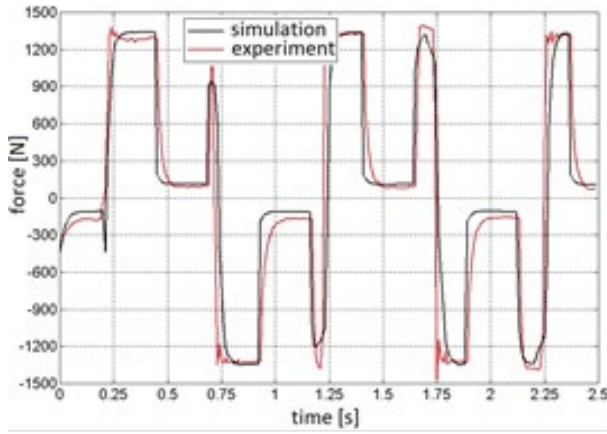


Figure 25 Force vs. time for the test load

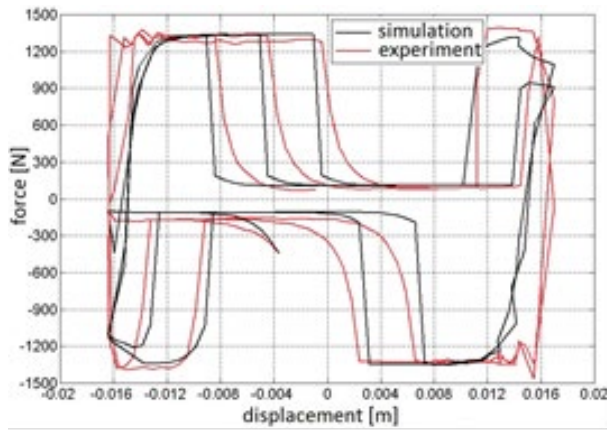


Figure 26. Work characteristic (F vs. d) for the test load

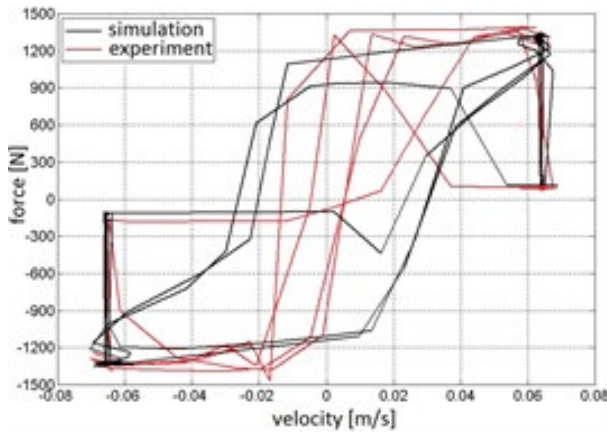


Figure 27. Hydraulic characteristic (F vs. v) for the test load

3. SELECTION OF THE MATHEMATICAL PARAMETRIC MODEL

To evaluate the accuracy with which the proposed models approximate the damper response, the chosen objective function is the Weighted Mean Absolute Percentage Error (WMAPE) of the difference

between the experimentally measured forces (F^{exp}) and the calculated forces (F^{sim}), based on the same displacement, velocity, and control current [36]:

$$WMAPE = \sum_{i=1}^N \frac{|F^{exp} - F^{sim}|}{\max(\lambda, |F^{exp}|)} \times 100\% \quad (9)$$

where $\lambda = 0.001$ is a smoothing term used to prevent division by zero.

For the two models, the following were obtained:

- the Modified Bingham Model:
WMAPE = 27.04%
- the Modified Bouc-Wen Model:
WMAPE = 18.28%

The RRMSE-type (*Relative Root Mean Square Error*) performance indicators are [22]:

- the normalized error over time:

$$E_t = \frac{\varepsilon_t}{\sigma_F} \quad (10)$$

- the displacement-normalized error:

$$E_d = \frac{\varepsilon_d}{\sigma_F} \quad (11)$$

where

$$\varepsilon_t^2 = \int_0^T (F^{exp} - F^{sim})^2 \cdot dt; \quad (12)$$

$$\varepsilon_d^2 = \int_0^T (F^{exp} - F^{sim})^2 \cdot \left| \frac{dx}{dt} \right| \cdot dt \quad (13)$$

$$\sigma_F^2 = \int_0^T (F^{exp} - \mu_F^{sim})^2 \cdot dt; \quad (14)$$

with

μ_F^{sim} - the average value of the simulated force;

F^{exp} - the measured force (experimental)

and

F^{sim} - the force calculated by the model (under the same displacement, velocity, and control current). It follows that:

$$E_d = \frac{\sqrt{\sum_{i=1}^N (F_i^{exp} - F_i^{sim})^2 \cdot |\dot{x}_i|}}{\sqrt{\sum_{i=1}^N \left(F_i^{exp} - \frac{1}{N} \sum_{i=1}^N F_i^{sim} \right)^2}} \quad (15)$$

$$E_d = \frac{\sqrt{\sum_{i=1}^N (F_i^{exp} - F_i^{sim})^2 \cdot |\dot{x}_i|}}{\sqrt{\sum_{i=1}^N \left(F_i^{exp} - \frac{1}{N} \sum_{i=1}^N F_i^{sim} \right)^2}} \quad (16)$$

– the Modified Bingham Model:

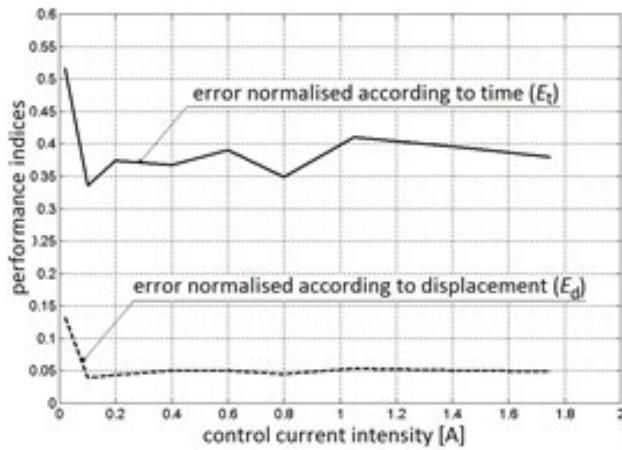


Figure 28. RRMSE vs. control current intensity for the Modified Bingham Model

– the Modified Bouc – Wen Model:

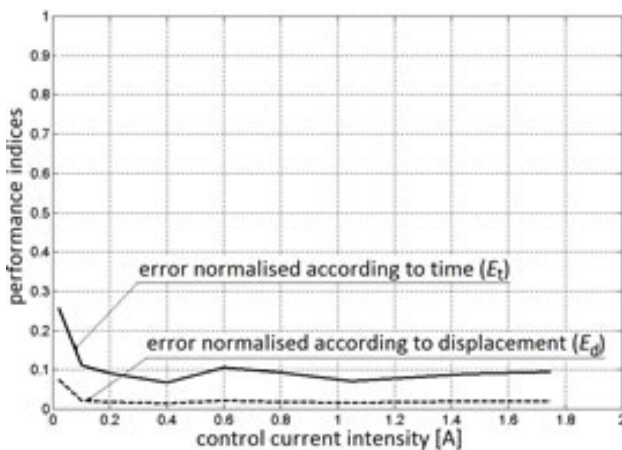


Figure 29. RRMSE vs. control current intensity for the Modified Bouc – Wen Model

The variations of the RRMSE performance indicators for the loading scenarios illustrated in Figure 3 and different constant control current levels are shown in Figures 28 and 29. The stability of these indices across the control current range highlights the robustness of the analyzed models.

The performance indicators for the two models on the test load are:

Table 3. The performance indicators (RRMSE) for the test load

	Modified Bingham Model	Modified Bouc-Wen Model
E_t	0,2712	0,1834
E_d	0,0685	0.0463

3. CONCLUSIONS

1. The proposed method can be used to select the appropriate mathematical model for various devices or processes, based on specific performance indices;
2. In the case study presented in this paper (an MR shock absorber), the performance indices refer to the difference between the damping forces (output quantity) measured experimentally and those calculated using two phenomenological models;
3. It should be noted that, although MR fluid **viscosity** is a function of both magnetic field intensity and flow velocity, the mathematical modeling assumes constant viscosity. Consequently, the control of the dissipative characteristic relies solely on the variation of the yield stress under the influence of the magnetic field;
4. For both models, parameter identification was performed using the same experimental dataset recorded on a test rig (sinusoidal periodic motion at various constant control currents). The predictive accuracy was then validated by comparing model responses to experimental data obtained under highly non-linear conditions, specifically involving triangular periodic displacement and pulse-type variations of the control current;
5. Parameter identification and optimization for both models were achieved through a genetic algorithm (GA) approach. A population of 60 individuals (chromosomes) and 300 generations were used for both models;
6. The findings demonstrate that both models remain consistent with experimental data, thereby validating the proposed methodology (see the performance indicator values in Section 3);
7. As the GA approach is a stochastic technique, the difference between the maximum experimental force and the Modified Bingham model simulated force (Figure 12) results from limiting the optimization to 300 generations before the two parameters could reach their optimal values (refer also to Figure 28 — the performance index varies between 0.3 and 0.5, indicating that the solution is suboptimal);
8. The Modified Bouc–Wen model provides a superior approximation compared to the Modified Bingham model, especially in capturing hysteresis effects;
9. Since the modified Bingham model cannot describe the pre-yield and post-yield regions, the **hysteresis** in these two regions does not appear in its hydraulic characteristic;
10. The Modified Bingham model is algebraic, making it extremely fast and ideal for real-time

control. However, its use of signum functions can cause numerical instability (chattering) or "sharp corners" in the output that do not exist in physical systems.

11. Although the Modified Bouc-Wen model requires higher computational time due to numerical integration of the internal state variable, it offers superior accuracy. Notably, due to the high number of parameters, multiple combinations can yield similar simulation results;

12. To address the issues mentioned above, the Genetic Algorithm ensures robust parameter identification, achieving a compromise between accuracy and computational speed;

13. As shown in Figure 12, the modified Bingham model does not account for MR fluid elasticity (rigid-plastic behavior), neglecting the response delay during abrupt control current variations. Conversely, the Bouc-Wen model captures this quasi-elastic response by incorporating an evolutionary hysteresis variable, z ;

14. The yield force f_c in the Modified Bingham model is current-dependent. While the classical linear approach for $f_c(i)$ depicts magnetic saturation, employing GA-based parameter identification from experimental data (Figure 10) effectively addresses this limitation;

15. By utilizing the evolutionary variable z and parameters β , γ , n , the Modified Bouc-Wen model maintains control over the hysteresis loop's shape in the saturation zone. As a result, GA optimization effectively captures the damping force saturation, providing a high-fidelity reproduction of the experimental 'capping' behavior;

16. In the presented examples, the initial population for all parameters was chosen randomly from the predefined value sets;

17. To improve approximation, it is recommended that the parameter range for the initial population be established based on a preliminary interpretation of experimental data;

18. To mitigate the current shortcomings of both models, a parallel computing algorithm is intended to be developed in order to reduce computational overhead;

As a future research direction, the GA-based identification for the two models could be re-implemented using a population of 90 individuals [37] and 600 generations to investigate the effects of magnetic saturation and fluid elasticity.

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