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# Damping Description in the Vibration Analysis of Structures Made of Composite Materials

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**Abstract:** - Damping in composite materials is an important factor in the proper functioning of structures that have composite materials as a component. As a general observation, it can be said that these structures absorb vibrations better because they have complex internal mechanisms for energy dissipation; however, this way of energy dissipation implies difficulties in performing calculations to determine the behavior of these materials. Models that help in modeling in this case are useful. In the paper, a method for determining the damping matrix using the least squares method for its approximation is presented. Once the damping matrix is obtained in this form, the calculation can be carried out using classical models and methods from modal analysis.

**Keywords:** - damping, composite material, dynamics, vibration, Lagrange equations, finite element method, FEM, modal analysis.

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## 1. INTRODUCTION

Damping in different types of industrial applications, made of very varied materials existing in practice, can sometimes represent a significant factor for the proper functioning of the machine or structure. There are quite a few machines that require a certain type of damping and for which the vibrations that occur and their effect must be carefully studied. Obviously, vibrations occur in most engineering applications and, sometimes, they can significantly influence the functioning of machines and structures in operation. They can be influenced and controlled both by a judicious design of the structure and by an adequate choice of materials. In this work, we deal with the study of systems in which damping is introduced due to the use of composite materials in manufacturing. It is known that for these materials, greater damping can be obtained and more useful energy dissipation can be achieved in certain applications.

These days, new and composite materials are crucial for achieving efficient damping in machines and structures because of two factors that can be used to their advantage: the way they are constructed and the characteristics of the materials that make up the composite. These materials are essentially composed of two or more components, and one of the key elements that contributes to energy

dissipation is the interface between them. The mode of propagation of vibration changes; the mechanical energy is no longer transmitted as in homogeneous materials (metals, for example), but is "fragmented" and then absorbed and transmitted at a microscopic level.

An important concept here is vibration damping. Composite materials have a greater capacity to transform vibration energy into heat, thus reducing the amplitude of oscillations. Another aspect that has an influence on vibration transmission is the viscoelastic behavior of the matrix (more accentuated in polymer composites), which involves the phenomenon of hysteresis (which, through the cyclic loss of mechanical energy at each cycle, directly contributes to damping). The orientation of the fibers in the matrix and the structure of the layers in the composite can be optimized for controlling vibration attenuation, and a layered structure (for example) of a composite material allows the design of zones with different damping in the same part. This results in better damping at lower weight and reduced noise and wear in mechanical systems. Obviously, the applications are immediate in the aerospace, automotive, and industrial equipment industries. The structures become more stable and durable.

The problem that it is presented in the paper is related to a proper description of the damping that

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occurs in new and composite materials, with the aim of facilitating the analysis of the dynamics and vibrations of such systems.

Different constructive solutions are used in the design of multilayer composites to ensure superior damping. For example, a composite damping layer composed of three materials is proposed for improved vibration behavior of a material [1]. It is composed of polyurethane rubber, graphite particles, and aluminum wafers for constraint. An experimental study validates the excellent behavior of this material. The results prove a significantly better damping performance of the material and a high damping capacity of the three-phase composite layer. To reduce the effect of low-frequency vibrations (which can affect human comfort and health), a hybrid composite is proposed in [2], resulting in a material named Ecoflex-Reinforced Fabric (ERF) by the authors that exhibits improved damping performance. Five proposed variants of this material are studied and evaluated at different frequencies to evaluate its vibration attenuation capacity. The highest damping ratio of 93.21% was achieved at 70 Hz, and the system maintained an effective damping bandwidth of approximately 55 Hz, defined as the range in which the damping ratio exceeded 50%. It is thus proven that it is possible to design efficient, lightweight, stable damping systems in the automotive industry in particular.

In all cases, the best method to determine the damping properties of composites remains experimental testing [3]. The analysis of the damping of a composite can be done on various established models. A micromechanical model of temperature-dependent vibration damping of fiber-reinforced ceramic matrix composites is developed to study the temperature-dependent damage mechanisms, interfacial delamination and sliding, and fracture and their role in vibration damping. The effects of the properties of the composite constituents (i.e., fiber volume, interfacial delamination energy, interfacial shear stress, and interfacial friction coefficient) and the composite damage state (i.e., matrix crack spacing) on vibration damping are thus analyzed.

The analysis of the damping of a composite can be done using various established models. A micromechanical model of temperature-dependent vibration damping of fiber-reinforced ceramic matrix composites is developed to study the temperature-dependent damage mechanisms, interfacial delamination and sliding, and fracture and their role in vibration damping [4]. The effects of the properties of the composite constituents (i.e., fiber volume, interfacial delamination energy, interfacial shear stress, and interfacial friction coefficient) and

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Civil engineering is a field in which structural damping can be an important factor in the design and manufacturing process. In this sense, monitoring and modernization represent a viable solution for extending the life cycle, as well as for the reuse of existing structures [5]. The two mentioned aspects are pursued, as well as the development of adaptive systems aimed at suppressing vibrations in structures. Specific objectives can be pursued, such as controlling the dynamic response of the structure to earthquakes, strong winds, or other dynamic loads, thereby increasing the dynamic stability of the structure, which will be able to meet the requirements of resisting external vibrations and meeting safety, service, and comfort requirements [6]. The internal viscosity of the material is an important factor in the characterization of composite materials, and dynamic damping can occur even when inelastic materials are used, when hysteresis exists [7]. In construction, specific applications emphasize the importance of damping. For example, the applicability of NiTi wires in damping forced oscillations produced by wind, rain, or traffic in cable-stayed bridges [8]. NiTi wires have proven to be of great interest in applications due to the large number of working cycles they can withstand, the reliable results, the long service life, and the relatively easy obtaining of similarity sets. Numerous composite applications that offer good damping properties are presented in the literature [9-11]

Modern industrial applications involve the use of composite materials in multibody systems (MBS) where they must provide the desired damping properties. A paper considering nonmodal model reduction techniques for flexible multibody systems presents methods from system dynamics for the process of system dimension reduction [12]. The technique of these methods is verified for active vibration damping of a scanning tunneling microscope. An active vibration control method for a multibody system with fast start and braking is presented in [13]. Closed-loop active vibration control is used. The method offered by the authors leads to better vibration reduction than the previously used methods. A description based on Kane's formalism is presented for the direct dynamics simulation of open- and closed-loop mechanical systems [14]. The method is presented in parallel with the use of the finite element method. The description is implemented in FLXDYN. Excellent performance is obtained in terms of the accuracy and efficiency of the solutions. Specific

methods used to analyze the effect of depreciation in MBS are presented in [15-20].

Research that may give a picture of these materials' behavior under various mechanical stress conditions is particularly necessary due to the advent of these novel materials, which have a wide range of applications. In order to do this, new mathematical models and techniques are created that are as exact as possible in describing the material's response to various mechanical stresses. This method includes the model created in this paper. This is accomplished by using modal analysis, which yields equations of motion in terms of nodal displacements. The damped vibrations of the structure can then be studied using traditional calculation techniques based on this modeling and description. Composite materials have complex internal energy dissipation mechanisms such as friction between fibers and matrix, micro-deformations in the matrix, or slippage at the interface. As a consequence, in real applications they reduce noise, increase comfort, and limit structural fatigue [21-23].

The damping of composites is therefore very important functionally (not just a side effect) and a decisive factor for the choice of material. A procedure describing the steps required for dynamic analysis can become important in the study of structures and devices in which the damping provided by the materials used can be a significant factor. Damping can also lead to more complex situations, such as the study of anisotropy that may occur in some composites, depending on the excitation frequency or the influence of additional factors such as temperature and humidity.

## 2. BEHAVIOR OF AN ELASTIC STRUCTURE

In the following, a mechanical system with elastic elements is studied using the modal analysis method [24]. In this instance, the equations of motion can be regarded as second-order differential equation systems with constant coefficients over very short time intervals. Usually, the coefficient matrices are symmetric, but there are cases when some of these coefficients are skew-symmetric matrices (for example, in the analysis of MBS, the symmetric viscous damping matrix is accompanied by a symmetric skew matrix, due to the Coriolis effects; or the finite element's altered geometry causes some term of the stiffness matrix to change) [25-31].

In the mechanics of discrete elastic systems, the equations that characterize the system's evolution are ultimately produced in the following form by

linearizing the forces that exist between the system's components:

$$[M]\{\ddot{\Delta}\} + [C]\{\dot{\Delta}\} + [K]\{\Delta\} = \{F\}; \quad (1)$$

The matrix coefficients in this equation are regarded as constants. In some cases, these coefficients may be directly or indirectly dependent on time through the parameters set by the system configuration, which is dependent on time. In these situations, the problem might be studied incrementally, by intervals where it is assumed that these matrix coefficients are constant.

In addition to being a numerical technique for resolving differential equations with constant coefficients, the modal superposition approach serves as an intuitive foundation for the physical comprehension of phenomena. A linear transformation of the following type is considered:

$$\{\Delta\} = [\Phi]\{q\}; \quad (2)$$

for transforming the system (1) to a simpler form (where the equations are decoupled).

Matrix  $[\Phi]$  is called the modal matrix. The system can be broken down into a series of  $n$  second-order differential equations with constant, independent coefficients.

Looking for harmonic form solutions:

$$\{\Delta(t)\} = \{X\} \cos(\omega t + \psi); \quad (3)$$

for Eq.(1) the condition results:

$$([K] - \omega^2[M])\{X\} \sin(\omega t + \psi) = 0; \quad (4)$$

at any moment of time  $t$ , which requires:

$$([K] - \omega^2[M])\{X\} = 0, \quad (5)$$

This linear homogeneous system admits nontrivial solutions if and only if:

$$\det([K] - \omega^2[M]) = 0. \quad (6)$$

If:  $P(\omega^2) = \det([K] - \omega^2[M])$  the characteristic polynomial, the equation  $P(\lambda) = 0$  is the characteristic equation, the frequency equation or the eigenvalue equation, and the values  $\omega_i = \sqrt{\lambda_i} = 2\pi f_i$  are called the eigenpulsations of the system and  $f$  the eigenfrequencies. The eigenpulsation with the lowest value is called the fundamental pulsation.

The homogeneous linear system's solution, into which it is introduced the previously established eigenvalue  $\omega_i$ , is represented by  $\{X_i\}$ :

$$([K] - \omega_i^2[M])\{X_i\} = 0; \quad i=1, n \quad (7)$$

The solution of the linear homogeneous system (7) is called the  $i$ -th rank eigenvector of the system. For such a system, the determinant must be equal to zero. So, the  $n$  components of the vector  $\{X_i\}$  will not be independent. It will not be possible to obtain the absolute magnitude of the vector, but only its direction. It is possible to introduce a method to scale these vectors. It is said that the  $i$ -rank *mode of motion* is obtained after the scaling procedure (normalization) of the vector  $\{X_i\}$ . Denote the eigenmodes with  $\{\Phi_i\} = \mu_i \{X_i\}$ . The ensemble of all eigenmodes forms the modal matrix  $[\Phi]$ :

$$[\Phi] = [\Phi_1 \ \Phi_2 \ \dots \ \Phi_n] \quad (8)$$

There are several methods presented in the literature for normalizing vibration modes  $\{X_i\}$  so as to provide the  $\mu$  scalar.

The following orthogonality conditions are fulfilled by the eigenmodes:

- if  $i \neq j$ :

$$\{\Phi_j\}^T [M] \{\Phi_i\} = 0 \quad (9)$$

and:

$$\{\Phi_j\}^T [K] \{\Phi_i\} = 0. \quad (10)$$

- if  $i=j$ , it is denoted:

$$m_i = \{\Phi_i\}^T [M] \{\Phi_i\}; \quad (11)$$

$$k_i = \{\Phi_i\}^T [K] \{\Phi_i\} = 0 \quad (12)$$

The eigenmodes are said to be orthogonal by  $[M]$  respectively by  $[K]$ .

These properties can be extended to obtain the generalized orthogonality relations.

Starting from:

$$[K]\{\Phi_j\} - \omega_j^2[M]\{\Phi_j\} = 0 \quad (13)$$

it is possible to express  $\{\Phi_j\}$  in two distinct ways:

$$\text{i) } \{\Phi_j\} = \frac{1}{\omega_j^2} [M]^{-1} [K] \{\Phi_j\} \quad (14)$$

$$\text{ii) } \{\Phi_j\} = \omega_j^2 [K]^{-1} [M] \{\Phi_j\} = \omega_j^2 ([M]^{-1} [K])^{-1} \{\Phi_j\}, \quad (15)$$

relationship valid for  $p=0,1,2,3,\dots$

Now, having these two Eq.(14) and (15), a single form can be obtained for the generalized orthogonality properties:

$$\{\Phi_i\}^T [M] ([M]^{-1} [K])^p \{\Phi_j\} = 0, \quad p \in \mathbb{Z}, \quad i \neq j. \quad (16)$$

So it is possible to obtain a definition relation for the damping matrix.

The damping matrix can be defined in a form that makes use of these orthogonality relations thanks to relation (16). Therefore, a linear combination of orthogonal matrices defines the Caughey damping matrix.

$$[C] = \sum_{r \in \mathbb{Z}} \alpha_r [M] ([M]^{-1} [K])^r \quad \text{with: } \alpha_r \in \mathbb{R}. \quad (17)$$

Usually, a number of terms equal to the number of degrees of freedom of the system,  $n$ , is used, and Eq. (17) becomes:

$$[C] = \sum_{r=0}^{n-1} \alpha_r [M] ([M]^{-1} [K])^r = [M] \sum_{r=0}^{n-1} \alpha_r ([M]^{-1} [K])^r \quad (18)$$

It is very clear that Eq. (18) represents only an approximation for the damping matrix. The problem is to obtain this approximation in the best mode possible. As an example, the Rayleigh damping matrix can be obtained as a particular case of the Caughey matrix, using only two terms:

$$[C] = [C_0] + [C_1] = \alpha_0 [M] + \alpha_1 [K] \quad (19)$$

Using this decomposition, it is possible to use the orthogonality relations to decouple the system of differential equations.

Considering Eq. (2), the system of differential equations becomes a system of differential equations of second order in  $\{q\}$ :

$$[M][\Phi]\{\ddot{q}\} + [C][\Phi]\{\dot{q}\} + [K][\Phi]\{q\} = \{F\} \quad (20)$$

If the damping matrix  $[C]$  admit a Caughey factorization as in Eq. (17), finally the system of Eq. (1) will be decoupled, by multiplying the Eq. (20) with the transpose of the modal matrix:

$$[\Phi]^T [M] [\Phi] \{\ddot{q}\} + [\Phi]^T [C] [\Phi] \{\dot{q}\} + [\Phi]^T [K] [\Phi] \{q\} = [\Phi]^T \{F\} \quad (21)$$

In this way, the matrices  $[\Phi]^T[M][\Phi]$ ,  $[\Phi]^T[C][\Phi]$  and  $[\Phi]^T[K][\Phi]$  become diagonal matrices, and it is possible to write:

$$[\backslash M^* \backslash] = [\Phi]^T[M][\Phi] \quad (22)$$

$$[\backslash C^* \backslash] = [\Phi]^T[C][\Phi] \quad (23)$$

$$[\backslash K^* \backslash] = [\Phi]^T[K][\Phi] \quad (24)$$

In the paper by  $m_i$ ,  $c_i$ , and respectively  $k_i$ , the diagonal components of the matrices were noted  $[\backslash M^* \backslash]$ ,  $[\backslash C^* \backslash]$  and  $[\backslash K^* \backslash]$ . So, it is noted:

$$m_i = [\Phi_i]^T[M][\Phi_i]; \quad (25)$$

$$c_i = [\Phi_i]^T[C][\Phi_i]; \quad (26)$$

$$k_i = [\Phi_i]^T[K][\Phi_i]. \quad (27)$$

(The other components are zero; if  $i \neq j$ , there exist the orthogonality relations  $\{\Phi_j\}^T[C]\{\Phi_i\} = 0$ ;

$\{\Phi_j\}^T[C]\{\Phi_i\} = 0$  ;  $\{\Phi_j\}^T[C]\{\Phi_i\} = 0$   $i \neq j$ .)

In the hypothesis that the eigenvalues are non-zero and distinct, the system of differential equations (5.1.a) is decoupled into  $n$  second-order differential equations, with constant, independent coefficients:

$$\begin{aligned} m_1 \ddot{q}_1 + c_1 \dot{q}_1 + k_1 q_1 &= F_1 \\ m_2 \ddot{q}_2 + c_2 \dot{q}_2 + k_2 q_2 &= F_2 \\ m_n \ddot{q}_n + c_n \dot{q}_n + k_n q_n &= F_n \end{aligned} \quad (28)$$

or, with the notations:  $2\beta_i = c_i / m_i$  ;  $\omega_i^2 = k_i / m_i$  ;  $f_i = F_i / m_i$  it results:

$$\begin{aligned} \ddot{q}_1 + 2\beta_1 \dot{q}_1 + \omega_1^2 q_1 &= f_1 \\ \ddot{q}_2 + 2\beta_2 \dot{q}_2 + \omega_2^2 q_2 &= f_2 \\ \ddot{q}_n + 2\beta_n \dot{q}_n + \omega_n^2 q_n &= f_n \end{aligned} \quad (29)$$

The problem is thus reduced to solving  $n$  independent second-order differential equations.

### 3. DECOMPOSITION OF THE DAMPING MATRIX

To obtain a decomposition for the damping matrix, we introduce the Frobenius norm for a matrix:

$$\|A\|_F = \sqrt{\sum_{i,j} |a_{ij}|^2} \quad (30)$$

If  $[C]$  it is the damping matrix, we look for a matrix  $[\tilde{C}]$  as close as possible to  $C$ , so that it admits the factorization (18). In this way, the system of differential equations obtained can be decoupled into a system of second-order differential equations, independent, with constant coefficients, which no longer poses a problem to solve. The condition that it is imposed for the damping approximation to be as close as possible to the real matrix is that the mean square deviation of the norm of the difference between the two matrices be as small as possible. The condition is therefore:

$$\|A\|_F = \sqrt{\sum_{i,j} |c_{ij} - \tilde{c}_{ij}|^2} = \min \quad (31)$$

or:

$$\sum_{i,j} |c_{ij} - \tilde{c}_{ij}|^2 = \min \quad (32)$$

$$[C] = [\tilde{c}_{ij}] = \sum_{r \in Z} \alpha_r [M] ([M]^{-1} [K])^r = \sum_r \alpha_r [\tilde{c}_{ij}^r] \quad (33)$$

The standard deviation is defined as

$$\begin{aligned} \delta^2 &= \sum_{i,j} \left| c_{ij} - \sum_r \tilde{c}_{ij}^r \right|^2 = \\ &= \sum_{i,j} (c_{ij} - \sum_r \alpha_r [\tilde{c}_{ij}^r])^2 = \min \end{aligned} \quad (34)$$

and it is imposed that the standard deviation be minimum. The set of values that fulfill these conditions is obtained by considering the conditions of minimum:

$$\begin{aligned} \frac{\partial(\delta^2)}{\partial \alpha_r} &= -2 \sum_{i,j} \left( c_{ij} - \sum_r \alpha_r \tilde{c}_{ij}^r \right) \tilde{c}_{ij}^r = 0; \\ i, j &= \overline{1, n}; \quad r = \overline{1, p} \end{aligned} \quad (35)$$

that represents a linear system of  $n$  equations. From this system result the values of  $\alpha_r$  that offer the best approximation for the damping matrix.

Consider, i.e., that it is known, the damping matrix  $[C]$  and we want to express this damping by an approximate matrix (Rayleigh damping):

$$[C] \cong [\tilde{C}] = [C_0] + [C_1] = \alpha_0 [M] + \alpha_1 [K] \quad (36)$$

so that the standard square deviation is minimized. Define the standard square deviation as:

$$\begin{aligned} \delta^2 = & (c_{11} - \alpha_0 m_{11} - \alpha_1 k_{11})^2 + \\ & 2(c_{12} - \alpha_0 m_{12} - \alpha_1 k_{12})^2 + \\ & (c_{22} - \alpha_0 m_{22} - \alpha_1 k_{22})^2. \quad i, j = \overline{1, n} \end{aligned} \quad (37)$$

where it is denoted:

$$\begin{aligned} [C] = [c_{ij}]; \quad [C_0] = [c_{ij}^0]; \quad [C_1] = [c_{ij}^1]; \\ [M] = [m_{ij}]; \quad [K] = [k_{ij}]; \end{aligned} \quad (38)$$

$$\begin{aligned} \frac{\partial(\delta^2)}{\partial \alpha_0} = & -2(c_{11} - \alpha_0 m_{11} - \alpha_1 k_{11})m_{11} - \\ & -4(c_{12} - \alpha_0 m_{12} - \alpha_1 k_{12})m_{12} - \\ & -2(c_{22} - \alpha_0 m_{22} - \alpha_1 k_{22})m_{22} = 0; \end{aligned} \quad (39)$$

$$\begin{aligned} \frac{\partial(\delta^2)}{\partial \alpha_1} = & -2(c_{11} - \alpha_0 m_{11} - \alpha_1 k_{11})k_{11} - \\ & -4(c_{12} - \alpha_0 m_{12} - \alpha_1 k_{12})k_{12} - \\ & -2(c_{22} - \alpha_0 m_{22} - \alpha_1 k_{22})k_{22} = 0; \end{aligned} \quad (40)$$

After some simple calculus, it results in:

$$\begin{aligned} \alpha_0(m_{11}^2 + 2m_{12}^2 + m_{22}^2) + \\ \alpha_1(m_{11}k_{11} + 2m_{12}k_{12} + m_{22}k_{22}) = \end{aligned} \quad (41)$$

$$\begin{aligned} c_{11}m_{11} + c_{12}m_{12} + c_{22}m_{22} \\ \alpha_0(m_{11}k_{11} + 2m_{12}k_{12} + m_{22}k_{22}) + \\ \alpha_1(k_{11}^2 + 2k_{12}^2 + k_{22}^2) = \end{aligned} \quad (42)$$

or:

$$[A]\{\alpha\} = \{B\} \quad (43)$$

where:

$$\begin{aligned} \{\alpha\} = \begin{Bmatrix} \alpha_0 \\ \alpha_1 \end{Bmatrix}; \quad \{B\} = \begin{Bmatrix} c_{11}m_{11} + c_{12}m_{12} + c_{22}m_{22} \\ c_{11}k_{11} + c_{12}k_{12} + c_{22}k_{22} \end{Bmatrix} \\ [A] = \begin{bmatrix} m_{11}^2 + 2m_{12}^2 + m_{22}^2 & m_{11}k_{11} + 2m_{12}k_{12} + m_{22}k_{22} \\ m_{11}k_{11} + 2m_{12}k_{12} + m_{22}k_{22} & k_{11}^2 + 2k_{12}^2 + k_{22}^2 \end{bmatrix} \end{aligned} \quad (44)$$

It results in:

$$\{\alpha\} = [A]^{-1}\{B\} \quad (45)$$

so, the coefficients proposed by Eq. (19).

The motion equation becomes:

$$\begin{aligned} [M]\{\ddot{\Delta}\} + (\alpha_0[M] + \alpha_1[K])\{\dot{\Delta}\} + \\ ([K])\{\Delta\} = \{F_{tot}\} \end{aligned} \quad (46)$$

Using Eq. (2) and pre-multiplying by  $[\Phi]^T$  it results in:

$$\begin{aligned} [\Phi]^T[M][\Phi]\{\ddot{q}\} + \\ (\alpha_0[\Phi]^T[M][\Phi] + \alpha_1[\Phi]^T[K][\Phi])\{\dot{q}\} + \\ [\Phi]^T[K][\Phi]\{q\} = [\Phi]^T\{F_{tot}\} \end{aligned} \quad (47)$$

and it obtains the final form of the equations:

$$\begin{aligned} [\backslash M^* \backslash]\{\ddot{q}\} + (\alpha_0[\backslash M^* \backslash] + \alpha_1[\backslash K^* \backslash])\{\dot{q}\} + \\ [\backslash K^* \backslash]\{q\} = [\Phi]^T\{F_{tot}\} \end{aligned} \quad (48)$$

so the form of  $n$  independent second-order differential equations that can be easily solved using classic procedures.

#### 4. DISCUSSION AND CONCLUSIONS

Damping is an important element in civil engineering, and the use of new and composite materials in this field opens a promising field of applications. For this reason, the study of damping in mechanical systems used in construction and subjected to vibrations becomes important for engineers, especially during the design period. Among them, composites with improved damping qualities are gaining increasing importance.

Composite materials have complex internal energy dissipation mechanisms such as friction between fibers and matrix, micro-deformations in the matrix, or slippage at the interface. As a consequence, in real applications they reduce noise, increase comfort, and limit structural fatigue. The damping of composites is therefore very important functionally (not just a side effect) and a decisive factor for the choice of material. A procedure describing the steps required for dynamic analysis can become important in the study of structures and devices in which the damping provided by the materials used can be a significant factor.

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