
Lagrange's Formulation for the Vibration Analysis of the Structures using Mycelium Composite

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Abstract: - The spectacular development of the use of mycelium composites in real-world applications has led to an increase in research into the properties of these materials and their behavior in different situations encountered in practice. An important direction of study is the response of structures in the manufacture of which such composites are developed to support mechanical loads. The use of mycelium composites in different applications was initially due to their ecological advantages, ease of obtaining them, and low price. As the complexity of applications has increased, the need to have appropriate models for the study of their behavior under complex mechanical loads has also increased. The present work aims to develop a model based on the Lagrangian description of the mechanical system in which the composite is incorporated in order to obtain the equations of motion of such a system in a finite element formulation. In this way, results can be obtained regarding the vibrations that the analyzed structure may have.

Keywords: - vibration, Lagrange equations, mycelium composite, finite element method, modal analysis

1. INTRODUCTION

The network of "roots" of mushrooms, known as mycelium, is used to create mycelium composite, a biological substance that develops and combines organic particles, such as sawdust and straw, into a solid structure. In essence, it "grows" into the required shape before being dried to make it sturdy

and hard. Currently, Mycelium composite (a material based on mycelium - the root network of mushrooms) is increasingly used in industrial applications as a sustainable and relatively cheap alternative to traditional materials. For example, it is used extensively in the manufacture of ecological packaging, being biodegradable and compostable. This type of composite successfully replaces

polystyrene and plastics. It has begun to be used in construction, in the manufacture of insulating panels or as bricks or lightweight structural elements (and, importantly, biodegradable). It can successfully replace plastic in the manufacture of interior furniture, such as tables, chairs, and decorative objects. The latest applications show that it can be used to create "vegan leather". Packaging using mycelium composite can be used as protection for fragile products and as a substitute for synthetic foams. Obviously, there is a wide range of applications, which is expanding day by day. A review of the main engineering applications will be made below. The main advantages of mycelium composites consist of reduced energy consumption during manufacturing (hence low price), recycling (100% biodegradable), and a low final impact on the environment.

This material has demonstrated a strength comparable to polystyrene foams and particle boards, thus becoming a sustainable and environmentally friendly alternative for many applications. In today's applications, resource depletion remains a major problem in the construction field. Due to this major reason, alternative sources are being sought. Thus, mycelium can represent a new and suitable solution for the sustainable construction industry. Numerous studies are being undertaken to analyze the mechanical properties of these materials. The mechanical properties of mycelium bricks and composites have been examined, and the methodology for producing mycelium has been studied. Substrates such as rice bran, coconut shells, and sawdust have been used by various researchers to develop mycelium composites and bricks. Based on the results obtained, it is found that mycelium bricks are a good thermal insulation material. Mycelium fibers reduce the brittle nature of bricks and increase ductile properties, have better fire resistance, and can be manufactured relatively easily. Mixing the mycelium with other products, such as *Miscanthus*, improves its properties, and the results obtained have proven to be satisfactory for transforming it into a sustainable and unconventional building material [1]. For this reason, there has been research to improve many of the properties of mycelium composites. Hybridization and reinforcement methods have been used to strengthen mycelium composite boards and foams. Although most methods have a limited effect on improving mechanical properties, other properties can lead to the successful use of these materials in applications [2].

Mycelium, the root structure of mushrooms, is combined with various natural fibers (NF) to obtain

a strong, lightweight, inexpensive material with environmental benefits. Incorporating NF such as hemp, jute, or bamboo into the mycelium matrix improves strength, flexibility, and durability. Mycelium composites offer sustainability by using agricultural waste, reducing the carbon footprint compared to conventional construction materials. Analysis of the mechanical properties and potential impact of these composites is presented in [3].

Their transition from niche innovations to mainstream applications depends on their ability to overcome a number of manufacturing challenges. The critical challenges facing the fabrication of mycelium composites are described in [4]. Key factors leading to fabrication failure, such as contamination, inconsistent growth, insufficient moisture, inappropriate pH, and improper substrate preparation, are rigorously presented with an emphasis on preventive and mitigation strategies. Potential strategies for refining the fabrication processes and the identification of promising research areas are also presented.

The methods used to produce these materials, their benefits and drawbacks, how they can be used to solve urgent environmental and economic issues, and how they are currently being used in a variety of industries, such as manufacturing and biomedicine, are all highlighted in [5]. Mycelium-based composites may be widely used as sustainable materials in the future by improving the material's qualities and streamlining the manufacturing procedures. Social problems involved in the use of these materials are presented in [6]. The potential of mycelium composites in different industries and their possible applications are presented in [7-9]. How the mechanical properties can be controlled is described in [10], and the basic application in construction is studied in [11].

Construction is an area where the use of mycelium composites can prove beneficial. For example, a new type of composite insulating brick was manufactured by cultivating a biocompatible, fast-growing mushroom, *Pleurotus ostreatus* [12]. The growth is done using rye as a raw material. Research on the effects of density and humidity on the mechanical characteristics of mycelium composites revealed that at both low and medium relative humidity (RH) levels, composites with higher density showed improved flexural strength. This can be achieved as a durable insulation material for buildings. A biomaterial called mycelium has the potential to be a novel substitute for current architectural solutions [13]. The relationships between mycelium and other building materials, strategies for shielding mycelium from outside influences, and aesthetic and finishing elements are

also studied. Mycelium-based composites (MBCs) are created by introducing fungal spores into a cellulosic matrix, allowing the mycelium to proliferate and bond the matrix particles. By essentially "killing" the fungus—typically by heating it to 80 degrees Celsius or higher—growth is halted. The low embodied carbon footprint of MBCs, the possibility of using wood and agricultural waste, the ability to produce these biocomposites locally, and the potential for the biocomposites to biodegrade at the end of their life (i.e., be composted) are some of their main advantages over conventional materials [14]. To present, the majority of mycelium composites have been utilized in packaging or insulation. More recently, a cotton fabric, hemp husks, and mycelium binder were combined to create an MBC composite that is said to have better mechanical qualities. Aspects of the use of mycelium biocomposites are studied in numerous works [15-19].

Some interesting results and possible developments are presented below. The investigation of the effects of material characteristics (physiological traits of fungi) on the mechanical properties of mycelium composites and the microscopic structure of the resulting material, for instance, may be an intriguing aspect [20]. When compared to the coarse mycelium networks produced by *Pleurotus ostreatus*, the dense and continuous mycelium networks supplied by *Trametes hirsuta* will enhance the mechanical qualities of the resulting composite. Because of the composite's superior form retention and great flexibility in the wet state, novel characteristics and industrial uses can be predicted. A series of studies is specifically aimed at studying the mycelium and its formation and growth conditions, as well as the differences that exist between different types of mushrooms and the conditions in which they develop [21-27].

In recent years, researchers have been constantly concerned with determining the mechanical properties of mycelium composites [29-32]. Within its filamentous network structure, the mycelium serves as a supporting matrix for the reinforcing particles in the composite. According to experimental findings, the composite resembles pure mycelium's soft elastic response at mild deformations. The densification of the rigid particles causes a noticeable stiffening at bigger deformations if more particles are added to the composite. Additionally, during cyclic compression, such a composite displays the typical stress-relieving action and hysteresis. Although mycelium composites offer a sustainable alternative to lightweight plastic products, creating a mycelium composite with

superior thermal insulation and mechanical strength remains a significant challenge, but one that researchers have engaged in, as it presents broad prospects in construction and packaging. Another area of significant practical interest is the impact of varying ratios of high-silica agricultural and industrial wastes on the flammability of mycelium composites in comparison to standard synthetic construction materials [33]. With significantly lower average and maximum heat release rates and a longer time to rapid combustion, the results demonstrate that mycelium composites are safer than the conventional building materials taken into consideration. Additionally, they emit far less CO₂ and smoke; however, the amount of CO produced varied. The fire performance was enhanced by the substantial amount of silica char and ash created by rice husks, but the composites with fine glass particles had the highest fire performance because of their much greater silica concentrations and lower proportion of combustible materials. Particular aspects of the use of these materials are reported in [34-40].

The use of composite mycelium in construction, in the manufacture of composite structures or bricks, has made it necessary to study their behavior under mechanical stresses of different types and intensities. It is therefore normal to develop models that allow the study of the static and dynamic response of these materials and structures manufactured from these materials under different stresses and tests. For modeling, the results of classical mechanics are used. A brief presentation of the methods used is outlined below.

Lagrange's book from 1788 [45] made it evident that the analytical methods of mechanics were different ways to formulate Newton-Euler's equations of motion (EoM). These techniques have shown their value when the complexity of the systems under study started to rise, despite the initial focus being more theoretical. Gibbs-Appell's (1879, 1899) methods [46,47], Hamilton's equations (1835) [48], Maggi's equations (1896) [49], and Kane's equations (1924) [50] (basically a new way of writing the Maggi's equations [51,52]) are just a few of the formulations that emerged in the context of the ongoing development of Lagrangian mechanics. For a long time, the potential to define a mechanical system's evolution equations from its basic principles was mostly of theoretical interest with little bearing on real-world applications.

Each of the stiff and deformable components that make up a flexible multibody system (FMS) is capable of significant translational and rotational movements. The elements could come into contact with the surroundings or with one another.

Cylindrical and spherical joints, prismatic couplings, flat connectors, screws, gears, or cams are the traditional kinematic linkages implied in engineering applications. These components may be linked in an open kinematic chain, as in the case of manipulators, or in a closed loop, as in the theory of classical mechanisms. Flexible multibody dynamics (FMD) of an MBS imposes computational procedures that are used to define the dynamic response. These strategies require knowledge about the system's mechanical behavior over time, deformations, stresses from external forces, liaisons, and boundary conditions.

However, the ongoing industrialization that resulted in the development of machinery and equipment that operated at high speeds and accelerations while exerting strong forces in recent decades necessitated the constant advancement of the models and computing techniques employed. As a result, the analytical techniques previously employed for the study of multibody systems were reexamined. The results showed that, in certain situations, using one of the aforementioned techniques may be more beneficial, both in terms of modeling and from the perspective of the calculation volume and, consequently, of the required computational time [51]–[53].

It is clear that the field and coordinated tactics have developed as a result of the necessity to address practical issues. The outstanding and thoroughly researched work [53] classifies and ranks the various techniques. One of the crucial phases in such a study is the modeling of flexible elements within an MBS system. An inertial reference system is employed as a global reference system in the analysis of the MBS to characterize the motion of the entire system. Each flexible element (component) in the system has an intermediary reference system linked to it. The general motion of this flexible elastic part in space is average. In this case, the movement of the flexible element in relation to the local reference system is due to elastic deformation.

The two primary local reference systems used by researchers are corotational frames and floating frames. The first kind, the floating frame, follows a hypothetical rigid body by taking into account an average rigid body motion. The second kind, known as the corotational frame, takes into account a single finite element and tracks its average rigid body motion. Nonetheless, it is feasible to report the deformations straight to the global coordinate system without the need for intermediate reference systems. The second kind of reference frame is used in our presentation. We provide a quick overview of the primary analytical techniques applied in FMD [62]. The following procedures must be taken in

order to algorithmize dynamics problems using Lagrange's equations [54,55]. The equations of motion of the mechanical system under study are then obtained by performing some pre-established differentiation computations. These equations are second-order differential equations that enable the identification of the unknown generalized coordinates. In conclusion, generalized coordinates (velocities), kinetic energy, potential energy, and work are the fundamental concepts covered by analytical mechanics. It is obvious that for the part describing the elastic behavior of composites, the classical results [56-60] are used, with minor adaptations.

Mycelium composites are increasingly used as interest in ecological materials has increased; they represent alternatives to plastic and polystyrene. Legislation in all developed countries imposes regulations on recyclable materials, and technology allows obtaining solutions with improved properties and reduced cost. Construction and packaging represent over 60% of practical applications, with the notable result that about 30% of eco-packaging companies already use mycelium. These materials are currently being intensively investigated globally, in the development and expansion phase, due to their ecological potential. They also represent viable alternatives to concrete or plastic. We are dealing with a modern field, in full development and expansion.

2. DESCRIPTION OF THE MYCELIUM COMPOSITE

The physical and mechanical properties (elasticity, strength) and homogeneity of mycelium bio-composites are inherited from the substrate, but are also determined by the fungal species, growing conditions, the processing of the material, and additives [61]. Differences in mycelial hyphal structures developed on different substrates are reflected as alterations in the morphology and mechanical properties of the biocomposites. Mycelium materials get stiffer when their feeding substrates are harder to digest, for example, cellulose-based substrates. Scientific experiments have shown that “the materials grown on cellulose contained more chitin and showed higher Young’s modulus and lower elongation than those grown on dextrose-containing substrates. The possibility of tailoring mycelium materials’ properties by choosing their nutrient substrates paves the way for their use in various scale applications” (Haneef et al. 2017).

Some mycelium-based composites display high impact resistance and compressive strengths of 6.4 MPa for rice straw and wheat straw mycelium

biocomposites [63] and 6 MPa for wild bush waste-based mycelium blocks developed by MycoHab Namibia [64], which is comparable to standard clay brick (7 MPa) and concrete blockwork - the minimum compressive strength for concrete blocks recommended by the UK's National House Building Council Standards is 7.3 MPa (NHBC Standards 2024).



Figure 1. Mycelium Composite

Mycelium composites have low thermal conductivity and therefore excellent thermal insulation properties. Thermal conductivity for these materials varies depending on the substrate properties and the thickness of the biocomposite, measuring roughly 0.028 W/mK at 100mm thickness. Mycelium biocomposites with low density and high porosity have demonstrated “outstanding thermal insulation, energy absorption, and hydrophobic properties” [65], comparable to or even better than expanded polystyrene (EPS).

The high porosity of these materials also traps and dissipates sound waves, offering them outstanding acoustic absorption properties. Research findings show that “sound can be successfully attenuated by the peculiar structure of mycelium, which is composed of an interconnected network of hyphae” [66].

Studies have shown that mycelium biocomposite materials have a noise-reduction coefficient of 0.53, meaning that 53% of the energy of incident sound waves reaching the material is absorbed and is not reflected back (an average of the material's sound absorption coefficients at 250, 500, 1000, and 2000 Hz), outperforming fossil-derived insulations like EPS with a coefficient between 0.10 and 0.32. Furthermore, the high porosity and natural fibers of the mycelium biocomposites have also been shown to perform great air filtration, absorbing atmospheric particulate matter for cleaner and healthier indoor environments [67].

Tests developed on various types of mycelium biocomposites demonstrate that they are hydrophobic materials, with water contact angles higher than 120°. However, these materials are not 100% waterproof and will lose rigidity and degrade when repeatedly exposed to moisture. While they have been temporarily used in outdoor applications, more research needs to be done to allow long-term continued exposure to moisture for external or wet area uses.

Mycelium-based composites exhibit superior fire performance compared to synthetic polymers, characterized by low heat release, minimal smoke production, and a high char yield that inhibits flame spread and provides a thermal insulation barrier. *Ganoderma lucidum*-based biocomposites show elevated chitin content, conferring “durability and fire-resistant properties, surpassing conventional synthetic foams in performance metrics” [68]. Mycelium biocomposites are fire-resistant at over 760°C. Some have even demonstrated self-extinguishing capabilities.

3. MODEL AND METHOD

In the following, a finite element model built for the study of composite mycelium used in mechanical structures, at the level of dynamic analysis, is described. The goal is to obtain the equations of motion in the case of the existence of mechanical vibrations in the analyzed structures.

The main objective in such an approach is to obtain the equations of motion for a finite element. Once these are obtained, the classical techniques of assembling the equations, introducing the connections between the elements, and the mathematical solution methods remain the classical ones, very well known and successfully applied for decades.

Numerous types of finite elements can be analyzed to obtain these equations, with the researcher ultimately using the type of elements that best fit the problem, also taking into account the researcher's previous experience. Different types of elements are presented in a rich literature (see, for example, [68-71]).

A MBS's bodies are meant to be linearly elastic. The independent nodal coordinates serve as a representation of the unknowns. Compared with classical FEM application cases, in the case of an analysis of MBS, additional terms will be obtained. Coriolis effects, the relative movement of the local reference frame (LRF) with respect to the global reference frame (GRF), and the change in stiffness, a phenomenon brought on by the acceleration field in rigid motion, provide these extra terms. The motion

of the finite element with respect to the LRF/GRF also results in the appearance of additional inertial terms. Following the determination of the motion equations, the equations pertaining to each element must be referred to the GRF while taking into account the relevant rotation matrix.

Below are some basic findings about the finite element's dynamics and kinematics. The LRF is represented by $Oxyz$, and the GRF by $OXYZ$. The motion equations will be found using the Lagrange equations [72,73].

The position vector of an arbitrary point is [74-78].

$$x_{M'i,G} = x_{Oi,G} + r_{ij}x_{j,L} + r_{ij}N_{jk}\delta_k \quad (1)$$

$$i,j=1,2,3; k=1,2,\dots,p,$$

Here notation $x_{M'i,G} / x_{M'i,L}$, $i=1,2,3$ represents the components of the position vector of an arbitrary deformed point in the G/L reference system. The components of the velocity of this point, M , having the components $\dot{x}_{M'i,G} / \dot{x}_{M'i,L}$ $i=1,2,3$, that becomes M' after deformation, if we take into account a single finite element in the LRF, are:

$$\dot{x}_{M'i,G} = \dot{x}_{Oi,G} + \dot{r}_{ij}x_{j,L} + \dot{r}_{ij}N_{jk}\delta_k + r_{ij}N_{jk}\dot{\delta}_k \quad (2)$$

$$i,j=1,2,3; k=1,2,\dots,p,$$

where a size (vector, matrix) expressed in the LRF/GRF is denoted with the index L/G.

- $\dot{x}_{Oi,G}$, $i=1,2,3$ represent the components of the velocity of the origin of the LRF expressed in the GRF.
- $[R]=[r_{ij}]$, $i,j=1,2,3$ represents the rotation matrix (his role is to transform the vector from the LRF to the GRF);
- N_{ij} , $i=1,2,3; j=1,2,\dots,p$ is the matrix of the shape functions. This matrix depends on the type of chosen finite element
- $x_{i,L}$, $i=1,2,3$ $\{r\}_L$ are the components of the position vector of a current point of a finite element expressed in the LRF;
- δ_i , $i=1,2,\dots,p$ represent the nodal coordinates expressed in the LRF;
- $\dot{\delta}_i$, $i=1,2,\dots,p$ represent the velocities of the nodal coordinates expressed in the LRF.

For a single finite element, the kinetic energy has the expression:

$$E_c = \frac{1}{2} \int_V \rho \dot{x}_{M'i,G} \dot{x}_{M'i,G} dV = \frac{1}{2} \int_V \rho \left(\dot{x}_{Oi,G} + \dot{r}_{ij}x_{j,L} + \dot{r}_{ij}N_{jk}\delta_k + r_{ij}N_{jk}\dot{\delta}_k \right)^2 dV \quad (3)$$

$$\left(\dot{x}_{Oi,G} + \dot{r}_{il}x_{l,L} + \dot{r}_{il}N_{lm}\delta_m + r_{il}N_{lm}\dot{\delta}_m \right) dV$$

$$i,j,l=1,2,3; k,m=1,2,\dots,p.$$

or:

$$E_c = \frac{1}{2} \int_V \rho \left(\dot{x}_{Oi,G} \dot{x}_{Oi,G} + \dot{r}_{ij}x_{j,L} \dot{r}_{il}x_{l,L} + \dot{r}_{ij}N_{jk}\delta_k \dot{r}_{il}N_{lm}\delta_m + r_{ij}N_{jk}\dot{\delta}_k r_{il}N_{lm}\dot{\delta}_m \right) dV +$$

$$2 \left(\dot{x}_{Oi,G} \dot{r}_{il}x_{l,L} + \dot{x}_{Oi,G} \dot{r}_{il}N_{lm}\delta_m + \dot{x}_{Oi,G} r_{il}N_{lm}\dot{\delta}_m \right) +$$

$$2 \left(\dot{x}_{Oi,G} \dot{r}_{il}x_{l,L} + \dot{x}_{Oi,G} \dot{r}_{il}N_{lm}\delta_m + \dot{r}_{ij}N_{jk}r_{il}N_{lm}\delta_k\dot{\delta}_m \right) dV$$

$$i,j,l=1,2,3; k,m=1,2,\dots,p,$$

where ρ is for the material density of the finite element.

For the potential energy there is the classic relation:

$$E_p = \frac{1}{2} \int_V k_{ij} \delta_i \delta_j dV ; \quad (5)$$

$$i,j=1,2,\dots,p,$$

Here k_{ij} represents the components of the rigidity matrix [74].

Damping offers a work:

$$W^a = \frac{1}{2} \int_V c_{ij} \delta_i \delta_j dV ; \quad (6)$$

$$i,j=1,2,\dots,p,$$

Concentrated forces (having the components q_i^c , applied in the nodes of the finite element give the work:

$$W^c = \frac{1}{2} \int_V q_i^c \delta_i dV ; \quad (7)$$

$$i,j=1,2,\dots,p,$$

The volume forces $p_i = p_i(x,y,z)$, reduced in the nodal points, give the work:

$$W^d = \frac{1}{2} \int_V q_i^d \delta_i dV ; \quad (8)$$

$$i, j=1,2,\dots,p,$$

The Lagrangian describing the considered finite element is:

$$L = E_c - E_p + W \quad (9)$$

where: $W = -W^a + W^c + W^d$ is the total work. Lagrange equations for a finite element are [79-85]:

$$\frac{d}{dt} \left\{ \frac{\partial L}{\partial \dot{\delta}_i} \right\} - \left\{ \frac{\partial L}{\partial \delta_i} \right\} = 0 ; \quad (10)$$

$$i=1,2,\dots,p.$$

Applying rel. (10) it obtained the motion equations for a single element finally:

$$m_{ij} \ddot{\delta}_{j,L} + c_{ij} \dot{\delta}_{j,L} + \left(k_{ij} + k_{ij}(\varepsilon) + k_{ij}(\omega) \right) \delta_{j,L} = q_{i,L}^c + q_{i,L}^d - q_{i,L}^{dyn}(\varepsilon) - q_{i,L}^{dyn}(\omega) - m_{ij,O}^{dyn} \ddot{x}_{Oj,L} \quad (11)$$

$$i, j=1,2 \dots p.$$

The *LRF* is the reference system where the equations of motion are expressed. They have some sizes that vary depending on the bodies' positions, but they can be linearized if the *LRF* is "frozen" and the known field of velocities and accelerations remains constant for brief periods of time.

4. ASSEMBLING THE MOTION EQUATIONS

The motion equations (12) are expressed in the *LRF*. The coordinates of the nodes are expressed in this system of coordinates. To have a unitary description of the evolution of the body, all the coordinates may be expressed in the same coordinate system, the *GRF/LRF*. For a single finite element *e*, the relation that makes this transformation is in the form:

$$\delta_{i,G} = T_{ij} \delta_{j,L} ; \quad (12)$$

$$i=1,2,\dots,p.$$

It exists too the relations:

$$\dot{\delta}_{i,G} = T_{ij} \dot{\delta}_{j,L} ; \quad (13)$$

$$i, j=1,2,\dots,p.$$

and:

$$\ddot{\delta}_{i,G} = T_{ij} \ddot{\delta}_{j,L} ; \quad (14)$$

$$i, j=1,2,\dots,p.$$

Pre-multiplying the system (11) with T_{ij} it results:

$$m_{ij} T_{jk} \ddot{\delta}_{k,G} + c_{ij} T_{jk} \dot{\delta}_{k,G} + \left(k_{ij} + k_{ij}(\varepsilon) + k_{ij}(\omega) \right) \delta_{j,L} = q_{j,L}^c + q_{i,L}^d - q_{i,L}^{dyn}(\varepsilon) - q_{i,L}^{dyn}(\omega) - m_{ij,O}^{dyn} \ddot{x}_{Oj,L} \quad (15)$$

$$i, j, k=1,2,\dots,p,$$

Equation (16) represents a force balance, written in the local reference system. To move to the global reference system, to which all finite elements will be referred after assembly, we must pre-multiply with T_{ij} :

$$T_{ij} m_{jk} T_{lk} \ddot{\delta}_{l,G} + T_{ij} c_{jk} T_{lk} \dot{\delta}_{l,G} + T_{ij} \left(k_{jk} + k_{jk}(\varepsilon) + k_{jk}(\omega) \right) T_{lk} \delta_{k,G} = T_{ij} q_{j,L}^c + T_{ij} q_{j,L}^d - T_{ij} q_{j,L}^{dyn}(\varepsilon) - T_{ij} q_{j,L}^{dyn}(\omega) - T_{ij} m_{jk,O}^{dyn} \ddot{x}_{Ok,L} \quad (16)$$

$$i, j, k, l=1,2,\dots,p.$$

Let us write the specific equation for an element that we number with *e*. Let *ne* be the total number of finite elements into which the structure is discretized.

$$m_{ik}^e = T_{ij} m_{jk} T_{lk} ; \quad c_{ik}^e = T_{ij} c_{jk} T_{lk} ;$$

$$k_{ik}^e = T_{ij} k_{jk} T_{lk} ; \quad k_{ik}^e(\varepsilon) = T_{ij} k_{jk}(\varepsilon) T_{lk} ;$$

$$k_{ik}^e(\omega) = T_{ij} k_{jk}(\omega) T_{lk} ; \quad q_{j,G}^{c,e} = T_{ij} q_{j,L}^c ; \quad (17)$$

$$q_{j,G}^{d,e} = T_{ij} q_{j,L}^d ; \quad q_{j,G}^{dyn,e}(\varepsilon) = -T_{ij} q_{j,L}^{dyn}(\varepsilon) ;$$

$$q_{j,G}^{dyn,e}(\omega) = -T_{ij} q_{j,L}^{dyn}(\omega) ; \quad q_{j,G}^{a,e} = -T_{ij} m_{jk,O}^{dyn} \ddot{x}_{Ok,L}$$

$$i, j, k=1,2,\dots,p .$$

so:

$$m_{ij}^e \ddot{\delta}_{j,G} + c_{ij}^e \dot{\delta}_{j,G} + \left(k_{ij}^e + k_{ij}^e(\varepsilon) + k_{ij}^e(\omega) \right) \delta_{j,G} = q_{j,G}^{c,e} + q_{j,G}^{d,e} + q_{j,G}^{dyn,e}(\varepsilon) + q_{j,G}^{dyn,e}(\omega) + q_{j,G}^{a,e} \quad (18)$$

$$i, j=1,2,\dots,p .$$

To assemble the equations of motion for the entire structure, we will have to take into account the nodes of the finite elements that introduce independent coordinates and the connections that occur between the finite elements considered (the contact between two or more finite elements through the nodes that define them). If it is denoted with:

$$\delta_{i,G}^e = G_{ij}^e \Delta_j ; \quad (19)$$

$$j=1,2,\dots,p ; j=1,2,\dots,np.$$

where $\Delta_{ij}, i=1,2,\dots,np$ represent the independent nodal coordinates for the entire structure and np the total number of the independent coordinates (the number of DOF). Using substitution (19), the motion equations (18) become:

$$m_{ij}^e G_{jk}^e \ddot{\Delta}_k + c_{ij}^e G_{jk}^e \dot{\Delta}_k + (k_{ik}^e + k_{ik}^e(\varepsilon) + k_{ik}^e(\omega)) G_{ij}^e \Delta_j = q_{j,G}^{c,e} + q_{j,G}^{d,e} + q_{j,G}^{dyn,e}(\varepsilon) + q_{j,G}^{dyn,e}(\omega) + q_{j,G}^{a,e} \quad (20)$$

$$i,j,k=1,2,\dots,p.$$

The equilibrium equations are written in the space of dependent coordinates. To bring them into the space of independent coordinates, they are multiplied by G_{kj}^e obtaining:

$$G_{ij}^e m_{jk}^e G_{LK}^e \ddot{\Delta}_L + G_{ij}^e c_{jk}^e G_{LK}^e \dot{\Delta}_L + G_{ij}^e q_{j,G}^{c,e} + G_{ij}^e q_{j,G}^{d,e} + G_{ij}^e q_{j,G}^{dyn,e}(\varepsilon) + G_{ij}^e (k_{jk}^e + k_{jk}^e(\varepsilon) + k_{jk}^e(\omega)) G_{kl}^e \Delta_l = G_{ij}^e q_{j,G}^{dyn,e}(\omega) + G_{ij}^e q_{j,G}^{a,e} \quad (21)$$

$$i,j,k=1,2,\dots,p.$$

It is denoted:

$$M_{ij} = G_{ij}^e m_{jk}^e G_{lk}^e ; \quad C_{ik} = G_{ij}^e c_{jk}^e G_{lk}^e ;$$

$$K_{ik}^e = G_{ij}^e k_{jk}^e G_{lk}^e ; \quad K_{ij}(\varepsilon) = G_{ij}^e k_{jk}^e(\varepsilon) G_{lk}^e ;$$

$$K_{ij}(\omega) = G_{ij}^e k_{jk}^e(\omega) G_{lk}^e ; \quad Q_i^c = G_{ij}^e q_{j,L}^c ; \quad (22)$$

$$Q_i^d = G_{ij}^e q_{j,G}^{d,e} ; \quad Q_i^{dyn,e}(\varepsilon) = -G_{ij}^e q_{j,G}^{dyn,e}(\varepsilon) ;$$

$$Q_i^{dyn}(\omega) = -G_{ij}^e q_{j,G}^{dyn}(\omega) ; \quad Q_i^a = -G_{ij}^e m_{jk}^{dyn} \ddot{x}_{Ok,L}$$

$$i,j,k,l=1,2,\dots,p.$$

So, the final form of the motion equations becomes

$$M_{ij} \ddot{\Delta}_j + C_{ij} \dot{\Delta}_j + (K_{ij} + K_{ij}(\varepsilon) + K_{ij}(\omega)) \Delta_j = Q_i^c + Q_i^d + Q_i^{dyn}(\varepsilon) + Q_i^{dyn}(\omega) + Q_i^a \quad (23)$$

$$i,j=1,2,\dots,np.$$

If viscous damping is included in the equations of motion, the previous reasoning still applies. However, the original equations (such as equation 23) will now have an additional term representing the viscous damping effect. This term is expressed in terms of the modal coordinates. Consequently, solving the system with viscous damping reduces to solving a modified system in modal coordinates, where the damping term is incorporated. This approach preserves the modal analysis framework while accounting for the damping forces.

$$M_1 \ddot{\phi}_1 + C_1 \dot{\phi}_1 + K_1 \phi_1 = Q_1$$

$$M_2 \ddot{\phi}_2 + C_2 \dot{\phi}_2 + K_2 \phi_2 = Q_2$$

$$\dots\dots\dots$$

$$M_n \ddot{\phi}_n + C_n \dot{\phi}_n + K_n \phi_n = Q_n \quad (24)$$

$$i,j,k=1,2,\dots,np.$$

and ϕ_1 represents the modal coordinates. Starting from Eq. (24) the classic modal analysis can be applied to obtain the solution motion equation to obtain the vibration of the structure.

The method proposed in this research can be used to quickly estimate the motion equations for a mycelium composite. The technique could be useful to designers in gathering the information needed for the design process. This method uses FEM, a computational method that designers are familiar with and that produces precise outcomes. The eigenpulsations for the specimen under examination are then used to compute the shear modulus. It is necessary to comprehend the properties of the phases that make up the composite. This method's speed and convenience of use are two reasons to employ it during the design phase. The assumptions and approximations used in the conventional calculation of the straight beam, as well as FEM errors, which are essentially an approximation method, have the largest influence.

5. DISCUSSION AND CONCLUSIONS

Current industrial development benefits from the use of new materials with new properties, recyclability, ease of manufacture, and low cost. Among them is the composite mycelium, described in this paper. The emergence of these new materials, with very varied possibilities of use, also requires

the development of research that can provide an image of the behavior of these materials under different conditions of mechanical stress. For this, new calculation models are developed that can describe as accurately as possible what happens to the material under different types of mechanical stress. The model developed in this paper is part of this line of research. A finite element model appropriate to the type of composite mycelium material is proposed to describe the behavior of the material in the event of vibrations that stress the structure. For this, Lagrange's equations are used, along with the classical finite element description of a linear elastic structure. The equations of motion in terms of nodal displacements are thus obtained. On this model and description, it is then possible to apply classical finite element calculation methods to study the vibrations of the structure.

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