
Inverse Vibration and Optimization Approach for Estimating Young's Modulus of Structural Steel

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Abstract: - Accurate estimation of Young's modulus is essential for reliable structural analysis and design. This study presents an integrated vibration-based inverse optimization methodology for estimating the Young's modulus of structural steel cantilever beams through the combined use of experimental modal analysis, analytical modelling, finite element analysis, and static deflection testing. Free vibration experiments were performed on nineteen steel cantilever beam specimens using an impact hammer and the FFT analyzer to determine the first and second bending natural frequencies. In addition, a static deflection experiment was conducted on a representative beam specimen to experimentally determine the Young's modulus based on Euler–Bernoulli beam theory. An analytical model was developed to predict the natural frequencies, while a finite element model was established to validate both the static deflection and modal responses. The experimentally measured natural frequencies were subsequently used in an inverse optimization framework to estimate the Young's modulus by minimizing the squared error between the measured and predicted frequencies. The analytical and finite element predictions showed good agreement with the experimental results for both vibration modes over the investigated range of beam lengths. The Young's modulus estimated using the proposed optimization approach was 177 GPa, compared with 181.7 GPa obtained from the static deflection experiment, corresponding to a percentage error of only 2.59%. The close agreement between the independent approaches demonstrates the accuracy and reliability of the proposed methodology. The developed framework provides a practical, computationally efficient, and non-destructive approach for material characterization and has potential applications in structural health monitoring and condition assessment of beam-like engineering structures.

Keywords: - Inverse vibration analysis; Young's modulus estimation; Euler–Bernoulli beam theory; Natural frequencies; Optimization; and Modal analysis.

1. INTRODUCTION

Structural steel is extensively used in engineering structures owing to its high strength, stiffness,

durability, and ductility. Accurate determination of its Young's modulus is essential for structural design, finite element analysis, vibration studies, and

structural health monitoring. Conventional techniques such as tensile and static bending tests are often destructive, time-consuming, and unsuitable for in-service structures. Consequently, vibration-based inverse approaches have gained considerable attention as reliable non-destructive methods for estimating material properties from measured dynamic responses. Finite element model updating has become an important tool for identifying unknown structural parameters. Friswell and Mottershead [1] established the fundamental framework for updating finite element models using measured vibration data, while Mottershead et al. [2] developed a sensitivity-based model updating technique that significantly improved the estimation of stiffness and mass parameters. Hemez and Doebling [3] further reviewed model updating strategies for nonlinear and transient dynamic systems, highlighting their importance in structural parameter identification. Reliable modal parameter extraction is essential for inverse vibration analysis. Reynders [4] reviewed operational modal analysis and output-only system identification methods suitable for in-situ structural evaluation. Ewins [5] presented the theoretical foundations and practical procedures of experimental modal analysis, whereas Bendat and Piersol [6] introduced advanced FFT-based signal processing techniques for accurate extraction of natural frequencies from vibration measurements. The theoretical development of inverse vibration methods has been widely investigated. Gladwell [7] established the mathematical foundation of inverse eigenvalue problems for structural parameter identification. Marinov and Vatsala [8] proposed inverse formulations based on the Euler–Bernoulli beam theory, while Huang [9] demonstrated optimization-based parameter estimation using measured vibration responses. Several researchers have employed vibration techniques for estimating Young's modulus. Chirikov et al. [10] determined the dynamic Young's modulus and Poisson's ratio using higher-order beam vibration frequencies, improving estimation accuracy. Kumar and Singh [11] experimentally estimated Young's modulus from natural frequency measurements using beam theory. Apsan et al. [12] proposed a non-destructive free vibration method for evaluating Young's modulus through inverse comparison of experimental and analytical frequencies, whereas Podgórski and Kawecki [13] successfully applied vibration analysis to determine the modulus of elasticity of non-uniform cantilever beams. Optimization techniques have further enhanced inverse vibration analysis. Balcı and Gündoğdu [14] combined finite element modeling with optimization

algorithms for material property estimation. Öz and Gündoğdu [15] simultaneously identified elastic modulus and axial force using modal information. Chiwiacowsky et al. [16] developed a variational inverse vibration methodology, while Smyl [17] proposed an inverse optimization framework for estimating elastic properties under multiple loading conditions. Recent studies have demonstrated the applicability of inverse methods for metallic structures. Faraji et al. [18] accurately determined the Young's modulus of steel alloys using acoustic emission measurements and inverse analysis, with excellent agreement with tensile tests. Loh et al. [19] integrated finite element analysis, digital image correlation, and modal testing to estimate the static and dynamic Young's modulus of cantilever beams using a non-contact approach. Barboni et al. [20] presented a precise vibration-based method for determining Young's modulus from dynamic measurements, whereas Baveja [21] demonstrated an early dynamic technique for measuring Young's modulus based on vibration characteristics. To improve the accuracy of natural frequency estimation, Mituletu et al. [22] proposed a swept-sine acoustic excitation technique that provides more accurate frequency identification than conventional FFT peak-picking methods.

Although recent studies have significantly improved the accuracy of Young's modulus and frequency estimation [20–22], relatively few investigations have integrated accurate experimental frequency extraction with a simple inverse optimization framework for estimating Young's modulus of structural steel using multiple beam specimens. Therefore, the present study proposes a vibration-based inverse optimization methodology for estimating the Young's modulus of ASTM A36 structural steel using nineteen cantilever beam specimens of different lengths. Experimental natural frequencies obtained from free vibration testing are combined with Euler-Bernoulli beam theory, and Young's modulus is identified through mean square error minimization using the Excel Solver optimization tool. The estimated Young's modulus is subsequently validated through static deflection experiments and finite element analysis to verify the reliability of the proposed methodology. The proposed approach provides a simple, accurate, computationally efficient, and non-destructive framework for material property estimation that can be readily applied in laboratory investigations as well as practical structural engineering applications.

1.1. Objectives of the study are as follows:

(1) to experimentally determine the Young's modulus of structural steel cantilever beams using

the static deflection method based on Euler–Bernoulli beam theory; (2) to develop a vibration-based inverse optimization methodology for estimating the Young's modulus from experimentally measured natural frequencies; (3) to establish analytical and finite element models for predicting the static and dynamic behaviour of structural steel cantilever beams; (4) to validate the proposed methodology by comparing the predicted natural frequencies and Young's modulus with analytical, finite element, and experimental results; and (5) to evaluate the feasibility of the proposed non-destructive vibration-based approach as an alternative to conventional static material characterization techniques.

1.2. The novelty and major contributions of the present study are as follows:

(1) a comprehensive framework integrating experimental modal analysis, analytical modelling, finite element analysis, inverse optimization, and static deflection testing is developed for the non-destructive estimation and validation of the Young's modulus of structural steel cantilever beams; (2) the proposed vibration-based inverse optimization framework estimates the Young's modulus by minimizing the discrepancy between experimentally measured and analytically predicted natural frequencies, while the predicted Young's modulus is independently verified through conventional static deflection measurements, providing dual experimental validation of the proposed methodology; and (3) the integrated analytical, finite element, and optimization framework provides a practical and computationally efficient methodology for non-destructive material characterization, with potential applications to beam-like engineering

components for structural health monitoring and condition assessment.

2. METHOD AND MATERIALS

2.1. Details of the Prepared Test Specimens

The present research study considered twenty specimens of different lengths of structural steel material grade ASTM A36. The lengths of the specimens varied from 0.336 m to 1.2 m at an interval of 0.048 m. The cross-sectional area of the specimens or beam is $0.016 \times 0.016 \text{ m}^2$, respectively. The set of nineteen specimens is shown in Figure 1. A specimen with a length of 1.130 m was used in the experiment to measure its static displacement and subsequently determine Young's modulus. The specimen is shown in Figure 1.

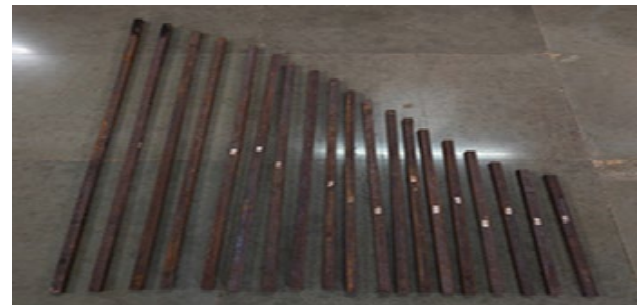


Figure 1. Steel Specimens with Different Lengths Used in the Experiment

Table 1 shows that ASTM A36 steel is primarily composed of iron with low carbon content and small amounts of manganese, phosphorus, sulfur, and silicon, making it suitable for structural applications due to its good strength and ductility.

Table 1. Chemical Composition of ASTM A36 Structural Steel (% by weight)

Element	Carbon (C)	Manganese (Mn)	Phosphorus (P)	Sulphur (S)	Silicon (Si)	Iron (Fe)
% (max/typical)	≤ 0.25–0.29	0.80–1.20	≤ 0.040	≤ 0.050	0.15–0.40	Balance (~98%)

2.2. Experimental Vibration Analysis

Free vibration experiments were conducted to determine the natural frequencies of the cantilever beam specimens. Each specimen was rigidly clamped at one end to establish cantilever boundary conditions. The overall experimental arrangement comprised a four-channel Brüel & Kjær FFT analyzer, a Dewesoft frequency response function (FRF) data acquisition system, an instrumented impact hammer, an accelerometer, and a laptop for signal acquisition and processing. The complete experimental setup used for the vibration measurements is shown in Figure 2 (a).

The beam specimens were excited in the bending direction using the instrumented impact hammer, while the corresponding vibration response was measured using an accelerometer attached to the beam surface with beeswax. To obtain reliable modal responses, the accelerometer was positioned at suitable locations along the beam during the measurements. A detailed view of the vibration measurement instruments employed in the experimental setup is presented in Figure 2 (b).



Figure 2a. Experimental setup for vibration analysis



Figure 2b. Detailed view of the instruments

Table 2. Specifications of the vibration instruments

Parameter	FFT Analyzer	Accelerometer	Impact Hammer
Make / Type	Brüel & Kjær, 4-channel FFT Analyzer	Type 4514 (S/N: 57125)	Type 8206 (S/N: 55940)
Frequency range	1 Hz–26.5 kHz	Mounted resonance frequency: 32 kHz	5 kHz
Frequency accuracy	0.00025%	—	—
Frequency resolution	1 mHz	—	—
Phase resolution	100 mdeg	—	—
Sensitivity	—	—	22.5 mV/N
Measurement range	—	—	Full scale: 222 N; Maximum force: 4448 N
Weight	—	8.7 g	Head mass: 100/140 g
Operating temperature	10–55 °C	–51 to +121 °C	–73 to +60 °C

The principal technical specifications of the vibration instruments are summarized in Table 2. The Brüel & Kjær FFT analyzer provides a frequency range of 1 Hz–26.5 kHz with a frequency resolution of 1 mHz and an accuracy of 0.00025%, ensuring precise acquisition of vibration signals. The lightweight accelerometer, having a mounted resonance frequency of 32 kHz and a mass of 8.7 g, minimizes mass-loading effects during measurements.

The instrumented impact hammer, with a sensitivity of 22.5 mV/N and a maximum force capacity of 4448 N, provides consistent excitation for modal testing.

The excitation and response signals were acquired over a frequency range of 0–1000 Hz using Pulse data acquisition software. The recorded time-domain signals were subsequently transformed into the frequency domain using the Fast Fourier Transform (FFT). The processed frequency spectra were then used to identify the natural frequencies of the cantilever beam specimens, which were subsequently employed for analytical, numerical, and optimization-based validation.

2.3. Experimental Setup and Measurement Procedure

The experimental procedure consisted of measuring the geometric dimensions, material density, and static end deflection of the cantilever beam specimens. Initially, the length (L), width (b), and thickness (t) of each steel beam specimen were measured using standard measuring instruments. These dimensions were subsequently used to calculate the cross-sectional area (A) and the second moment of area (I) of the beam cross-section.

The material density (ρ) was determined by measuring the mass (m) of each specimen using the precision weighing instrument shown in Figure 3. The density of each beam specimen was calculated using

$$\rho = \frac{m}{b t L} \quad (1)$$

where (m), (b), (t), and (L) represent the mass, width, thickness, and length of the beam specimen, respectively. Three identical beam specimens were tested, and the average density was used for all subsequent calculations to minimize experimental uncertainty. The experimentally determined average density of the steel specimens was 7895.8 kg/m³.



Figure 3. Photograph of the precision weighing instrument used for measuring the mass of the beam specimens

The static deflection experiment was performed using the experimental setup shown in Figure 4, which consisted of a rigid clamping fixture, a steel beam specimen, a dial gauge with a least count of 1 μm , and a dead load. One end of the beam specimen was rigidly clamped to establish cantilever boundary conditions, while the dial gauge was positioned at the free end to measure the vertical deflection.

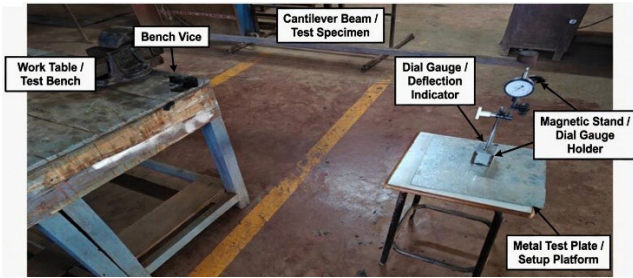


Figure 4. Experimental setup for static deflection measurements

Initially, the deflection produced solely by the self-weight of the beam was measured. Subsequently, a concentrated load of 0.165 kg (1.608 N) was applied at the free end of the beam, and the corresponding total deflection due to the combined effect of self-weight and the applied load was recorded. The measured self-weight deflection was 0.00437 m, whereas the total deflection under the combined action of self-weight and the applied load was 0.00480 m.

2.4. Static Deflection-Based Determination of Young's Modulus

The Young's modulus of the beam specimens was determined using the experimentally measured parameters together with Euler-Bernoulli beam theory. Considering the beam as a cantilever subjected to its self-weight and a concentrated load at the free end, the deflection due to the self-weight is expressed as

$$\delta_{selfweight} = \frac{\rho A g L^4}{8EI} \quad (2)$$

whereas the deflection due to the concentrated end load is given by

$$\delta_{pointload} = \frac{PL^3}{3EI} \quad (3)$$

Accordingly, the total deflection of the cantilever beam can be expressed as

$$\delta_{totaldeflection} = \frac{\rho A g L^4}{8EI} + \frac{PL^3}{3EI} \quad (4)$$

where (P) is the applied point load, (ρ) is the material density, (A) is the cross-sectional area, (g) is the acceleration due to gravity, (L) is the beam length, (I) is the second moment of area of the beam cross-section, and (E) is the Young's modulus.

Since all the parameters in Eq. (4), including the experimentally measured total deflection, are known, Young's modulus is the only unknown parameter. Therefore, Eq. (4) was rearranged to obtain

$$E = \frac{\left(\frac{\rho A g L^4}{8}\right) + \left(\frac{PL^3}{3}\right)}{(I * \delta_{total deflection})} \quad (5)$$

Finally, the experimentally measured geometric dimensions, average material density, and total deflection were substituted into Eq. (5) to determine the Young's modulus of the beam specimens.

The uniformly distributed load (UDL) acting on the cantilever beam due to its self-weight was determined from the beam mass. The beam mass was calculated from its measured geometric dimensions and material density using Eq. (6):

$$m = L b t \rho \quad (6)$$

The corresponding self-weight (W) of the beam was then determined as

$$W = m g \quad (7)$$

The equivalent uniformly distributed load (UDL) acting along the entire beam length was subsequently calculated as

$$w = (m g) / (L) \quad (8)$$

where (w) denotes the equivalent uniformly distributed load applied along the beam length in the finite element model

2.5. Procedure for Estimation of Young's Modulus of Steel Specimens Using Vibration-Based Optimization

In this study, the Young's modulus of the steel specimens was estimated using an inverse vibration-based optimization approach. The method is based on the relationship between the elastic stiffness of a beam and its natural frequencies. Since the dynamic response of a beam is strongly influenced by its material stiffness, the Young's modulus was identified by minimizing the difference between the

experimentally measured and theoretically predicted natural frequencies.

The discrepancy between the experimental and analytical natural frequencies was quantified using the sum of squared errors (SSE), which was adopted as the objective function:

$$SSE = \sum_{i=1}^n (f_i^{exp} - f_i^{theo})^2 \quad (9)$$

Here, f_i^{exp} and f_i^{theo} represent the experimental and theoretical natural frequencies, respectively, corresponding to the i^{th} bending mode. The optimum Young's modulus was obtained by minimizing this error function.

The theoretical natural frequencies were calculated using Euler–Bernoulli beam theory, assuming linear elastic material behavior, a uniform cross-section, and negligible shear deformation. The first and second bending natural frequencies were determined using:

$$f_{n1} = \left(\frac{1.875^2}{2\pi L^2} \right) * \sqrt{\frac{EI}{\rho A}} \quad (10)$$

$$f_{n2} = \left(\frac{4.694^2}{2\pi L^2} \right) * \sqrt{\frac{EI}{\rho A}} \quad (11)$$

where f_{n1} and f_{n2} are the first and second natural frequencies [24], respectively. L is the beam length; E is Young's modulus; I is the second moment of area; ρ is the material density; and A is the cross-sectional area of the specimen. For each steel specimen, the measured geometric dimensions and material density were incorporated into the analytical model. Young's modulus was treated as the unknown design variable, while all other parameters were maintained at their experimentally measured values. The theoretical natural frequencies were initially computed using an estimated value of Young's modulus and subsequently compared with the corresponding experimental frequencies.

The optimization was carried out using the Solver tool in Microsoft Excel. During each iteration, the Solver adjusted the value of Young's modulus to minimize the SSE between the experimental and analytical natural frequencies. The Generalized Reduced Gradient (GRG) nonlinear optimization algorithm was employed because of the nonlinear

relationship between Young's modulus and natural frequency. Suitable upper and lower bounds were imposed on Young's modulus to ensure a physically meaningful solution. The optimization process was considered to have converged when successive changes in both the objective function and the design variable became negligible. The final optimized value represents the Young's modulus of the steel specimens corresponding to the minimum value of the objective function. The optimized Young's modulus was subsequently compared with the Young's modulus obtained using the experimental static deflection method to validate the proposed vibration-based optimization approach.

2.6. Finite Element Modeling

Finite element simulations were performed using ANSYS Mechanical APDL [23]. A finite element model of the cantilever beam was developed in ANSYS Classic using the BEAM3 element. The beam geometry was defined according to the experimental specimen, with a length of 1.130 m and a square cross-section of 0.016 m $\times$ 0.016 m. The corresponding cross-sectional area and second moment of area were specified as the real constants of the BEAM3 element. The finite element model was discretized into 50 BEAM3 elements to achieve adequate numerical accuracy. The beam material was assumed to be homogeneous, isotropic, and linearly elastic. The experimentally determined Young's modulus obtained from the static deflection method was assigned as the elastic modulus of the material. A Poisson's ratio of 0.30 was adopted for structural steel, and the material density was specified according to the experimental specimen. One end of the beam was fully constrained to simulate the cantilever boundary condition. The beam mass was calculated from its geometric dimensions and material density, and the corresponding self-weight was applied as an equivalent uniformly distributed load (UDL) of 19.82 N/m along the entire beam length. In addition, a concentrated load of 1.60884 N was applied at the free end to replicate the experimental loading condition.

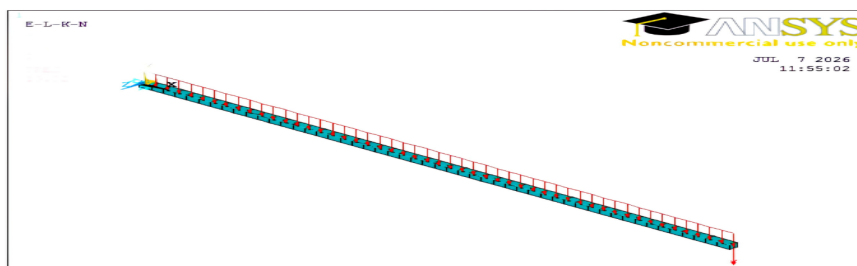


Figure 5. Finite Element Model of a 1.130 m Long Cantilever Beam with Boundary Condition and Applied Loads

Static structural and modal analyses were subsequently performed using the developed finite element model. Figure 5 illustrates the finite element model, including the cantilevered boundary conditions and the applied loading.

3. RESULTS AND DISCUSSION

The proposed vibration-based methodology was validated through experimental measurements, analytical calculations, optimization, and finite element analysis. Free vibration tests were performed on twenty steel cantilever beam specimens using a Dewesoft frequency response function (FRF) system equipped with a four-channel FFT analyzer to determine the first and second bending natural frequencies. The experimental natural frequencies of nineteen specimens were used in the optimization procedure for estimating Young's modulus. The remaining specimen, having a length of 1.130 m, was excluded from the optimization and used exclusively for validation of the analytical and finite element predictions of the first and second bending natural frequencies. Subsequently, the same specimen was subjected to static deflection tests under self-weight and an end point load to determine Young's modulus experimentally. The experimentally measured natural frequencies were compared with the corresponding analytical and finite element results to assess the accuracy and reliability of the proposed methodology. The

following subsections present the validation of the first and second bending natural frequencies, the optimization results, the experimental FFT responses, and the finite element analysis.

3.1. Validation of the First Bending Natural Frequency

The first bending natural frequencies obtained from the experimental, analytical, and finite element approaches are presented in Table 3. The finite element predictions show excellent agreement with the experimentally measured frequencies over the entire range of beam lengths investigated. The maximum absolute percentage error between the finite element and experimental results is 6.92%, while the difference between the finite element and analytical predictions remains nearly constant within the range of 1.33–1.37%.

As expected from Euler–Bernoulli beam theory, the first bending natural frequency decreases with increasing beam length owing to the reduction in flexural stiffness. The close agreement among the three approaches demonstrates that the developed finite element model accurately predicts the fundamental vibration characteristics of the cantilever beam. Furthermore, the small deviation between the analytical and numerical results confirms the validity of the analytical formulation adopted in the present study.

Table 3. Validation of the finite element model through comparison of the first bending natural frequencies obtained from finite element, experimental, and analytical approaches

Specimen Id	Beam Length, L (m)	f ₁ FEA(Hz)	f ₁ Exp (Hz)	f ₁ Analytical (Hz)	Error (FEA-Exp.) (%)	Error (FEA-Analytical) (%)
1	0.336	109.78	107	108.34	2.598	1.331
2	0.384	84.056	78.75	82.95	6.738	1.338
3	0.432	66.419	67.5	65.54	-1.601	1.345
4	0.48	53.802	53.75	53.09	0.097	1.350
5	0.528	44.467	44.5	43.87	-0.074	1.356
6	0.576	37.365	39	36.86	-4.192	1.357
7	0.624	31.839	32	31.41	-0.503	1.361
8	0.672	27.453	29	27.08	-5.334	1.361
9	0.72	23.915	23.75	23.59	0.695	1.363
10	0.768	21.019	22	20.74	-4.459	1.362
11	0.816	18.619	18.5	18.37	0.643	1.363
12	0.864	16.608	17	16.38	-2.306	1.365
13	0.912	14.906	15.5	14.71	-3.832	1.366
14	0.96	13.453	14	13.27	-3.907	1.368
15	1.008	12.202	12.5	12.04	-2.384	1.367
16	1.056	11.118	11.5	10.97	-3.322	1.367
17	1.104	10.172	10.25	10.04	-0.761	1.364
18	1.152	9.3424	9.75	9.22	-4.181	1.369
19	1.2	8.61	9.25	8.49	-6.919	1.369

3.2. Validation of the Second Bending Natural Frequency

The comparison of the second bending natural frequencies obtained using the experimental, analytical, and finite element methods is presented in Table 4. Similar to the first vibration mode, the finite element predictions closely match the experimental measurements throughout the investigated beam lengths. The maximum absolute percentage error between the experimental and numerical results is 6.99%, whereas the difference between the analytical and finite element results

varies only between 1.06% and 1.34%. The second bending mode follows the same trend as the first mode, exhibiting a gradual reduction in natural frequency with increasing beam length. The consistently low percentage errors indicate that the developed finite element model accurately captures the higher-order dynamic behaviour of the cantilever beam. The agreement obtained for both vibration modes confirms the reliability of the analytical formulation and numerical modelling procedure adopted in this investigation.

Table 4. Validation of the finite element model through comparison of the second bending natural frequencies obtained from finite element, experimental, and analytical approaches

Specimen Id	Beam Length, L (m)	f_2 FEA(Hz)	f_2 Exp (Hz)	f_2 Analytical (Hz)	Error (FEA-Exp.) (%)	Error (FEA-Analytical) (%)
1	0.336	686.16	673	678.99	1.955	1.056
2	0.384	525.72	497.5	519.85	5.672	1.129
3	0.432	415.58	420	410.75	-1.052	1.177
4	0.48	336.74	336.3	332.70	0.131	1.213
5	0.528	278.37	283	274.96	-1.636	1.239
6	0.576	233.96	244	231.04	-4.115	1.262
7	0.624	199.38	198	196.87	0.697	1.277
8	0.672	171.93	180	169.75	-4.483	1.286
9	0.72	149.79	148.5	147.87	0.869	1.299
10	0.768	131.66	137.5	129.96	-4.247	1.306
11	0.816	116.63	117	115.12	-0.316	1.309
12	0.864	104.04	107	102.69	-2.766	1.318
13	0.912	93.381	96	92.16	-2.728	1.323
14	0.96	84.28	88	83.18	-4.227	1.327
15	1.008	76.447	78.5	75.44	-2.615	1.331
16	1.056	69.658	73	68.74	-4.578	1.335
17	1.104	63.734	65.25	62.89	-2.323	1.337
18	1.152	58.535	60.75	57.76	-3.646	1.340
19	1.2	53.947	58	53.23	-6.988	1.342

3.3. Validation of the Optimization Approach (Squared Error Analysis)

The effectiveness of the proposed optimization approach was evaluated by minimizing the squared error between the experimentally measured and analytically predicted natural frequencies. The optimized results for both the first and second bending modes are summarized in Table 5.

The squared error values remain relatively small for most beam specimens, indicating that the optimization algorithm accurately predicts the natural frequencies over the entire range of beam

lengths considered in this study. Although slightly larger squared errors are observed for a few specimens, particularly for the second bending mode, the overall agreement between the experimental and predicted frequencies demonstrates that the optimization procedure successfully minimizes the objective function and provides reliable estimates of the beam's dynamic characteristics. These results confirm the suitability of the optimization technique for estimating the effective Young's modulus from experimentally measured vibration data.

Table 5. Squared Error Minimization for Experimental and Predicted Natural Frequencies

Beam Sample	Beam length L (m)	f ₁ Exp. (Hz)	f ₁ pred. (Hz)	Squared Error	f ₂ Exp. (Hz)	f ₂ pred. (Hz)	Squared Error
1	0.3360	107.00	108.3377	1.7894	673.00	678.9886	35.8638
2	0.3840	78.75	82.9460	17.6068	497.50	519.8507	499.552
3	0.4320	67.50	65.5376	3.8509	420.00	410.7462	85.6325
4	0.4800	53.75	53.0855	0.4416	336.30	332.7044	12.9281
5	0.5280	44.50	43.8723	0.3940	283.00	274.9623	64.6039
6	0.5760	39.00	36.8649	4.5586	244.00	231.0447	167.838
7	0.6240	32.00	31.4115	0.3463	198.00	196.8665	1.2848
8	0.6720	29.00	27.0844	3.6694	180.00	169.7472	105.120
9	0.7200	23.75	23.5935	0.0245	148.50	147.8686	0.3986
10	0.7680	22.00	20.7365	1.5964	137.50	129.9627	56.8113
11	0.8160	18.50	18.3687	0.0172	117.00	115.1226	3.5245
12	0.8640	17.00	16.3844	0.3790	107.00	102.6866	18.6058
13	0.9120	15.50	14.7051	0.6318	96.00	92.1619	14.7311
14	0.9600	14.00	13.2714	0.5309	88.00	83.1761	23.2699
15	1.0080	12.50	12.0375	0.2139	78.50	75.4432	9.3441
16	1.0560	11.50	10.9681	0.2829	73.00	68.7406	18.1426
17	1.1040	10.25	10.0351	0.0462	65.2500	62.8931	5.5550
18	1.1520	9.75	9.2162	0.2849	60.7500	57.7612	8.9330
19	1.2000	9.25	8.4937	0.5720	58.0000	53.2327	22.7271

3.4. Experimental FFT Response

The experimentally measured frequency responses obtained using the FFT analyzer are presented in Figures 6–9. Figures 6 and 7 illustrate the first and second bending natural frequencies of the cantilever beam having a length of 1.20 m, while Figures 8 and 9 present the corresponding FFT autospectrum responses for the 1.130 m long cantilever beam specimen. The dominant resonance peaks observed in the FFT spectra clearly identify the first and second bending vibration modes of the beam. These resonance frequencies were extracted

directly from the frequency response functions and used for subsequent analytical and numerical validation. The 1.130 m long beam specimen was also employed in the static deflection experiment for determining the experimental Young's modulus, thereby providing a common basis for validating both the static and dynamic analyses. The clear identification of the resonance peaks demonstrates the effectiveness of the experimental vibration testing procedure and provides reliable input data for the vibration-based estimation of the beam's mechanical properties.

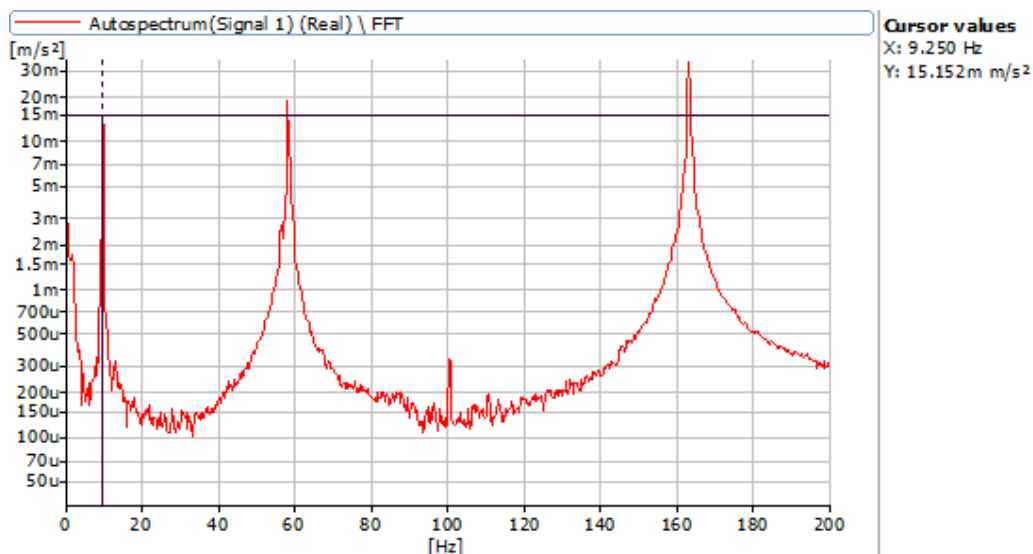


Figure 6. Experimental FFT spectrum of the 1.2 m long cantilever beam, with the cursor positioned at the first resonance peak to identify the first natural frequency

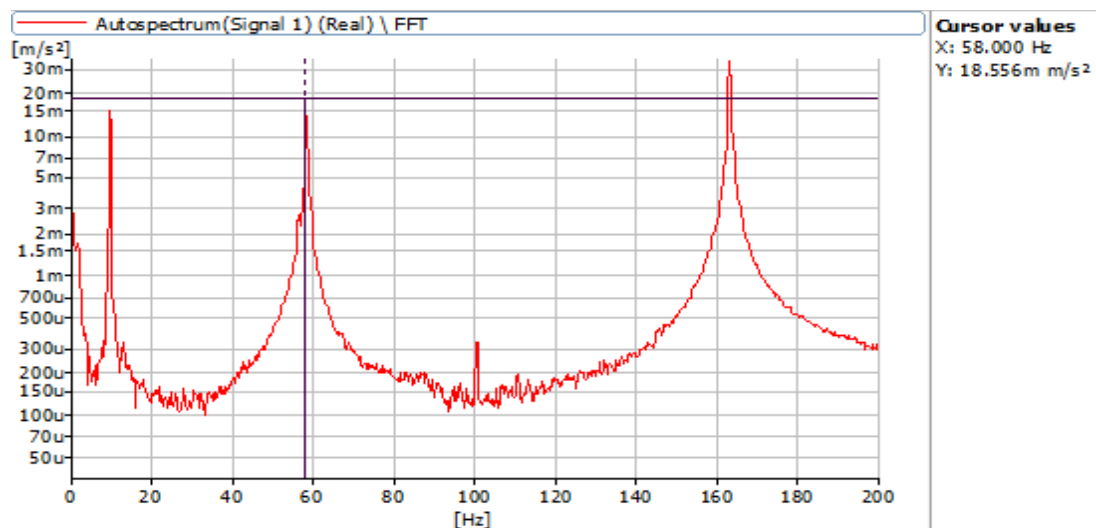


Figure 7. Experimental FFT spectrum of the 1.2 m long cantilever beam, with the cursor positioned at the second resonance peak to identify the second natural frequency

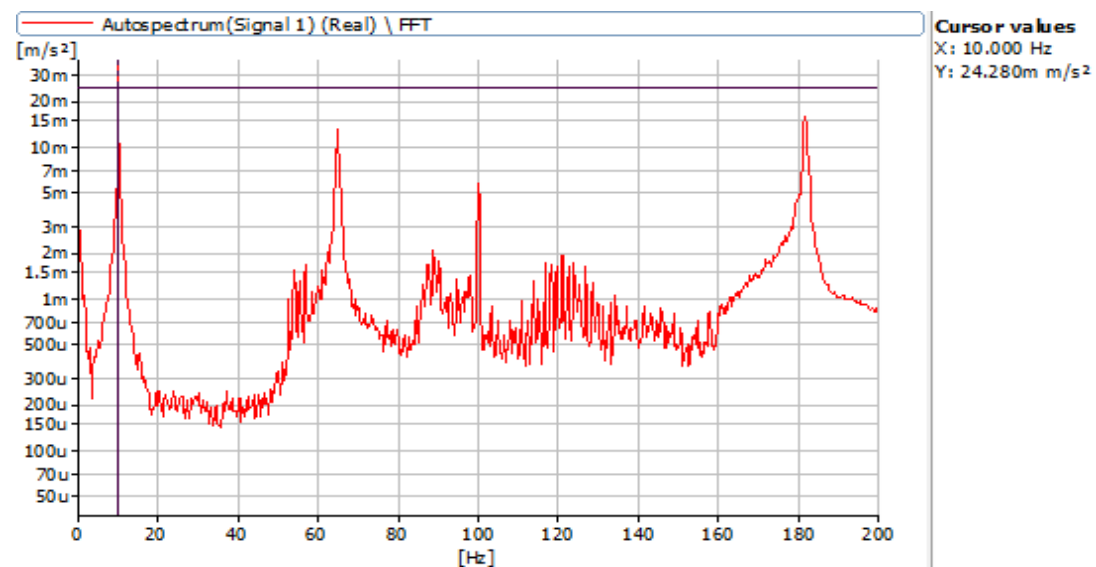


Figure 8. Experimental FFT spectrum of the 1.130 m long cantilever beam, with the cursor positioned at the first resonance peak to identify the first natural frequency

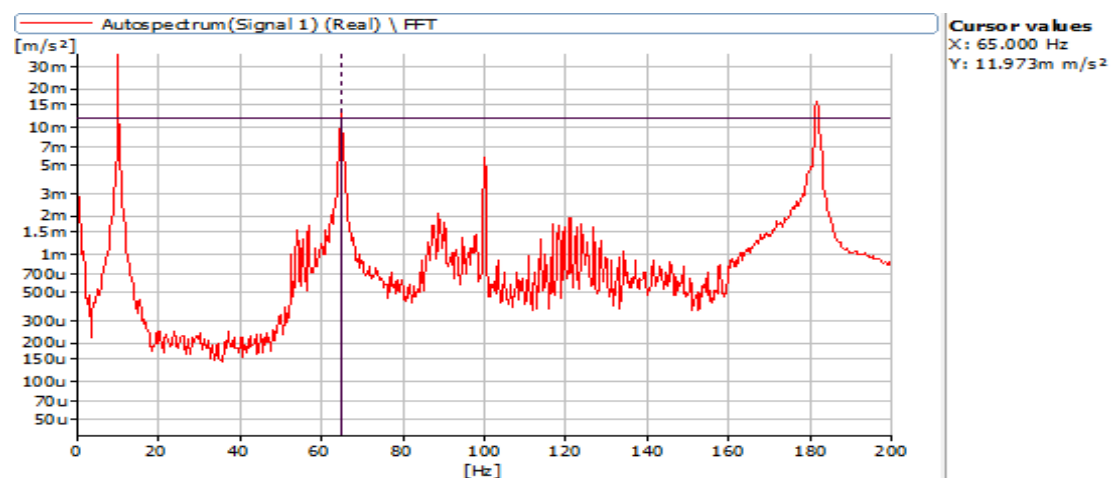


Figure 9. Experimental FFT spectrum of the 1.130 m long cantilever beam, with the cursor positioned at the second resonance peak to identify the second natural frequency

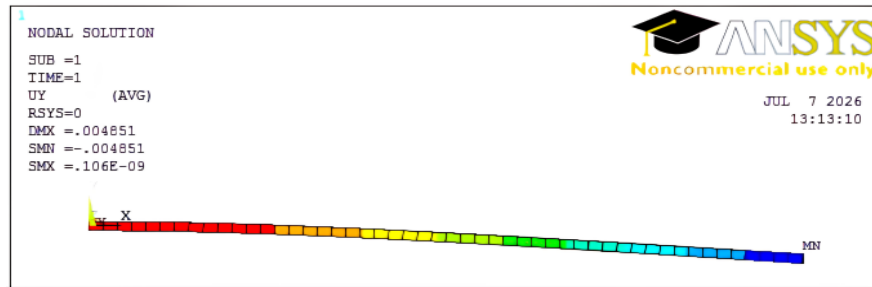


Figure 10. Static Deflection Response of a 1.130 m Long Cantilever Beam under Combined Self-Weight and End Point Loading

3.5. Finite Element Validation

3.5.1. Static Structural Validation

The developed finite element model was initially validated through static structural analysis by considering the combined effects of beam self-weight and an externally applied end load. Figure 10 presents the predicted static deflection contour of the cantilever beam having a length of 1.130 m.

The numerical and analytical predictions of the maximum deflection were compared with the corresponding experimentally measured value, and the results are summarized in Table 6.

Table 6. Comparison of experimental, numerical, and analytical maximum deflections of the 1.130 m long steel cantilever beam

Method	Maximum deflection δ (m)
Experimental	0.0048
Numerical (ANSYS)	0.004851
Analytical	0.004981
Error between Experimental and Numerical (%)	1.06
Error between Experimental and Analytical (%)	3.77

As shown in Table 6, the finite element analysis predicted a maximum deflection of 0.004851 m, whereas the analytical model predicted a maximum deflection of 0.004981 m, compared with the experimentally measured value of 0.004800 m. The corresponding percentage errors for the numerical and analytical predictions were 1.06% and 3.77%, respectively. The numerical prediction exhibited closer agreement with the experimental measurement than the analytical prediction. Nevertheless, the small deviations obtained from both approaches demonstrate that the finite element and analytical models accurately represent the static behaviour of the cantilever beam. Therefore, both models can be reliably employed for subsequent modal analysis and prediction of the beam's natural frequencies.

3.5.2. Modal Validation

Following the successful validation of the static response, modal analysis was performed using the same finite element model to determine the first and second bending natural frequencies of the cantilever beam. The computed first bending mode shape is presented in Figure 11.

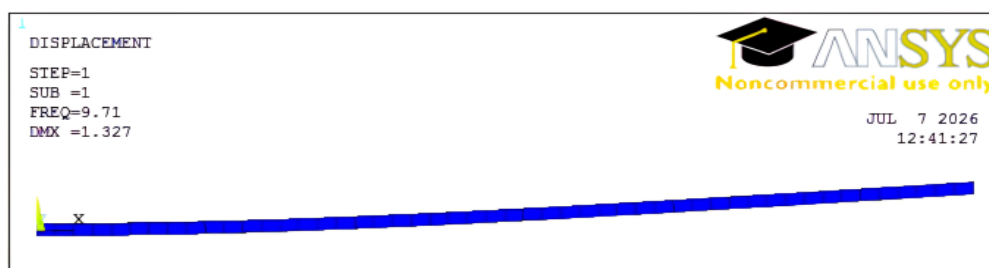


Figure 11. First Bending Mode Shape of a Cantilever Beam with Length 1.130 m

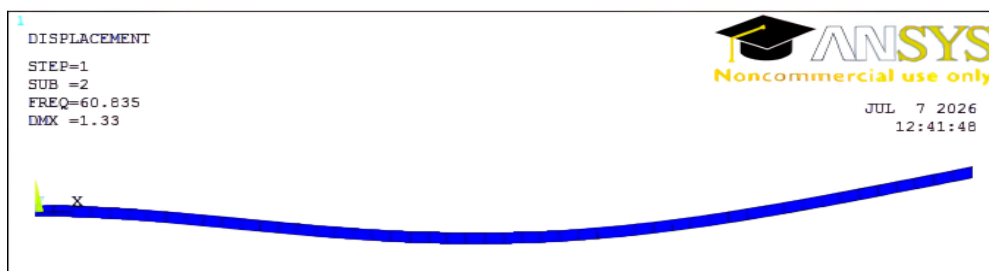


Figure 12. Second Bending Mode Shape of a Cantilever Beam with Length 1.130 m

The corresponding second bending mode shape obtained from the finite element analysis is shown in Figure 12. The first and second natural frequencies of the 1.130 m long steel cantilever beam predicted using the finite element model and the analytical model were compared with the corresponding experimental values obtained from the FFT analysis.

Table 7. Comparison of experimental, numerical, and analytical natural frequencies of the 1.130 m long steel cantilever beam

Method	First natural frequency f_{n_1} (Hz)	Second natural frequency f_{n_2} (Hz)
Experimental	10	65
Numerical (ANSYS)	9.71	60.835
Analytical (Optimization)	9.58	60.06
Error between Experimental and Numerical (%)	2.986	6.846
Error between Experimental and Analytical (%)	4.384	8.225

As shown in Table 7, the numerical model predicted the first and second natural frequencies with percentage errors of 2.99% and 6.85%, respectively, while the analytical model yielded corresponding errors of 4.38% and 8.23%. In both vibration modes, the numerical predictions exhibited closer agreement with the experimental measurements than the analytical predictions. The relatively larger deviations observed for the second natural frequency may be attributed to modelling assumptions, idealized boundary conditions, and unavoidable experimental uncertainties. Nevertheless, the errors obtained from both the numerical and analytical approaches remain within acceptable limits, demonstrating that both models accurately capture the dynamic behaviour of the 1.130 m long steel cantilever beam. The close agreement among the experimental, numerical, and analytical results further validates the proposed vibration-based methodology for predicting the natural frequencies of steel cantilever beams.

3.6. Validation of the Estimated Young's Modulus

The primary objective of the present investigation was to develop and validate a vibration-based inverse optimization methodology for estimating the Young's modulus of structural steel using experimentally measured natural frequencies. To assess the accuracy of the proposed approach, the optimized Young's modulus obtained through the inverse optimization procedure was compared with the experimentally determined value

obtained from the static deflection test. The comparison is presented in Table 8.

Table 8. Comparison of Young's modulus (E) obtained using optimization-based prediction and experimental measurements

Method	Young's Modulus E (GPa)
Optimization	177
Experimental	181.7
Percentage Error	2.59

The optimization procedure estimated the Young's modulus of the structural steel as 177 GPa, whereas the experimentally determined value was 181.7 GPa, resulting in a percentage error of only 2.59%. This close agreement demonstrates that the proposed optimization framework can accurately identify the elastic modulus of the material by minimizing the discrepancy between the experimentally measured and analytically predicted natural frequencies. The low percentage error confirms that the developed objective function effectively converges towards the optimum material property while preserving the dynamic behaviour observed during the experimental vibration tests.

3.7. Overall Discussion

The experimental, analytical, finite element, and optimization results consistently demonstrate the effectiveness of the proposed vibration-based inverse methodology for estimating the Young's modulus of structural steel. The close agreement among the different approaches confirms that the developed analytical formulation, finite element model, and optimization framework accurately capture the static and dynamic behaviour of the cantilever beam. The experimental modal analysis provided reliable natural frequency measurements, while the optimization procedure successfully identified the material's elastic modulus by minimizing the discrepancy between the measured and predicted responses. The finite element model effectively reproduced both the static deflection and modal characteristics of the beam, further validating the adopted modelling approach. Minor deviations between the experimental and predicted results can be attributed to unavoidable measurement uncertainties and idealized modelling assumptions; however, these differences remain within acceptable engineering limits. Overall, the integration of experimental vibration testing, analytical modelling, finite element analysis, and inverse optimization provides a reliable and non-destructive methodology for material property identification. The proposed framework offers a practical alternative to conventional mechanical testing and has significant potential for applications in structural health monitoring, material characterization, and condition assessment of beam-like engineering structures.

4. CONCLUSIONS

A vibration-based inverse optimization methodology was developed for estimating the Young's modulus of structural steel by integrating experimental modal analysis, analytical modelling, finite element analysis, and numerical optimization. The proposed approach utilizes experimentally measured natural frequencies to identify the elastic modulus through minimization of the error between the measured and predicted dynamic responses. The analytical and finite element models accurately predicted the first and second bending natural frequencies and showed good agreement with the experimental measurements. The optimization procedure successfully minimized the prediction error, and the estimated Young's modulus closely matched the experimentally determined value, with a percentage error of only 2.59%. These results confirm the accuracy and reliability of the proposed methodology for non-destructive estimation of material properties. Overall, the developed framework provides a simple, reliable, and computationally efficient alternative to conventional destructive testing for determining the elastic modulus of beam-like structures. The methodology has potential applications in structural health monitoring, material characterization, and condition assessment, and can be extended to other structural materials and beam configurations with appropriate modifications.

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