Features of the Dual-Frequency Acoustic Signal Velocity in the Shallow Sea

Olekcii KORZHYK
National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute", Kyiv, Ukraine,email: kafedra@aae.kpi.ua, avkorzhyk@gmail.com

Vitalii DIDKOVSKYI
National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute", Kyiv, Ukraine,email: kafedra@aae.kpi.ua

Sergii KURDIUK
Institute VMS National University "OMA", serega15507@ukr.net

Olha PAVLENKO
National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute", Kyiv, Ukraine,email: pavlenko.olga.v@gmail.com

Abstract: - The paper considers the case of wave pocket propagation in a dual-frequency signal form in the shallow sea. The shallow sea is represented by a flat regular waveguide with combined boundaries. As the result of solving the case to determine the energy characteristics of the acoustic field, the basic analytical expressions for estimating the wave packet signal propagation velocity of the indicated type are obtained on the basis of the energy interpretation of the group velocity. The wave packet propagation velocity is described using frequency and coordinate dependencies of the dual-frequency signal velocity, as well as the concurrent observed time-average power stream density and acoustic energy density. The aim of the paper is to determine the peculiarities of the wave packet propagation velocity and main energy characteristics of the acoustic field in the plane regular waveguide with combined boundaries representing the shallow sea. The field is formed by a fixed source.

Keywords: - Acoustic fields; wave pocket; propagation velocity waveguide, combined boundaries energy characteristics

1. INTRODUCTION

By now, in cases of sound signals propagation under the conditions of waveguide representation of the shallow sea, there has been a situation that requires clarification of the physical, model and terminological interpretation of such an important characteristic as the propagation velocity of the broadband acoustic signals. The representation of broadband signals in the form of a wave packet has made it necessary to change the approaches to estimating the transmission rate of such an information message under the frequency dispersion conditions. In this connection, the aim of the paper is to determine the features of the wave packet velocity propagation and the main energy characteristics of the acoustic field in a flat regular waveguide with combined boundaries representing the shallow sea. The field is formed by a fixed source.

In the article under dispersion conditions for the low-frequency region and the lowest modes of the waveguide, the features of signal propagation velocity and the energy characteristics of the acoustic field formed in the waveguide are calculated and analyzed. It is shown that the propagation velocity of a signal is characterized by variability in its propagation path, depending on the values of the frequencies used, the bandwidth of the source packet, the distance to the observation point and mode number.

2. CASE STUDY OF THE "GROUP VELOCITY PROBLEM"

In accordance with the definition in [1], "a wave packet can be considered a signal of finite duration, whose frequency spectrum, in this connection, is not monochromatic." Each frequency component of a packet of a certain specific profile, taken after the Fourier expansion with a time factor that corresponds to the root of the dispersion equation, due to the difference in the phase velocities of the packet components v_{kph}, arrives at the observation point at different moments of time. Thus, the interference of different in frequency "out-of-phase fluctuations" leads to the initial wave packet profile shape change at the observation point. At the same time [1], in case of "significant fronts steepness" of the packet, the propagation velocity was approximately described by the group velocity v_{gr}. However, the group velocity (in accordance with the kinematic interpretation) characterizes the velocity of the profile's envelope of the transmitted wave packet and the wider the spectrum of the packet, the more distortions the signal undergoes, and the less confidence it is that exactly this signal has reached the observation point and, accordingly, the worse v_{gr} describes the message propagation velocity. In other words, the kinematic interpretation, which corresponds to a situation of a harmonic nature, becomes unreliable. It should be noted that in [1] the term "signal velocity" was proposed, which better describes the signal arrival rate of to the observation point if there is narrower-band the signal and the change in the amplitude of its envelope is sharper.

In turn, L. Brillouin [2] introduces the following concepts for the signal velocity:

− **.group" Relley velocity"** [3], defined as oscillations of close frequencies, "moving one by one in a regular sequence" or "the velocity of a group of waves," which is "less than the velocity of individual waves forming a group" represented by "... the superposition of the infinite sequences of waves of a slightly different wavelength ... ". The main difficulty in this case is the consideration of the form of the group itself. Relley's assumptions, in this connection, were not seen as a true value due to the experimental difficulties;

− ***"signal" velocity according to Sommerfeld*** [4] (or simply the **"*signal velocity*"** according to [1]) given in the form of the assumption that v_{gr} can approximately correspond to the velocity of the finite signals. The approximate determination of the signal velocity v_{cv} is associated with the ratio $v_{cv} = x / t, \quad t -$ is the packet propagation time from the source to the receiving point upon arrival. Here, the arrival criterion (by Voight) is the fixation of the "maximum intensity of the main signal", in contrast to determining the arrival of "extremely small" signal components (or the presence of "precursors"), which is associated with v_{gr} i.e. a distinction was made between these velocities.

However, in the future, it was recognized that the definition of "... velocity through the medium of the final signal" is very difficult, which is due to the change in waveform and the difficulty of separating v_{gr} and v_{cv}.

Both the first and second concepts are oriented to the case of small difference in frequencies constituting the group. Nevertheless, for any ratio of the wavelengths of the group and the velocities of the component waves, for two sources with wavelengths λ_1, λ_2 and phase velocities v_{ph1}, v_{ph2}, the following relation [5] remains valid:

$$v_{гр} = v_{ф2} - \lambda_2 \frac{v_{ф1} - v_{ф2}}{\lambda_1 - \lambda_2},$$

being the basic from the point of view of the general interpretation of group velocity.

− ***velocity "of the transformation (transfer) of energy."*** This concept is well-derived from the introduced by L.Weinstein [6] term the "center of energy":

$$x_e = \frac{\int xEdx}{\int Edx}, \quad E - \text{ energy density},$$

for which the velocity of the energy center is defined as: $v_e = \frac{dx_e}{dt}$. At the same time, the regions of low velocity frequencies, v_{gr} and v_{cv} coincide (if, of course, the dispersion is low).

In the general case, according to [1], accurate calculation of the signal at the receiving point is impossible, therefore, it was suggested to introduce and use the term "effective group velocity" $v_{ef\ gr}$ also oriented on preserving the shape of the packet or its small changes. Reference should also be made to [7], in which it was noted that "the packet size and its duration are connected with v_{gr} " and indicated that the dispersion may lead to changes in the direction of motion of the wave packet in a homogeneous medium. However, a change in the direction of motion of the package components does not always lead to a change in the motion direction of its energy center, which, apparently, is due to the impossibility to accurately determine its location in view of the packet deformation during propagation.

Such a variety of interpretations of the notion of the ***propagation velocity of complex signals*** under conditions of frequency dispersion is somewhat discouraging and makes it necessary to single out the following circumstances that can directly affect the quality of the transmission of sound messages over the waveguide:

- changing the shape of the envelope and the nature of the frequency filling of the packet as it propagates;
- elimination of the role of "precursors";
- selection of the center of the signal energy as an integral value at the signal observation intervals;
- establishing the features of the energy characteristics of the acoustic field associated with the wave packet propagation velocity;

In this case, the properties of the ***"signal propagation velocity" (SPV)*** should be detailed in order to compare with the traditional "group velocity", proceeding from the fact that it is the group velocity that characterizes the energy transferring velocity of the acoustic wave in the presence of dispersion. In this regard, using the positions of energy interpretation v_{gr} (as the most general and successful for studying), we will determine the features of the velocity of propagation of complex sound signals (***SPV***) and the accompanying energy characteristics of the acoustic field formed by them in a regular waveguide with combined boundaries.

3. STATEMENT AND SOLUTION OF THE PROBLEM

Let us consider a plane regular waveguide representing a shallow sea model with an acoustically rigid bottom surface (Fig. 1). The waveguide is defined in rectangular coordinates xOz (O-coordinate center) and contains a dual-frequency source forming two combination sound waves. The operating frequencies of the sound source - ω_1, and ω_2. $\omega_2 > \omega_1$. We use the general provisions of the energy interpretation of the group velocity [5],[8].

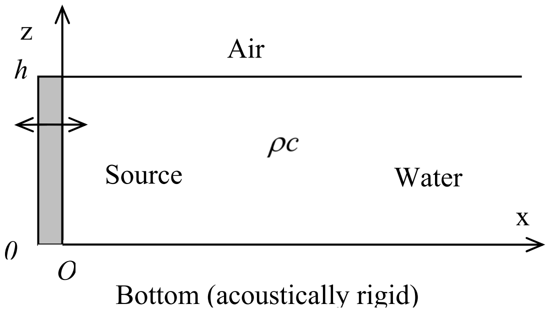


Figure 1. Waveguide representation of the shallow sea

In a plane regular waveguide, the velocity potential of the acoustic field produced by some monochromatic source in accordance with [8] can be represented in the form:

$$\varphi_n(x,z,t) = B_n cos\left[\frac{(2n+1)\pi}{2h}z\right]e^{-i(\omega t - k_n x)}, \quad (1)$$

where B_n is the coefficient determined by the source; h - the depth of the sea (distance from bottom $z=0$ to surface $z=h$); k_n - the wave number of the n-th mode (= 0, 1, ..); k - is the wave number for the free field:

$$k_n = k\sqrt{1-\left(\frac{\omega_{ncr}}{\omega}\right)^2};\ \omega_{ncr} = \frac{\pi(2n+1)c}{2h};\ , \quad (2)$$

$$k = \frac{\omega}{c} = \frac{2\pi f}{c}.$$

ω_{ncr} - the critical frequency, ω - the circular frequency, c - the sound velocity in the free medium.

Using the transfer equations (for example, [8]) to the pressure p and the components of the oscillating velocity v_x, v_z

$$p(x,z,t) = \rho\frac{\partial(\varphi(x,z,t))}{\partial t},\ v_x(x,z,t) = -\frac{\partial(\varphi(x,z,t))}{\partial x},$$

$$v_z(x,z,t) = -\frac{\partial(\varphi(x,z,t))}{\partial z},$$

as well as Euler's formulas, allows us to write (1) in the form:

$$p_n(x,z,t) = -\omega\rho_0 B_n cos\left[\frac{(2n+1)\pi}{2h}z\right]sin(\omega t - k_n x),$$

$$v_{nx}(x,z,t) = -k_n B_n cos\left[\frac{(2n+1)\pi}{2h}z\right]sin(\omega t - k_n x), \quad (3)$$

$$v_{nz}(x,z,t) = B_n\left(\frac{(2n+1)\pi}{2h}\right)sin\left[\frac{(2n+1)\pi}{2h}z\right]cos(\omega t - k_n x)$$

It is clear that the expressions (1)-(3) should be written in groups for frequencies ω_1, and ω_2. We also believe that the waves created by the source propagate in one direction. For this case, it is necessary to determine v (***SPV***) for that n – mode and its dependence on frequency f and coordinate, using such characteristics of the acoustic field as the acoustic energy density $E_{n\Sigma}(x,z,t)$, the resulting average value of the power stream density (resulting intensity) $\overline{W}_{n\Sigma}(x,z,t) = J_{n\Sigma}(x,z,t)$, and the average power stream through the cross section of the waveguide $\overline{P}_{n\Sigma}(x,z,t)$.

3.1. Acoustic energy density $E_{n\Sigma}(x,z,t)$

Taking into account the fact that the waves of different frequencies formed by the source propagate co-directively, the expression for the acoustic energy density [7], [8]

$E_n(x,z,t) = \rho_0\frac{v_{nx}^2}{2} + \rho_0\frac{v_{nz}^2}{2} + \frac{p_n^2}{2\chi}$, where ρ_0 – medium density, $\chi = \rho_0 c^2$ - medium elasticity,

we write in the form:

$$E_n(x,z,t) = \rho_0\frac{\left(v_{nx}^I + v_{nx}^{II}\right)^2}{2} + $$
$$+\rho_0\frac{\left(v_{nz}^I + v_{nz}^{II}\right)^2}{2} + \frac{\left(p_n^I + p_n^{II}\right)^2}{2\chi}, \quad (4)$$

where $p_n^I, v_{nx}^I, v_{nz}^I$ - pressure and components of the oscillating velocity with frequency ω_1;

$p_n^{II}, v_{nx}^{II}, v_{nz}^{II}$ - pressure and components of the oscillating velocity with frequency ω_2.

Altering relations (3), writing them for frequencies ω_1, ω_2, substitute the expressions for the components $p_n^I, v_{nx}^I, v_{nz}^I$ и $p_n^{II}, v_{nx}^{II}, v_{nz}^{II}$ in (4), from which after a series of transformations taking into account equality $k^2 - k_n^2 = \left(\frac{\pi(2n+1)}{2h}\right)^2$ and carrying out integration over the vertical section of the waveguide, we determine the energy of the $n-$mode per unit length of the waveguide channel:

$$
\begin{aligned}
&E_n(x,t) = \int_0^h E_n(x,z,t)\,dz = \\
&= A_n^2 \left(k_n^I\right)^2 sin^2\left(\varphi_n^I(x,t)\right) + \\
&+ A_n^2 \left(k_n^{II}\right)^2 sin^2\left(\varphi_n^{II}(x,t)\right) + \\
&+ 2A_n^2 \left(k_n^I\right)^2 \left(k_n^{II}\right)^2 sin\left(\varphi_n^I(x,t)\right) sin\left(\varphi_n^{II}(x,t)\right) + \\
&+ A_n^2 (a_n)^2 cos^2\left(\varphi_n^I(x,t)\right) + \\
&+ A_n^2 (a_n)^2 cos^2\left(\varphi_n^{II}(x,t)\right) + \\
&+ 2A_n^2 \left(a_n\right)^2 \left(a_n\right)^2 cos\left(\varphi_n^I(x,t)\right) cos\left(\varphi_n^{II}(x,t)\right) + \\
&+ A_n^2 \left(k^I\right)^2 sin^2\left(\varphi_n^I(x,t)\right) + \\
&+ A_n^2 \left(k^{II}\right)^2 sin^2\left(\varphi_n^{II}(x,t)\right) + \\
&+ A_n^2 \left(k^I\right)\left(k^{II}\right) sin\left(\varphi_n^I(x,t)\right) sin\left(\varphi_n^{II}(x,t)\right),
\end{aligned} \tag{5}
$$

where $A_n^2 = \rho \frac{B_n^2}{2}\frac{h}{2}$, k_n^I, k_n^{II} - wave numbers for $n-$mode and frequencies ω_1, ω_2, respectively (see equation (3)); $\varphi_n^I(x,t) = \omega_1 t - k_n^I x,\ \varphi_n^{II}(x,t) = \omega_2 t - k_n^{II} x$ - phase functions;

$$
(a_n)^2 = (a_n^I)^2 = (a_n^{II})^2 = \left(\frac{\pi(2n+1)}{2h}\right)^2 .
$$

We find the average energy density over the observation interval in the waveguide sector $\bar{E}_n(x,t)$, as:

$\bar{E}_n(x) = \frac{1}{T}\int_0^T E_n(x,t)\,dt,$ where $T = \frac{2\pi}{\omega_2 - \omega_1} = \frac{2\pi}{\Delta\omega}$ - observation interval, $\Delta\omega$ - difference combinational frequency.

The determination of the average energy density in the observation interval is, in essence, reduced to the time integration of the expression (5) containing the phase functions $\varphi_n^I(x,t),\ \varphi_n^{II}(x,t)$, which leads to the result:

$$
\begin{aligned}
&\bar{E}_n(x) = A_n^2 \left(k_1\right)^2 - A_n^2 \left(k_n^I\right)^2 C_1 cos\left(\varphi_1(x)\right) + \\
&+ A_n^2 \left(k_2\right)^2 - A_n^2 \left(k_n^{II}\right)^2 C_2 cos\left(\varphi_2(x)\right) - \\
&- A_n^2 \Delta_{nk}^{I,II} C_3 cos\left(\varphi_3(x)\right)
\end{aligned} \tag{6}
$$

where

$$
C_1 = \frac{sin(\omega_1 T)}{\omega_1 T},\ C_2 = \frac{sin(\omega_2 T)}{\omega_2 T},\ C_3 = \frac{sin\left(\frac{\Omega T}{2}\right)}{\frac{\Omega T}{2}},
$$

$$
\varphi_1(x) = \omega_1 T - 2k_n^I x,\quad \varphi_2(x) = \omega_2 T - 2k_n^{II} x,\quad \varphi_3(x) = \frac{\Omega T}{2} - K_n^{I,II} x,
$$

$$
\Omega = \omega_1 + \omega_2,\quad K_n^{I,II} = k_n^I + k_n^{I,II}\quad ,
$$

$$
\Delta_k^{I,II} = k_1 k_2 + \left(k_n^I\right)\left(k_n^{II}\right) - \sqrt{k_1^2 - \left(k_n^I\right)^2}\sqrt{k_2^2 - \left(k_n^{II}\right)^2} . \tag{7}
$$

Continuing to introduce substitutions $R_1 = \left(k_n^I\right)^2 C_1,\quad R_2 = \left(k_n^{II}\right)^2 C_2,\quad R_3 = \Delta_k^{I,II} C_3$, we get:

$$
\begin{aligned}
&\bar{E}_n(x) = A_n^2 \left(k_1\right)^2 + A_n^2 \left(k_2\right)^2 - \\
&- A_n^2 \left(R_1 cos\left(\varphi_1(x)\right) + R_2 cos\left(\varphi_2(x)\right)\right) - \\
&- 2A_n^2 R_3 cos\left(\varphi_3(x)\right)
\end{aligned} \tag{8}
$$

Let us find the result of the summation $R_1 cos\left(\varphi_1(x)\right) + R_2 cos\left(\varphi_2(x)\right)$ for the third term of expression (8) and get the final result for the energy density at the waveguide sector in the following form:

$$
\begin{aligned}
&\bar{E}_n(x) = A_n^2 \left(k_1\right)^2 + A_n^2 \left(k_2\right)^2 - \\
&- A_n^2 \sqrt{R_1^2 + R_2^2 + 2R_1 R_2 cos\left(\Delta\varphi(x)\right)}\, cos\left(\varphi_1(x) + \psi(x)\right) - , \\
&- 2A_n^2 R_3 cos\left(\varphi_3(x)\right)
\end{aligned}
$$

where $\Delta\varphi(x) = \varphi_2(x) - \varphi_1(x)$,

$$
\psi(x) = arctg\left(\frac{R_2 sin\left(\Delta\varphi(x)\right)}{R_1 + R_2 cos\left(\Delta\varphi(x)\right)}\right) \tag{9}
$$

The first two terms of (9) do not depend on x coordinate and represent the constant density values $\bar{E}_n(x) = \bar{E}_n$ produced by each wave separately. The third and fourth terms (9) characterize the alternating additive and depend on the coordinate x. At the same time, for waves with frequencies ω_1, and ω_2, mutual phase shift $\Delta\varphi(x)$ determines the variation range of the energy density when the condition is $cos\left(\varphi_1(x) + \psi(x)\right) = 1$. The fourth term containing the combinational frequency of the sum $\Omega = \omega_1 + \omega_2$, does not describe the envelope (profile) of the signal that interests us, and it could be ignored. However, it performs the role of a peculiar frequency key and becomes equal to zero at a multiplicity of combinational frequencies:

$$
m = \frac{\Omega}{\Delta\omega} = \frac{\omega_1 + \omega_2,}{\omega_2 - \omega_1,} .
$$

Thus, for instance, for the observation interval $T = 2\pi / \Delta\omega$:

- if $\Delta\varphi(x)=1$, then the oscillations at the observation point are added so that the resulting value $\bar{E}_n(x)$ – the greatest and is R_1+R_2. This is the lowest limit of the change $\bar{E}_n(x)$. Such a maximum contribution will provide a minimum (relatively average) value $\bar{E}_n(x)$ (9) with a spatial period T_x.

$$T_x=\frac{2\pi m}{2\left(k_n^{II}-k_n^{I}\right)},\quad meter,$$

where $m=0,2,4,...$ (10)

- if $\Delta\varphi(x)=-1$, then at the observation point the resulting value is $\bar{E}_n(x)$ – the lowest and is R_1-R_2. Such a minimum contribution will provide a maximum (relatively average) value $\bar{E}_n(x)$, which is the upper limit of the change $\bar{E}_n(x)$. Spatial period in this case is:

$$T_x=\frac{2\pi(2m+1)}{2\left(k_n^{II}-k_n^{I}\right)},\quad meter, \qquad (11)$$

where $m=0,1,2,3,...$

The presence in the third term of $cos(\varphi_1(x)+\psi(x))$, providing alternating variability, introduces additional variability of design situations.

3.2. The average stream of power density (intensity) $\bar{W}_{n\Sigma}(x,z,t)=J_{n\Sigma}(x,z,t)$.

Let us consider the average power density stream (i.e., the intensity $J_{n\Sigma}(x,z,t)$) along coordinate axes Ox ($\bar{W}_{n\Sigma x}(x,z,t)$) and Oz ($\bar{W}_{n\Sigma z}(x,z,t)$). Here (say, in contrast to the situation with using a monochromatic source in a waveguide with acoustically rigid boundaries), the intensity component along the axis Oz is not equal to zero.

Thus, along the axis Ox:

$$\begin{aligned}&W_{n\Sigma x}(x,z,t)=W_{nx}^{I}(x,z,t)+W_{nx}^{II}(x,z,t)+\\&+W_{nx}^{I,II}(x,z,t)=\\&=\rho c\left(v_{nx}^{I}\right)^2+\rho c\left(v_{nx}^{II}\right)^2+2\rho c\left(v_{nx}^{I}\right)\left(v_{nx}^{II}\right),\end{aligned} \qquad (12)$$

where the velocity components v_{nx}^{I}, v_{nx}^{II} must be written with the expression (3) for frequencies ω_1, and ω_2.

$$\begin{aligned}&W_{nx}^{I}(x,z,t)=\rho cB_n^2\left(k_n^{I}\right)^2cos^2\left(a_n^{I}z\right)sin^2\left(\varphi_n^{I}(x,t)\right),\\&W_{nx}^{II}(x,z,t)=\rho cB_n^2\left(k_n^{II}\right)^2cos^2\left(a_n^{II}z\right)sin^2\left(\varphi_n^{II}(x,t)\right),\\&W_{nx}^{I,II}(x,z,t)=2\rho cB_n^2\left(k_n^{I}\right)\left(k_n^{II}\right)cos\left(a_n^{I}z\right)\times\\&\times cos\left(a_n^{II}z\right)sin\left(\varphi_n^{I}(x,t)\right)sin\left(\varphi_n^{II}(x,t)\right),\end{aligned} \qquad (13)$$

and along the axis Oz:

$$\begin{aligned}&W_{n\Sigma z}(x,z,t)=W_{nz}^{I}(x,z,t)+W_{nz}^{II}(x,z,t)+W_{nz}^{I,II}(x,z,t)=\\&=\rho c\left(v_{nz}^{I}\right)^2+\rho c\left(v_{nz}^{II}\right)^2+2\rho c\left(v_{nz}^{I}\right)\left(v_{nz}^{II}\right),\end{aligned} \qquad (14)$$

where the velocity components v_{nz}^{I}, v_{nz}^{II} must be written with the expression (3) for frequencies ω_1 and ω_2

$$\begin{aligned}&W_{nz}^{I}(x,z,t)=\rho cB_n^2\left(a_n^{I}\right)^2sin^2\left(a_n^{I}z\right)cos^2\left(\varphi_n^{I}(x,t)\right),\\&W_{nz}^{II}(x,z,t)=\rho cB_n^2\left(a_n^{II}\right)^2sin^2\left(a_n^{II}z\right)cos^2\left(\varphi_n^{II}(x,t)\right),\\&W_{nz}^{I,II}(x,z,t)=2\rho cB_n^2a_n^{I}a_n^{II}\,sin\left(a_n^{I}z\right)sin\left(a_n^{II}z\right)\times\\&\times cos\left(\varphi_n^{I}(x,t)\right)cos\left(\varphi_n^{II}(x,t)\right).\end{aligned} \qquad (15)$$

The average power flux density over T intervals along the mentioned axis is determined by the expressions:

$$\bar{W}_{nx}(x,z)=J_{nx}(x,z)=\frac{1}{T}\int_0^T W_{nx}(x,z,t)dt,$$

$$\bar{W}_{nz}(x,z)=J_{nz}(x,z)=\frac{1}{T}\int_0^T W_{nz}(x,z,t)dt$$

and the total intensity will be written as:

$$J_{n\Sigma}(x,z)=\sqrt{\left(J_{nx}(x,z)\right)^2+\left(J_{nz}(x,z)\right)^2}\ . \qquad (16)$$

After a series of transformations, for the intensity component $J_{nx}(x,z)$ we get:

$$\begin{aligned}&J_{nx}(x,z)=\tilde{A}_n^2\cdot c\cdot cos^2\left(a_n^{I}z\right)\left(k_n^{I}\right)^2+\\&+\tilde{A}_n^2\cdot c\cdot cos^2\left(a_n^{II}z\right)\left(k_n^{II}\right)^2-\\&-\tilde{A}_n^2\cdot c\cdot\sqrt{D_1^2+D_2^2+2D_1D_2\,cos\left(\Delta\varphi(x)\right)}\times\\&\times cos\left(\varphi_1(x)+\vartheta(x)\right)cos\left(a_n^{I}z\right)cos\left(a_n^{II}z\right)+\\&+2\tilde{A}_n^2D_3\,cos\left(\varphi_3(x)\right),\end{aligned} \qquad (17)$$

where

$$\tilde{A}_n^2=\rho\frac{B_n^2}{2},\qquad D_1=\left(k_n^{I}\right)^2C_1,\quad D_2=\left(k_n^{II}\right)^2C_2,$$

$$D_3=\left(k_n^{I}\right)\left(k_n^{II}\right)C_3,\ \Delta\varphi(x)=\varphi_2(x)-\varphi_1(x),$$

$$\vartheta(x)=arctg\left(\frac{\tilde{R}_2\,sin\left(\Delta\varphi(x)\right)}{\tilde{R}_1+\tilde{R}_2\,cos\left(\Delta\varphi(x)\right)}\right),$$

and for the component $J_{nz}(x,z)$ respectively:

$$J_{nz}(x,z) = \tilde{A}_n^2 \cdot c \cdot sin^2\left(a_n^I z\right)\left(k_1^2 - k_n^I\right) +$$
$$+\tilde{A}_n^2 \cdot c \cdot sin^2\left(a_n^{II} z\right)\left(k_2^2 - k_n^{II}\right)^2 -$$
$$-\tilde{A}_n^2 \cdot c \cdot \sqrt{\tilde{R}_1^2 + \tilde{R}_2^2 + 2\tilde{R}_1\tilde{R}_2 cos\left(\Delta\varphi(x)\right)} \times \quad (18)$$
$$\times cos\left(\varphi_1(x) + \vartheta(x)\right) sin\left(a_n^I z\right) sin\left(a_n^{II} z\right) +$$
$$+2\tilde{A}_n^2 \cdot c \cdot \tilde{R}_3 \, sin\left(a_n^I z\right) sin\left(a_n^{II} z\right) cos\left(\varphi_3(x)\right),$$
$$\tilde{R}_1^2 = (a_n)^2 C_1, \; \tilde{R}_2^2 = (a_n)^2 C_2,$$

It should be noted that the use of the explanatory formulas of relation (5) substantially simplifies expressions (13), (15), (17), (18).

The value of the average power stream through the cross section x (which is necessary for finding the required velocity) determines the energy transfer in the direction of interest (along the axis Ox). Therefore, to determine the average power stream through the cross section of the waveguide, we use the final equations (16)- (18) and the dependence:

$$\overline{P}_n(x) = \int_0^h J_{n\Sigma}(x,z)\,dz. \quad (19)$$

As a result of integration (17), (18), we obtain the coefficient $h/2,$ the use of which in (17), (18) transforms the coefficient $\tilde{A}_n^2$ to A_n^2 as $A_n^2 = \tilde{A}_n^2 h/2$. The form of expressions (17), (18) does not change at the same time.

Keeping the general approach (for example [5], [6]. [8], [9]) to the issue of energy transfer of n - mode in an acoustic waveguide with combined boundaries, the ratio of the average over the observing interval of the power stream to the average energy density at the wavelength of the difference combination frequencies $\lambda_\kappa = c/\Delta\omega$ of the following equation:

$$v(x) = \frac{\overline{P}(x)}{\overline{E}(x, x+\lambda_\kappa)}\lambda_\kappa = \frac{\int_0^h J_{n\Sigma}(x,z)\,dz.}{\overline{E}(x, x+\lambda_\kappa)}\lambda_\kappa \quad (20)$$

can be considered as satisfying the interpretation of the energy transfer rate in the non-harmonic wave of a dual-frequency signal or ***SPV***. Calculation of the velocity v as a function of frequency f and coordinate x is performed by substituting in (16) the relations (17), (18), integrating by (19) and further applying (9) and (19) in formula (20).

4. ANALYSIS OF RESULTS

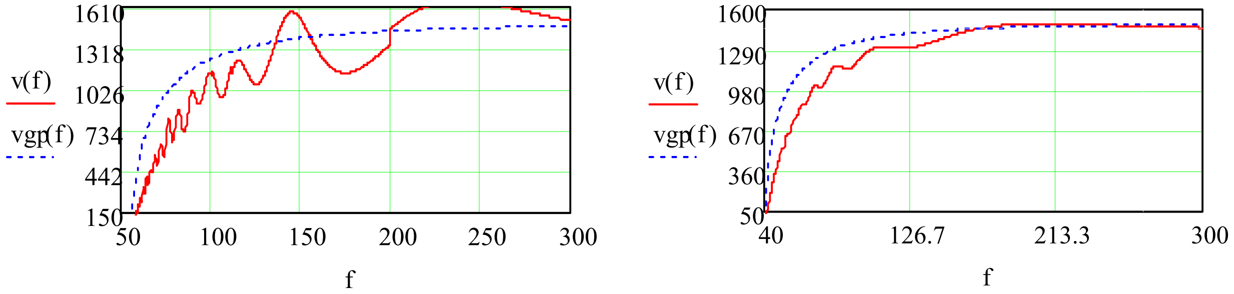
Calculations were made for a regular waveguide (with depth $h = 55\,m$) and some fixed source (located in the waveguide cross section $x = 0$) and forming a dual-frequency (f_1, f_2) signal. At the same time, the dependences of the ***SPV*** on the frequency f and the coordinate x were calculated, as well as the accompanying distributions of the acoustic energy density E_{nN} and the average power stream density W_{nN}. The change in the current frequency occurred within the range, bounded from below by the value of the critical frequency of the first mode $\omega_{1cr} = 3\pi c/2h, f_{lo} = f_{1cr} \approx 40Hz$ and the frequency $f_{up} = 300\,Hz$ from the top. The fixed source frequencies f_1, f_2 were selected from the region of the lower fundamental frequencies ($\approx (85-155)\,Hz$).

The frequency dependences of ***SPV***, $v(f)$ and group velocity $v_{gp}(f), v, \; m/s$; f(f)$, Hz$ for the mode $n = 1$ are shown in Fig. 2 - Fig.4.

As can be seen from the obtained results, the ***SPV*** is different from the ***group velocity*** (see Fig. 2 - Fig. 4). The main differences are in the presence of anomalies of the form of significant (up to 4 dB) oscillations in individual sections of the curve in the investigated frequency range. The smoothness and monotonicity of the frequency response, which accompany the dependence $v_{gp}(f)$ for close frequencies f_1, f_2, disappear. In this case, there are alternating aperiodic local maxima and minimums of the velocity, increasing with increasing frequency practically up to the upper limit of the chosen frequency range. In the frequency range $\geq 200\,Hz$, the ***SPV*** curve is smoothed, the oscillation occurrence interval is increased, their amplitude decreases and tends to a value $c = 1500\,m/s$.

As can be seen, the frequency and amplitude of the oscillations depends on the degree of difference in the source frequencies i.e. the greater the relative, the larger the amplitude and the interval of oscillations, and the sooner they appear. As the value decreases, the oscillation disappears and the ***SPV*** curve merges with the group velocity curve. At the same time, it can be noted that:

- the velocity of the wave packet along the waveguide (***SPV***) depends on the frequency band of the transmitted message and, accordingly, the mode number;
- an increase in the amplitude and the interval of manifestation of the oscillations of the ***SPV*** depends on the distance (cross section $x = 50\; m, x = 100\,m,$ $x = 180\,m$), which is explained by an increase in the distortion of the initial packet with an increase in the part of the loss of its original shape. The deformation of the packet occurs due to a change in the transfer time of the packet components propagating with different phase velocities from the source to the observation point [8],[9],[10].[11],[12],[13].

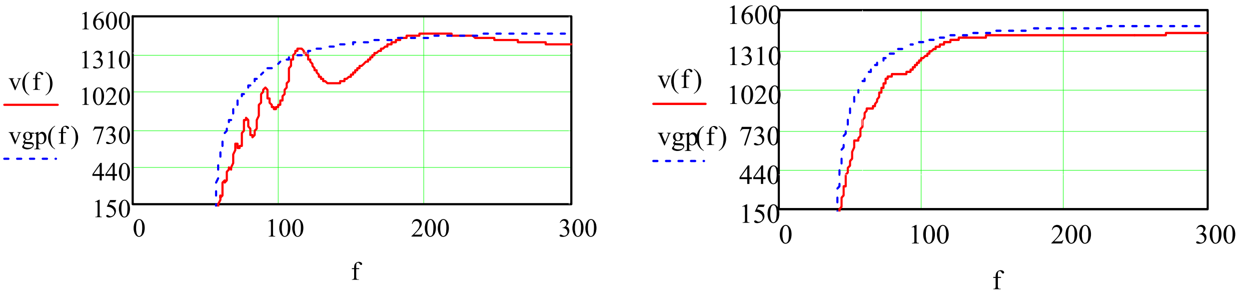


(a) (b)

Figure 2. Frequency characteristics of the ***SPV*** in the cross section of the waveguide $x = 180\ m$

(a) $f_1 = 67 Hz$, $f_2 = 91\ Hz$ and (b) $f_1 = 67 Hz$, $f_2 = 79\ Hz$

(dotted straight line is sight)

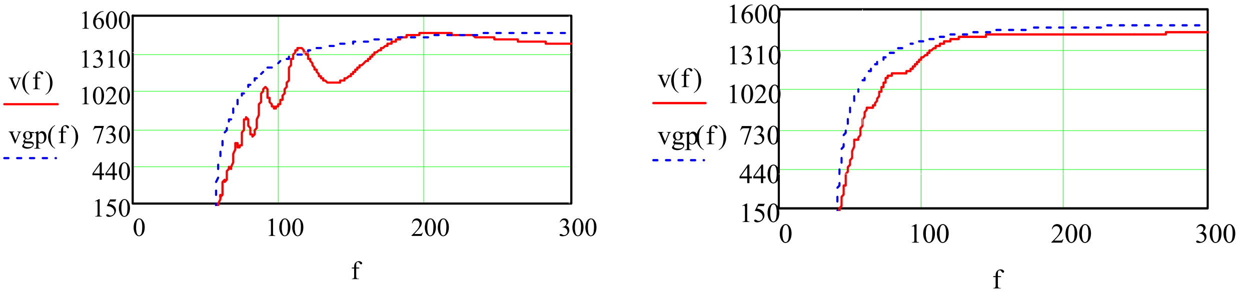


(a) (b)

Figure 3. Frequency characteristics of the ***SPV*** in the cross section of the waveguide $x = 100\ m$

(a) $f_1 = 67 Hz$, $f_2 = 91\ Hz$ and (b) $f_1 = 67 Hz$, $f_2 = 79\ Hz$

(dotted straight line is sight)



(a) (b)

Figure 4. Frequency characteristics of ***SPV*** in the cross section of a waveguide $x = 50\ m$

(a) $f_1 = 67 Hz$, $f_2 = 91\ Hz$ and (b) $f_1 = 67 Hz$, $f_2 = 79\ Hz$

(dotted straight line is sight)

The deformation of the packet is also accompanied by a change in the duration and frequency composition of the signal, the transformation and shift of the envelope maximum, leading to "dephasing" of the packet components [11], 12], [13] other, the change in the interference conditions of the signal components at the observation point and, hence, the conditions for identifying the ***"group velocity"*** with the ***"envelope transferring velocity"*** reduces the reliability of the application of such traditional concepts to a more significant extent than the deformation of the packet along the propagation path.

The wave packet of the presented type follows a waveguide with a variable velocity, varying in amplitude from section to section within local maxima and minima, the values and the intervals of their manifestation are determined by the frequency value from the range specified above.

It is obviously that the primary cause of such features is dispersion, which, in turn, is due to the presence of boundaries. The consequence of the dispersion is energy displacement of total complex oscillation with the group velocity. However, the energy transfer by the deformed packet, which was obtained by adding co-directional coherent oscillations with frequencies ω_1 and ω_2, and taking into account alternating additives for streams $W_{nx}^{I,II}$, $W_{nz}^{I,II}$ (13), (15), must occur at a different velocity ***SPV***. As calculations show, it is the SPV that is associated with both the deformation and the redirection of the power streams i.e. in the waveguide sections and from the cross section to the cross section. The effect of redirection of streams is shown in [14][15][16][17] and is stipulated in the provisions of [7].

Fig. 5 shows the normalized frequency dependences of the alternating stream ($W_{nz}^{I,II}$ ($W_{nzN}(f)$) and the acoustic energy density $E_n(f)$ in section $x = 180\ m$.

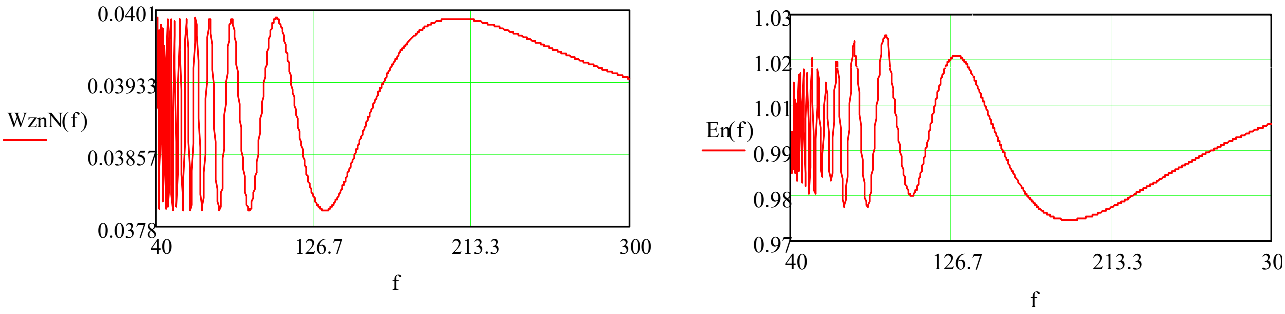


Figure 5. Normalized frequency dependencies of the stream $W_{nz}^{I,II}$ ($W_{nzN}(f)$) and acoustic energy density $E_n(f)$, $f_1 = 67\ Hz$, $f_2 = 79\ Hz$ $x = 180\ m$, (dotted line is the line of sight).

It can be seen from Fig. 5 that the frequency values determining the position of oppositely oriented aperiodic local maxima and minima of the curves $W_{nzN}(f)$ and $E_n(f)$ coincide with the values of the frequencies determining the position of the local maxima and minima of the SPV curves (Fig. 2, b.)).

As in the case of the frequency dependence of the velocity, the conditions for the interference of the streams from section to section vary, causing a change in the energy density along the waveguide. This situation can also be explained by using the field representation in the waveguide in the form of a set of Brillouin waves. It is known [2],[8],[9],[13] that any normal wave in a waveguide can be represented as a sum of a pair of plane waves whose exit angles from the source are equal in magnitude and depend on the frequency. Then, the above dual-frequency source is the source of two pairs of plane waves emitted at different angles. Undergoing multiple reflections from the channel boundaries, the waves interfere in the cross sections and on the axis of the waveguide, providing an increase or decrease in the sound intensity in the interference regions following one another with a certain spatial period. In this case, the type of the waveguide boundary used determines the changes in the phases of the oscillating velocities and pressures on them, which also translates phase exclusions into the foreseen streams of power density. Obviously, such considerations can be extended to the distribution of the sound energy density along the channel due to the interaction of power streams in the conditions specified above.

It should be noted that the questions of the intensity vector field formation in a waveguide with respect to space-time and frequency properties are only partially covered in papers [13] [14] [15] [16] [17] and require further study.

Let us illustrate this situation by spatial dependencies of the average power stream density $W_{n\Sigma}(x)$ and acoustic energy density $E_{n\Sigma}(x)$ along the waveguide channel. The results of calculations of the normalized $E_{nN}(x)$ and $W_{n\Sigma}(x)$, $W_{xnN}(x)$, $W_{znN}(x)$ are shown in Fig. 6.

The change of E_{nN} and $W_{n\Sigma}$ along x represents the distribution along the waveguide of amplitude-modulated oscillations with a spatial period determined by the difference frequency and the ratio of the wave numbers k_n^I, k_n^{II}. In our design situation, such a spatial period was $T_x \approx 103\ м$ (see Fig. 6 and ratios (10), (11)). The mutual displacement of the maxima and minima of the streams $W_{n\Sigma}(x)$, and $W_{xnN}(x)$, $W_{znN}(x)$ and $E_{n\Sigma}(x)$ by half of the spatial period, is due to the fact that the reflection from the boundaries of the waveguide occurs in different ways (for example, for the oscillation velocity, on the surface, without changing the phase and with a shift by π in reflection from the bottom).

As can be seen, in the case under consideration the acoustic energy density depends on the coordinate. In this case, each vertical section of the waveguide has its own value and direction of the intensity vector.

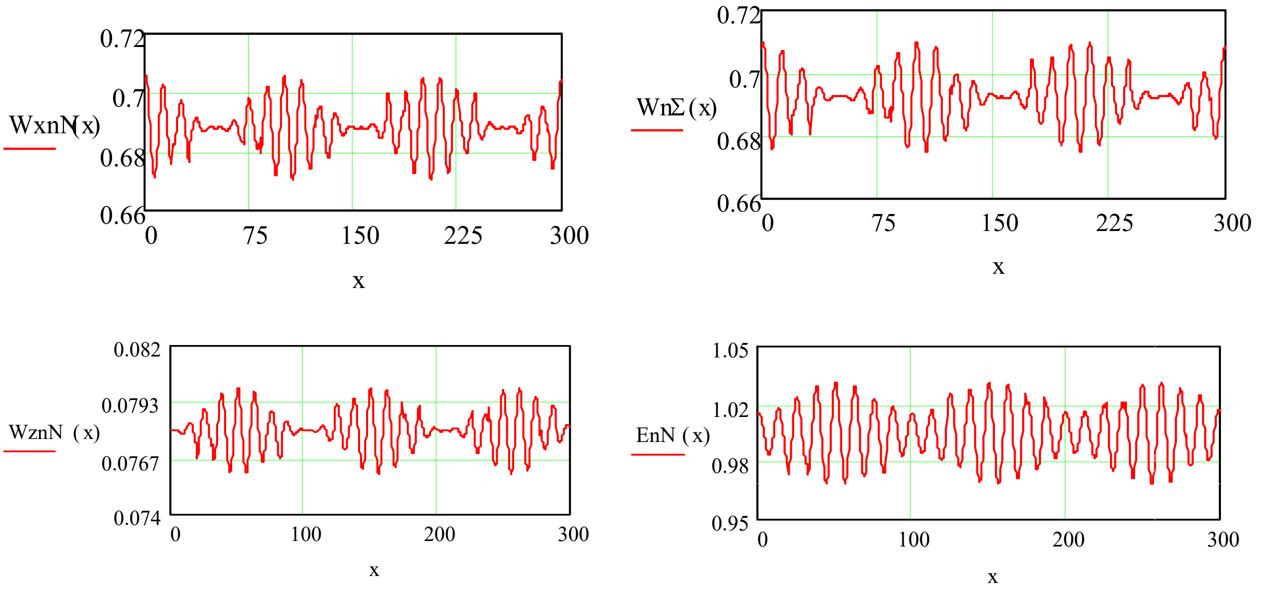


Figure 6. Normalized coordinate dependences of the acoustic energy density $E_{n\Sigma}(x)$, average power stream density $W_{n\Sigma}(x)$ and its components $W_{xnN}(x)$, $W_{znN}(x)$, (x - varies along the waveguide), $f_1 = 67\ Hz$, $f_2 = 79\ Hz$

Concerning the last position, it should be noted that in the considered design case the contributions of $W_{xnN}(x)$, $W_{znN}(x)$ to the total stream $W_{n\Sigma}(x)$ are not the same, and $W_{xnN}(x) > W_{znN}(x)$. In addition, standing waves in the vertical sections of the waveguide (due to the use of a complex nonharmonic signal) are not formed. The corresponding intensity component, varying from section to section, is sensitive to a change in the phase relationships of the interfering oscillations i.e. both due to a change in the width of the cross section (depth) and the type of the waveguide boundaries, and also by changing the values used in the frequency packet. Taking as a criterion of the singularity of the SPV, we can assume that the character of the energy distribution along the waveguide depends on the type of the boundaries, in fact, the SPV and the degree of packet deformation, which is being the larger, demonstrates the wider the frequency band of the original wave packet [9],[11] [13] (in our case, the greater the difference between the frequencies ω_1 and ω_2).

A comparison of the obtained results with the data of sources [9], [13] on modeling the situation of “wave packet” propagation in a waveguide shows:

- the form of the frequency and coordinate dependences of the group velocity, energy, and power stream density, which was obtained by us, confirms the presence of oscillating (steps) elements of the velocity graphs due to the redirection of the power streams in the channel;
- the effect of redirection of power flows in turn depends on the longitudinal coordinate

5. CONCLUSIONS

Let us generalize the results obtained.

In the operation of a dual-frequency source in a shallow sea represented by a planar waveguide with combined boundaries, the propagation velocity of the wave packet and the distribution of the energy characteristics of the acoustic field formed are characterized by the following features:

- in the frequency range 2-3 octaves relative to the boundary frequency of the lower modes of the waveguide, the SPV is different from the group velocity. The main differences are in the presence of anomalies in the form of velocity oscillations in certain sections of the curve of the SPV frequency dependence in the investigated frequency range;
- the increase in the amplitude and the interval of oscillations occurrence of the SPV depends on the distance from the source to the selected vertical section of the waveguide;
- the wave packet of the specified type moves along a waveguide with a variable velocity, varying in amplitude from section to section within local maxima and minima, the values and the development intervals of which are determined by the frequency value from the range specified above;
- SPV is related both to deformation and redirection of power fstreams in the waveguide sections and from section to section;
- energy transfer by a deformed packet formed by adding co-directional oscillations with frequencies

ω_1 and ω_2, and taking into account alternating additives for streams, occurs at speed - v (SPV) ;

- the change $E_{n\Sigma} = E_{nN}$ and $W_{n\Sigma}$ depends on the SPV and represents the spatial distribution of amplitude-modulated oscillations with a spatial period determined by the difference frequency and the ratio of the wave numbers k_n^I, k_n^{II} .

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